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- 2D Discrete Random Variable

$p(x_i, y_j) \geq 0$ for all i, j $\sum^{\infty}\sum^{\infty} p(x_i, y_j) = 1$

joint probability mass fn.

- Questions

1. The joint probability of distribution of X and Y is given by $P(X = x, Y = y) = (x + 3y)/24$, $x = 1, 2$; $y = 1, 2$. Find the joint pmf of X and Y. Find Marginal probability distributions of X and Y

→

X \ Y	1	2	Row sum
1	4/24	7/24	11/24
2	5/24	8/24	13/24
col. sum	3/8	5/8	1

Marginal probability dist. of X :-

X	1	2
P_i	11/24	13/24

" " " of Y :-

Y	1	2
P_i	3/8	5/8

2. In a bag with 3 red and 2 white balls, a man has to draw 2 balls at random without replacement. We will get ₹20 for each red ball and ₹10 for each white ball we draw, then find the expected amount we will get.

→ Red : 3 White : 2 ₹20 : Red White : ₹10

X :	0	1	2
P_i :	1/10	6/10	3/10

: no. of balls

$$\text{prob}(0 \text{ Red}) = \frac{n(A)}{n(S)} = \frac{{}^{2}C_2}{{}^{5}C_2} = \cancel{\frac{2}{20}} = \frac{1}{10}$$

$$P(1 \text{ Red}) = 1R, 1W = \frac{{}^{3}C_1 \times {}^{2}C_1}{{}^{5}C_2} = \frac{3 \times 2}{10} = \frac{6}{10}$$

$$P(2 \text{ Red}) = \frac{{}^{3}C_2}{{}^{5}C_2} = \frac{3}{10}$$

0 red ⇒ 2 white, so ${}^{2}C_2$ → drawing 2 white ; ^{2} → total 2 white

${}^{5}C_2$: total ; ^{5} → 3+2 ; C_2 → drawing 2 balls

1 red 1 white → ${}^{3}C_1 \times {}^{2}C_1$; ^{3} → total R ; C_1 → drawing 1 R ; ${}^{2}C_1$ → for W

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Y :	20	30	40
P_i :	1/10	6/10	3/10

Expected amount $E(Y) = \sum P_i Y_i = 1/10 * 20 + 6/10 * 30 + 40/10 * 3 = 32$

3 A and B throw a pair of die. A player who throws a 6 first will win ₹44 if A starts, find the expected amount for A as well as B.

→ P(A wins) cases : A, $\bar{A}\bar{B}A$, $\bar{A}\bar{B}\bar{A}\bar{B}A$ + $(\bar{A}\bar{B})^3 A$ + ...

$r = ar^3/ar^2$: common ratio : $\bar{A}\bar{B}$ a: first term : A

GP : $a, ar, ar^2, ar^3, ar^4, \dots, ar^{n-1}$ $a_n = ar^{n-1}$: n^{th} term

Sum $S_n = a[(r^n - 1)/(r-1)]$

In the question (ans) : $S_n = [1/(1-r)]$

$P(A \text{ wins}) = 1/6 + 5/6 \times 5/6 \times 1/6 + (25/36)^2 \times 1/6 + (25/36)^3 \times 1/6 + \dots$

$= 1/6 \; [1 + 25/36 + (25/36)^2 + (25/36)^3 + \dots]$

$= \frac{1}{6}\left[\frac{1}{1-25/36}\right] = \frac{1}{6}\left[\frac{36}{11}\right] = \frac{6}{11}$

$P(B \text{ wins}) = 1 - P(A \text{ wins}) = 5/11$

$E(A) = P(A \text{ wins}) \times 44 = 24$

$E(B) = P(B \text{ wins}) \times 44 = 20$

4 3 balls drawn at random from box with 2 white, 4 black and 3 red balls. If X denotes the no. of whites and Y denotes the no. of reds, find -

i) Joint probability distribution of X & Y.

ii) $P(X \le 1)$, $P(X \le 1, Y \le 2)$, $P(Y \le 2 / X \le 1)$, $P(X+Y \le 2)$

iii) Marginal prob. dist. of X   iv) " " " of Y

v) Conditional dist. of X given Y = 1. Also check whether X and Y are independent variables.

