PSOT : Mod 1

- **Probability**

Probability of an event $= n(A)/n(S) = P(A)$

1. De-Morgan's Law :-

$P(\overline{A \cup B}) = P(\bar{A} \cap \bar{B})$ $\quad P(\overline{A \cap B}) = P(\bar{A} \cup \bar{B})$

2. Addⁿ rule :-

$P(A \cup B) = P(A) + P(B) - P(A \cup B)$

$P(A \cup B \cup C) = P(A) + P(C) + P(B) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(B \cap A \cap C)$

$P(A \cup B \cup C \cup D)$ = 1 add, 2 elements : '-', 3 elements : '+', 4 : '-'

- **Conditional Probability**

S : sample space

A : event $\quad$ B : base event (alr. ocurred)

Condⁿ prob. of A when B has alr occurred :-

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

- **Questions**

1. Data on readership of a certain magazine show that proportion of male reader under 35 is 0.40 and over 35 is 0.20. If readers' propⁿ under 35 is 0.70, find propⁿ of subscribers that are female over 35. Also calculate the prob. that random male subscriber is under 35 years of age.

→ Let A be event where reader is male., $\bar{A}$ for female.
Let B be event where reader's age is below 35,
and $\bar{B}$ for above 35.

Given :-

$P(A \cap B) = 0.4 \quad P(A \cap \bar{B}) = 0.2 \quad P(B) = 0.7$

Find : $P(\bar{A} \cap \bar{B})$ and $P(B|A)$

$A = (A \cap B) \cup (A \cap \bar{B}) = 0.4 + 0.2 = 0.6$

$P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B}) = 1 - P(A \cup B)$

$= 1 - \{P(A) + P(B) - P(A \cap B)\} = 1 - \{0.6 + 0.7 - 0.4\} = 0.1$

- Conditional Probability :-

$P(A/B) = \frac{P(A \cap B)}{P(B)}$ $\quad$ $P(B/A) = \frac{P(A \cap B)}{P(A)}$

- Multiplication Theorem :-

$P(A \cap B) = P(A/B) \times P(B) = P(B/A) \times P(A)$

If A and B are independent $(A \cap B) = \phi$ and

$P(A/B) = P(A)$, $P(B/A) = P(B)$

A and B' are also independent. Even A' and B, A' and B'

- Questions

1. 2 dice are thrown simultaneously. If at least one of the dice shows 5, what is the probability that sum of the nos of both the dice is 9.

→ event A : on dice shows 5 $\quad$ B : sum of numbers is 9.

$A = \{(1,5), (2,5), (3,5), (4,5), (6,5), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6)\}$

$B = \{(3,6), (4,5), (5,4), (6,3)\}$

$A \cap B = \{(4,5), (5,4)\}$ $\quad$ $n(A) = 11$ $\quad$ $n(A \cap B) = 2$

$P(B/A) = \frac{n(A \cap B)}{n(A)} = \frac{2}{11}$

2. $P(A') = 0.7$ $\quad$ $P(B) = 0.7$ $\quad$ $P(B/A) = 0.5$

Find $P(A/B)$ and $P(A \cup B)$

→ $P(A/B) = \frac{P(A \cap B)}{P(B)}$ $\quad$ $P(B/A) = \frac{P(A \cap B)}{P(A)} = 0.5$

$P(A \cap B) = P(A) \times P(B/A) = 0.3 \times 0.5 = 0.15$

$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{0.15}{0.7} = 0.214$

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.3 + 0.7 - 0.15 = 0.85$

Page No.
Date

5 In an examination, 30% of the students have failed in Maths, 20% in Chem and 10% in both Maths and Chem. A student is selected at random, what is probability that the student :
i) has failed in maths, if its known that he has failed in Chem
ii) has failed in at least one subject.
iii) has failed in exactly one subject.

→ M: failed in maths C: failed in Chem
given: $P(M) = 0.3$ $P(C) = 0.2$ $P(M \cap C) = 0.1$
To find: i) $P(M|C)$ ii) $P(M \cup C)$ iii) $P(M \cup C) - P(M \cap C)$

i) $P(M/C) = \frac{P(M \cap C)}{P(C)} = \frac{0.1}{0.2} = 0.5$

ii) $P(M \cup C) = P(M) + P(C) - P(M \cap C) = 0.3 + 0.2 - 0.1 = 0.4$

iii) $P(M \cup C) - P(M \cap C) = 0.4 - 0.1 = 0.3$

6 From a city population of selecting i) a male & a smoker is 0.70 ii) a male smoker: 0.40 and iii) male, if smoker are selected = 2/3
Find prob of selecting a) non smoker b) male c) smoker if male selected

→ M: male S: smoker $\bar{S}$: non-smoker
given: i) $P(M \cup S) = 0.7$ ii) $P(M \cap S) = 0.40$
iii) $P(M/S) = 2/3 = 0.666$
To find: a) $P(\bar{S})$ b) $P(M)$ c) $P(S/M)$

a) $P(M/S) = \frac{P(M \cap S)}{P(S)} \longrightarrow 0.66 = \frac{0.4}{P(S)}$

$P(S) = \frac{0.4}{0.66} = 0.6$

$P(\bar{S}) = 1 - 0.6 = 0.4$

b) $P(M \cup S) = -P(M \cap S) + P(M) + P(S)$

$0.7 = -0.4 + P(M) + 0.6$

$P(M) = 0.5$

c) $P(S/M) = P(M \cap S)/P(M) = 0.4/0.5 = 0.8$