ITC Mod 3

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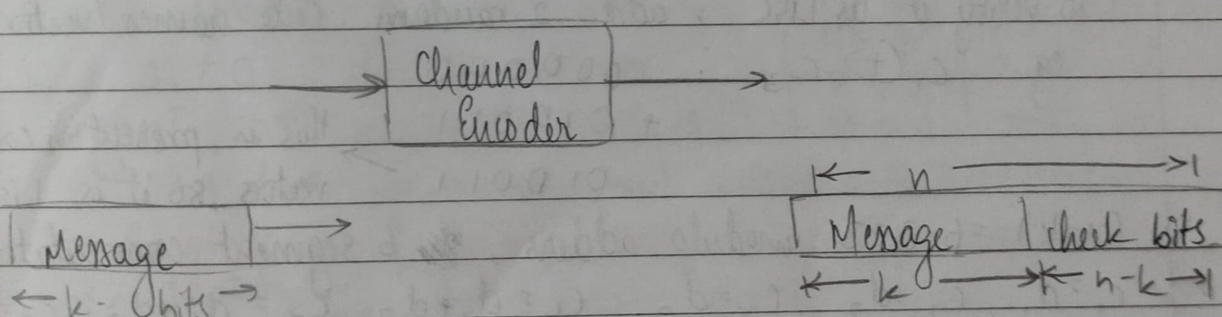
- **Types of Errors :-**

1. Random Error : Transmission errors occuring due to presence of Gaussian noise (sources : thermal & shot noise in transmitting & receiving equipment). Only 1 bit gets altered or corrupted
2. Burst Error : generated by impulsive noise in channel due to lightning, switching, transients and man-made noise etc. This affects more than one symbol.

- **Types of Codes :-**

1. Block Codes : Have 'n' no. of bits in one block or codeword. This codeword has 'k' message vectors / bits and (n-k) redundant bits
2. Convolution Codes : Check bits are continuously interleaved with info bits, which verify info bits not only in the block immediately preceeding but in other blocks as well.

- **Linear Block Code Formation**



In a channel encoder, a block of 'k' message bits is encoded into block of 'n' bits by adding (n-k) number of check-bits

$n > k$, (n, k) : block code

$C = \{c_1, c_2, c_3 \dots c_n\}$

$[D] = [d_1, d_2 \dots d_k]$

$[C] = [D][G]$

↳ check bits ↳ generator matrix

$[G] = [I_k \vdots P]$

↳ parity matrix

Questions

1. For a systematic (6,3) LBC, the parity matrix is given by

$$P = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

Find all possible code vectors.

→ $n=6$ $k=3$, 2^k distinct message vectors $\Rightarrow 2^3 = 8$

(000), (001), (010), (011), (100), (101), (110), (111)

$$[G] = [I_k \,\vdots\, P] = \begin{bmatrix} 1 & 0 & 0 & \vdots & 1 & 0 & 1 \\ 0 & 1 & 0 & \vdots & 0 & 1 & 1 \\ 0 & 0 & 1 & \vdots & 1 & 1 & 0 \end{bmatrix}$$

$$[C] = [D][G] = [d_1 \; d_2 \; d_3][G]$$
$$= [d_1, d_2, d_3, d_1+d_3, d_2+d_3, d_1+d_2]$$

Code-name	Message-vectors	Code-vector for (6,3) LBC
C_1	000	000000
C_2	001	001110
C_3	010	010011
⋮	⋮	⋮

