PSOT Formulas

- Integrals

1 $x^n : x^{n+1}/n+1$ $1 : x$ $1/x : \log x$

$a^x : a^x/\log a$ $e^{ax} : e^{ax}/a$

2 $\int uv\, dx = u\int v\, dx - \int (du/dx \int v\, dx)\, dx$

$\int uv\, dx = u \int v\, dx - u' \int v_1\, dx + u'' \int v_2\, dx + \ldots$

→ vanishing derivative

3 $\int_a^b f(x)\, dx = -\int_b^a f(x)\, dx$ $\int_a^b f(x)\, dx = \int_a^c f(x)\, dx + \int_c^b f(x)\, dx$

$\int_a^b f(x)\, dx = \int_a^b (a+b-f(x))\, dx$ $\int_a^{2a} f(x)\, dx = \int_a^a f(x)\, dx + \int_a^a f(x)\, dx$

4 $\int_0^{\infty} e^{-x} x^n\, dx = \Gamma{n+1}$

- Gamma Function :-

$\Gamma n = 1$ $\Gamma 0 = \infty$ $\Gamma 1/2 = \sqrt{\pi}$ $\Gamma n = (n-1)\Gamma{n-1}$

$\Gamma p = \Gamma{1-p} = \pi/\sin p\pi$

- Correlation

Mean $= \bar{x} = \dfrac{\Sigma x}{N} = \mu$ $Cov(x, y) = \dfrac{\Sigma (x-\bar{x})(y-\bar{y})}{N}$

Karl Pearson's coeff of Correlation :-

$$r = \frac{\Sigma d_x d_y - N\bar{d_x}\bar{d_y}}{\sqrt{(\Sigma d_x^2 - N\bar{d_x}^2)(\Sigma d_y^2 - N\bar{d_y}^2)}}$$

$d_x = x - a$ $d_y = y - b$

a: nearest int to $\bar{x}$ b: nearest int to $\bar{y}$

→ for some questions, replace with $x, \bar{x}$ and $y, \bar{y}$

Spearman Rank Correlation :-

$$R = 1 - \left(\frac{6\Sigma d_i^2}{N^3 - N}\right) = 1 - 6\,\frac{[\Sigma d_i^2 + 1/12(m_1^3 - m_1) + 1/12(m_2^3 - m_2) + \ldots]}{N^3 - N}$$

for duplicates $d_i = R_1 - R_2$

$$r = \frac{Cov(x, y)}{\underset{SD}{\sigma_x}\ \underset{SD}{\sigma_y}}$$

m_1: no. of similar terms in x

m_2: no. of similar terms in y

Values :	1	1	2	3
Ranks :	1	1	3	4

Page No.
Date

- Regression

Normalisation :-

$y = a + bx$

$\sum y = aN + b\sum x$

$\sum xy = a\sum x + b\sum x^2$

Calculate a, b from the 2 eqn

The b here is b_{yx}

as this is y on x

$x = p + qy$ (or a, b)

$\sum x = aN + b\sum y$

$\sum xy = a\sum y + b\sum y^2$

same for this.

The b here is b_{xy}

as this is x on y

$r = \sqrt{b_{xy} b_{yx}}$

put $x = ?$ (given in question) in $y = a + bx$

put $y = ?$ in $x = a + by$

- Relation :-

$\tan\theta = \left(\frac{1-r^2}{r}\right)\left(\frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2}\right)$

θ is the angle

$\theta = \tan^{-1} ?$

$(y - \bar{y}) = \frac{r\sigma_y}{\sigma_x}(x - \bar{x})$ when x is given

$(x - \bar{x}) = \frac{r\sigma_x}{\sigma_y}(y - \bar{y})$ when y is given

- Question : $6Y = 5X + 90$ $15X = 8Y + 130$ $\sigma_x^2 = 16$

Find : i) $\bar{x}$ and $\bar{y}$ ii) r iii) σ_y^2

i) Solve for X and Y $10Y = 40$ (from eqn) $\bar{Y} = 40 \rightarrow X = 30$ $\bar{X} = 30$

ii) X on Y : $X = 6/5\, Y - 18$ $b_{xy} = 6/5$

Y on X : $Y = 15/8\, X - 130/8$ $b_{yx} = 15/8$

$r = \sqrt{b_{xy} b_{yx}} = \sqrt{2.25} = 1.5$

iii) $b_{yx} = r\sigma_y / \sigma_x \rightarrow \sigma_y = 5$

[last question from notes]

questions type : one event is asked to be calculated in presence of others. Ex. failed in maths, knowing that failed in Chem. $P(M/C)$

Page No.
Date

- De Morgan's law, Probability, Addition Rule

$P(\overline{A \cup B}) = P(\bar{A} \cap \bar{B})$ $\quad$ $P(\overline{A \cap B}) = P(\bar{A} \cup \bar{B})$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

Probability of an event $= n(A)/n(S) = P(A)$

Conditional Probability : $P(A/B) = P(A \cap B) / P(B)$

B : base event, alr occured.