To verify it as LBC, add 2 random code names/vector,

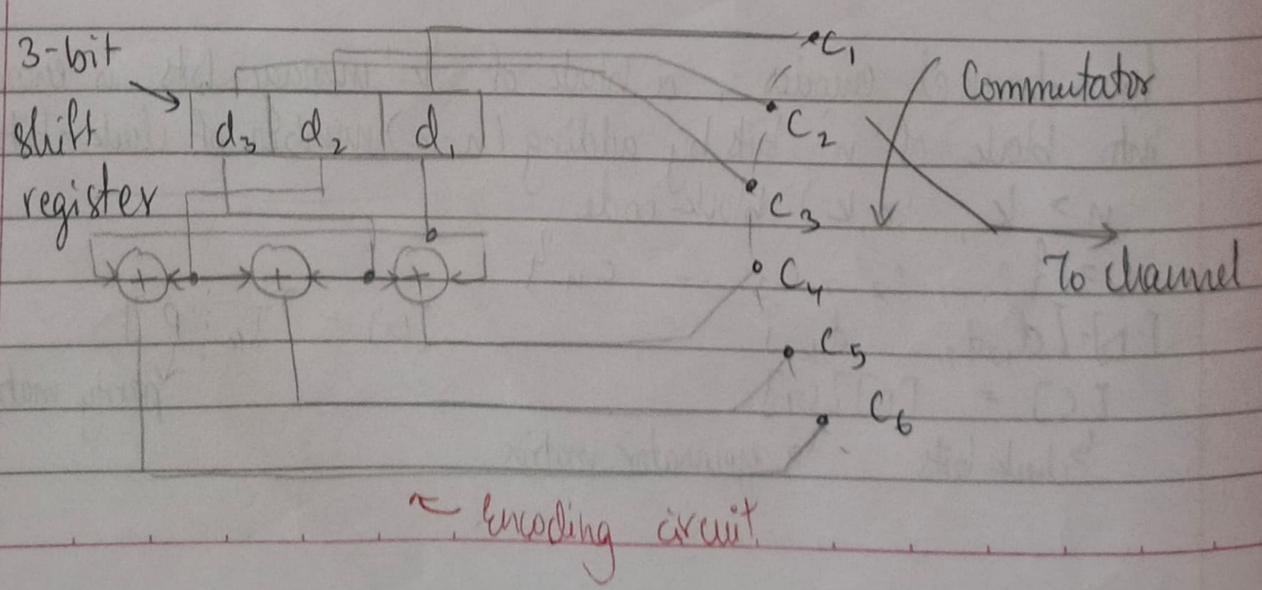
eg. $C_1 \oplus C_3$:

$$\begin{array}{r} 000000 \\ +\,010011 \\ \hline 010011 \end{array}$$

→ this is present in code-vectors, so it is LBC

$(n-k) = 6-3 = 3$ modulo adders, 6 segment commutator

$c_1 = d_1$, $c_2 = d_2$, $c_3 = d_3$, $c_4 = d_1+d_3$, $c_5 = d_2+d_3$, $c_6 = d_1+d_2$



← Encoding circuit

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Parity Check Matrix, Syndrome and Error Correction

Parity Check Matrix $[H] = [P^T \,\vdots\, I_{n-k}]$
Syndrome $[S] = [R][H]^T$
received vectors

- Questions

1. Consider (7,4) LBC with the parity matrix as $P = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$
i) find $[G]$, $[H]$ — sometimes given as C
ii) 3 received vectors : $R_1 = 0100000$ $R_2 = 1110111$
$R_3 = 1100100$, determine if there was an error during transmission, also detect and correct the errors.

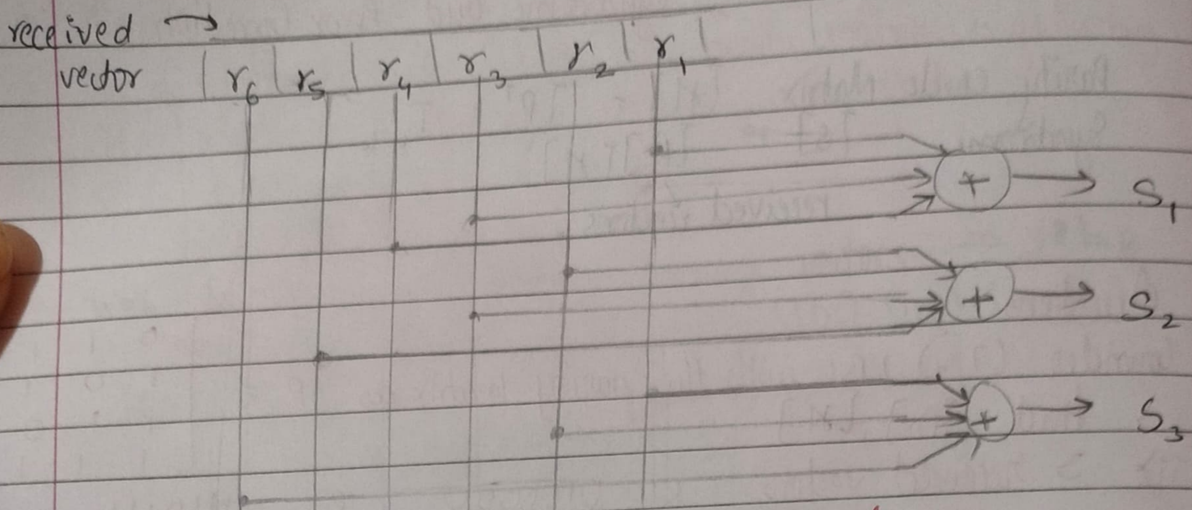
→ $H = [P^T \,\vdots\, I_{n-k}] = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}_{3\times 7}$

$[S] = RH^T$
$R_1 = 0100000$ $[S] = [0100000] \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{7\times 3}$
$S_1 = [101]$
$S_1 \neq 0$ there is an error
In H^T, 2^{nd} row matches with S_1
2^{nd} bit of R_1 is in error and corrected $C_1 = 0000000$

2. For the given parity matrix, construct syndrome circuit
→ $H = [P^T \,\vdots\, I_{n-k}] = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}_{3\times 6}$

$S = RH^T = [R_1 R_2 R_3 R_4 R_5 R_6] H^T$
$S_1 = r_1 + r_3 + r_4$ $S_2 = r_2 + r_3 + r_5$
$S_3 = r_1 + r_2 + r_6$

$n = 6$
$n-k$ = modulo - adder = 3 syndrome



- Hamming Weight, distance, min^m dist. of LBC

1. Hamming Weight : measure of total no. of non-zero elements (1's) in a given code vector. eg. $C_2 = 0110110$, $H_w = 4$

2. Hamming Distance : HD b/w 2 code-vectors C_1 and C_2 is the no. of components in which they differ.

   eg. $C_1 = 0\ 0\ 1\ 0\ 1\ 1\ 0$ $H_D = 4$
   $C_2 = 1\ 0\ 0\ 1\ 1\ 0\ 0$

   lowest $H_{w\,min} = \dfrac{d_{min} - 1}{2}$

3. Minimum Distance [d_{min}] : smallest H_D b/w any 2 code vectors in a code

   (or)

   d_{min} can also be measured by knowing all H_w of a given code and $d_{min} = H_{w\,min}$

   eg. if $C_3 = 0110100 \rightarrow H_w = 3$
   $C_4 = 1100110 \rightarrow H_w = 4$ $d_{min} = 2$
   $C_5 = 0001100 \rightarrow H_w = 2$

- Questions (Standard Array) :-

1. Construct standard Array for (6,3) LBC given $P = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

   Detect and correct errors for $R_1 = 100100$ and $R_2 = 000011$. Draw error correction for same

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$C = [D][G]$ $\qquad 2^3 = 8$

$[G] = [I_k : P] = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$ $\qquad H^T = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$[H] = [P^T : I_{n-k}]$

$[C] = [d_1\ d_2\ d_3]\ [G]$

$c_1 = d_1 \quad c_2 = d_2 \quad c_3 = d_3$

$c_4 = d_1 + d_3 \quad c_5 = d_2 + d_3 \quad c_6 = d_1 + d_2 + d_3$

D	[C]
000	000000
001	001111
⋮	⋮

Standard Array :-

Syndrome	Coset Leader	C_1	C_2	C_3	C_4	C_5	C_6	C_7
000	000000	001111	010011	011100 --				
101	100000	101111	110011	111100 ---				
011	010000	011111	$\boxed{000011}$	001100 --				
⋮	⋮	⋮	⋮	⋮				

H^T

$S = RH^T \quad = [100100][H^T]$ $\qquad$ match S with H^T, its in 6^{th} row

$= [001]$ $\qquad$ 6^{th} bit is in error

Corrected code vector $= [100101]$

Match R_1 with the elements of the table. The received vector R_1 is present in the 4^{th} column of 7^{th} row, hence the corrected code vector corresponds to the entry in top most row of the column 4 i.e. 100101