$P(A \cap B) = P(A/B) \times P(B) = P(B/A) \times P(A)$

If A and B are independent $P(A \cap B) = \phi$ and

$P(B/A) = P(A)$ $\quad$ $P(B/A) = P(B)$ and

A and B' are also independent, even A' and B, A' and B'

draw graphs

- Baye's Theorem :

$$P(A_i/B) = \frac{P(A_i)\,P(B/A_i)}{\sum_{i=1}^{n} P(A_i)P(B/A_i)} \Big/ \sum P(A_i \cap B)$$

$i = 1, 2, 3 \ldots n$

questions type probabilities of events are given for each person and another base event alr happened probabilities for each person is given. They'll ask to find P(base/active) and $\cap$

- Expectation and Variance :

Mean : $E(x) = \sum P_i x_i$ $\quad$ $\sum P_i = 1$: discrete

Variance : $V(x) = E(x^2) + [E(x)]^2$

$E(x) = \int_{-\infty}^{\infty} x f(x)\,dx$: continuous

$V(ax) = a^2 V(x)$ $\quad$ $V(x+b) = V(x)$

$E(ax+b) = aE(x) + b$ $\quad$ $V(ax+b) = a^2 V(x)$

$V(ax_1 + bx_2) = a^2 V(x_1) + b^2 V(x_2)$

$\hookrightarrow -$ $\quad$ $\hookrightarrow +$

Median : M : $\int_0^M f(x)\,dx = \int_M^b f(x)\,dx = \frac{1}{2}$

Page No.
Date / /

- **Joint Probability**

JPD table $x \backslash y$ ... Total

Total 1

Marginal prob of x :-

x : 1 2 ...

P_i :

To calculate probability :- $P(E_1, E_2) = \frac{{}^{n_1}C_1 \times {}^{n_2}C_2}{{}^{n}C_3} = \frac{n(A)}{n(S)}$ → Example

${}^{n_1}C_1$ → the active thingie? → total of E_1

${}^{n}C_3$ → total events

Marginal Probability in Continuous :-

$$\int_{-\infty}^{\infty} f_{xy}(x,y)\,d_x\,d_y = 1 = \int_{x=a}^{b}\int_{y=c}^{d} f_{xy}(x,y)\,d_x\,d_y$$

$\nearrow$ 2nd, $\nearrow$ 1st

draw graphs

$f_x(x) = \int_{y=c}^{d} f_{xy}(x,y)\,dxy$ for MPD of x, we use y stuff

$f_y(y) = \int_{x=a}^{x=b} f_{xy}(x,y)\,d_x$ for MPD of y, we use x stuff

- **Binomial Distribution**

success : p failure : q : probabilities no. of trials : n

$p+q=1$ $q=1-p$ $P(X=x) = {}^nC_x\,p^x\,q^{n-x}$

$$\sum_{x=0}^{n} p(x) = \sum_{x=0}^{n} {}^nC_x\,p^x\,q^{n-x} = (p+q)^n = 1$$

Frequency : $N\,P(X=x)$ $SD = \sqrt{\sigma^2}$

Mean = np Variance : npq mode = $\lfloor (n+1)$

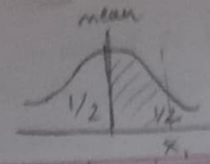
for fair coins : $p = 1/2 = q$: children - boy or girl

- **Poisson Distribution** [Used when p is too small, like 0.002 etc]

$$P(X=x) = \frac{e^{-m}\,m^x}{x!}$$ where m is mean

$$P(x+1) = \frac{m}{(x+1)}\,P(x) \qquad f(x+1) = \frac{m}{x+1}\,f(x)$$

$$V(\alpha x + \beta Y) = \alpha^2 V(x) + \beta^2 V(Y)$$



$P(m < x < x_1)$

$z_1 = \frac{x_1 - m}{\sigma}$

- **Normal Distribution**

$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-1/2\left(\frac{x-m}{\sigma}\right)^2}$ $X \sim N(m, \sigma)$ continuous r.v. X

$z = (x-m)/\sigma$: normal variate

Median = Mean = Mode = m

Quartile deviation : $[Q_3 - Q_1]/2 = 2/3\,\sigma$ where $Q_1 = m - 2/3\,\sigma$

$Q_3 = m + 2/3\,\sigma$

Mean deviation : $4/5\,\sigma$

$\sigma = \sqrt{npq}$ then $z = (x-np)/\sqrt{npq}$ $np > 15$ $nq > 15$

ND can also be obtained from Poisson when $m \to \infty$

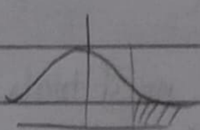
- **Exponential Distribution**

$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$: $\boxed{\text{pdf of } x}$ mention this!

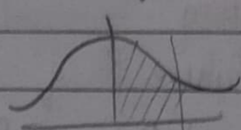
Mean $E(x^r) = \underbrace{\Gamma}_{\text{gamma fn.}}(r+1)/\lambda^r = \int_{-\infty}^{\infty} x f(x)\,dx$

Var $\sigma^2 = E(x^2) - [E(x)]^2 = 1/\lambda^2 = \int_{-\infty}^{\infty} x^2 f(x)\,dx$

use binomial distribution whenever required



convert these to



$P(\text{cond}^n) = \int_{\text{cond}}^{\text{cond}} \lambda e^{-\lambda x}\,dx$

- **Uniform Distribution** : Discrete [calculated like the simplest prob.]

$E(x) = \sum P_i x_i$ $P(X = x) = 1/n$ $U(1, n)$

$E(x) = (n+1)/2$ $Var(x) = (n^2-1)/12$

- **Uniform Distribution** : Continuous

$f(x) = \begin{cases} 1/(b-a) & a \leq x \leq b \\ 0 & \text{otherwise} \end{cases}$: density

$E(x) = (b+a)/2$ $Var(x) = (b-a)^2/12$

Keep the units (years, miles, etc.) in mind.