Error correction circuit :-

$s_1 = r_1 + r_3 + r_4$

$s_2 = r_2 + r_3 + r_5$

$s_3 = r_1 + r_2 + r_3 + r_6$

getting these from H^T

$\bar{S} = 0$

$e_1 = s_1 \bar{s}_2 s_3$

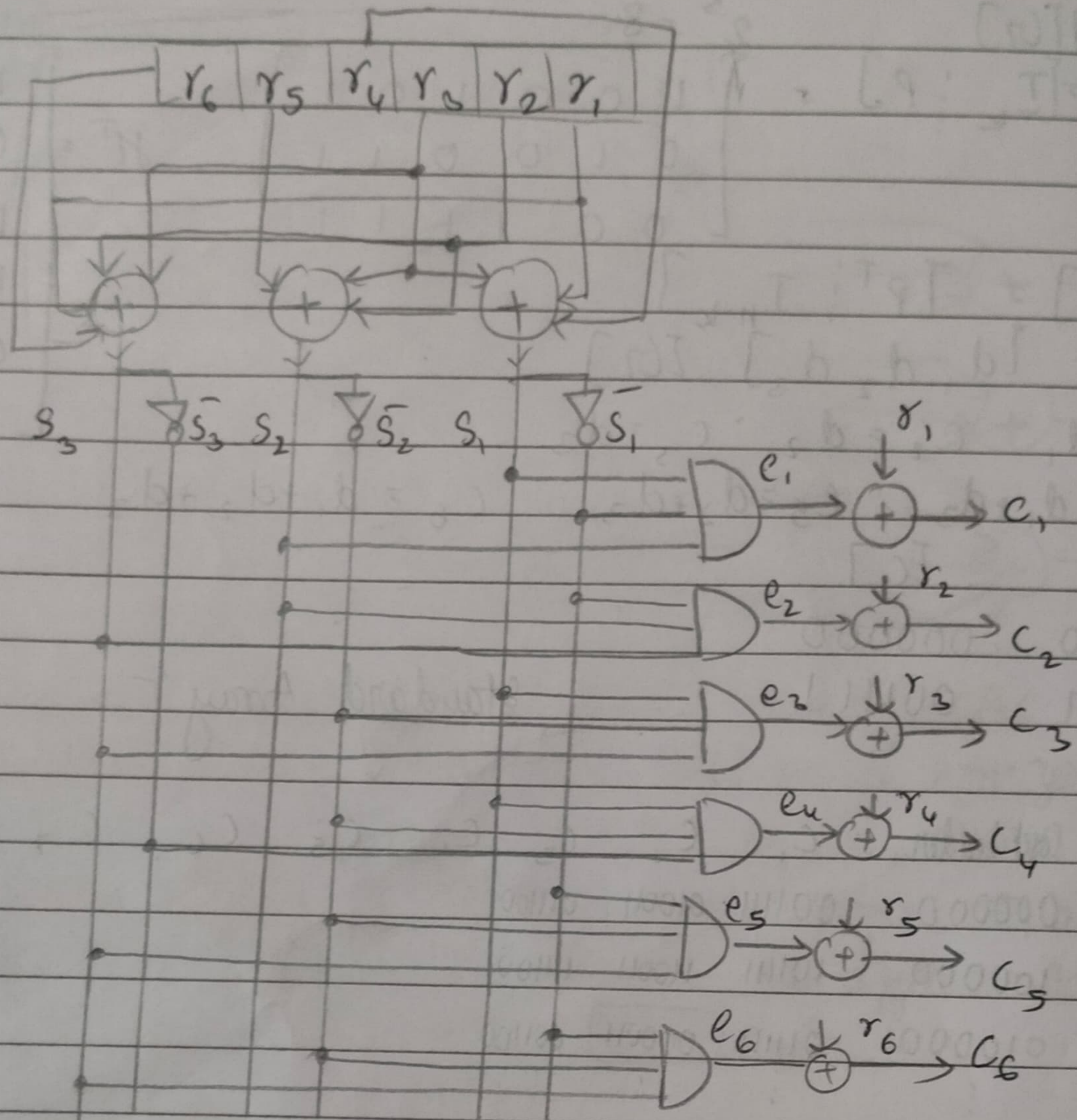
$e_2 = \bar{s}_1 s_2 s_3$

$e_3 = s_1 s_2 s_3$

$e_4 = s_1 \bar{s}_2 \bar{s}_3$

$e_5 = \bar{s}_1 s_2 \bar{s}_3$

$e_6 = \bar{s}_1 \bar{s}_2 s_3$


r6 r5 r4 r3 r2 r1
S3 S̄3 S2 S̄2 S1 S̄1
e1 r1 c1
e2 r2 c2
e3 r3 c3
e4 r4 c4
e5 r5 c5
e6 r6 c6