→

X \ Y	Y=0	1	2	3	Total sum
X=0	1/21	3/14	1/7	1/84	35/84
X=1	1/7	2/7	1/14	0	42/84
X=2	1/21	1/28	0	0	7/84
Total sum	5/21	15/28	3/14	1/84	1

$P(X=0, Y=0) = 0W, 0R, 3B) = {}^4C_3 / {}^9C_3 = 1/21$

$P(X=0, Y=1) = 0W, 1R, 2B) = \frac{{}^3C_1 \times {}^4C_2}{{}^9C_3} = \frac{3}{14}$

ii) $P(X \le 1) = P(X=0) + P(X=1) = 35/84 + 42/84 = 77/84 = 11/12$

$P(X \le 1, Y \le 2) = P(X=0, Y=0) + P(X=0, Y=1) + P(X=0, Y=2)$
$+ P(X=1, Y=2) + P(X=1, Y=1) + P(X=1, Y=0)$

$= \frac{1}{21} + \frac{2}{14} + \frac{1}{7} + \frac{1}{7} + \frac{2}{7} + \frac{1}{14} = \frac{19}{21}$

$P(Y \le 2 / X \le 1) = \frac{P(Y \le 2, X \le 1)}{P(X \le 1)} = \frac{19/21}{11/12} = \frac{76}{77}$

$P(X+Y \le 2) = P(X=0, Y=0) + P(X=0, Y=1) + P(X=0, Y=2)$
$+ P(X=1, Y=0) + P(X=1, Y=1)$

$= 1/21 + 3/14 + 1/7 + 1/7 + 2/7 = 37/42$

iii) Marginal probability distribution of X:

X:	0	1	2
P:	5/12	1/2	1/12

iv) Marginal probability distribution of Y:

Y:	0	1	2	3
P:	5/21	15/28	3/14	1/84

v) $P(X/Y=1) = P(X=0/Y=1) + P(X=1/Y=1) + P(X=2/Y=1)$

$= \frac{P(X=0, Y=1)}{P(Y=1)} + \frac{P(X=1, Y=1)}{P(Y=1)} + \frac{P(X=2, Y=1)}{P(Y=1)}$

$= \frac{3/14}{15/28} + \frac{2/7}{15/28} + \frac{1/21}{15/28} = 1$

X and Y are independent if $P(X, Y) = P(X) \cdot P(Y)$

$P(X=1, Y=2) = P(X=1) \cdot P(Y=2)$

$1/14 \ne 42/84 \times 3/14$

- Marginal Probability Distribution

$$\int_{-\infty}^{\infty} f_{xy}(x, y)\, dx\, dy = 1 \qquad P(a < x < b, c < y < d)$$

$$= \int_{x=a}^{b} \int_{y=c}^{d} f_{xy}(x, y)\, dx\, dy$$

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Marginal prob of x :-

$$f_x(x) = \int_{-\infty}^{\infty} f_{xy}(x,y)\,dy$$

of y :-

$$f_y(y) = \int_{-\infty}^{\infty} f_{xy}(x,y)\,dx$$

- Questions :-

1 The joint probability density of 2 r.v is given by

$$f_{xy}(x,y) = \begin{cases} 15e^{-3x-5y}, & x>0,\ y>0 \\ 0, & \text{elsewhere} \end{cases}$$

Find the probability that :-

i) $1 < x < 2$ and $0.2 < Y < 0.3$ ii) $x < 2$ and $Y > 0.2$

→ i) $P(1 < x < 2,\ 0.2 < Y < 0.3)$

$$= \int_{x=1}^{2} \int_{y=0.2}^{0.3} 15e^{-3x-5y}\,dx\,dy$$

$$= 15\left(\int_{x=1}^{2} e^{-3x}\,dx\right) \times \left(\int_{0.2}^{0.3} e^{-5y}\,dy\right)$$

$$= 15\left[\frac{e^{-3x}}{-3}\right]_1^2 \times \left[\frac{e^{-5y}}{-5}\right]_{0.2}^{0.3} = 0.00684$$

ii) $P(x < 2,\ y > 0.2) = P(0 \le x \le 2,\ \infty \ge Y \ge 0.2)$

$$= \int_{x=0}^{2} \int_{y=0.2}^{\infty} 15e^{-3x} e^{-5y}\,dx\,dy = 0.367$$

2 Given $f_{xy}(x,y) = \begin{cases} cx(x-y), & 0 < x < 2,\ -x < y < x \\ 0, & \text{otherwise} \end{cases}$

i) Evaluate c ii) Find $f_x(x)$ iii) Find $f_{y/x}(y/x)$

→ i) $\iint f_{xy}(x,y)\,dx\,dy = 1 \rightarrow \int_0^2 \int_{-x}^{x} cx(x-y)\,dx\,dy = 1$

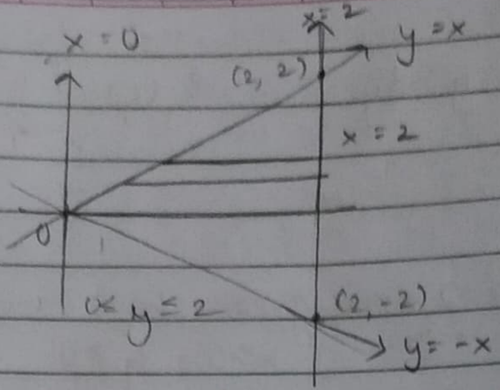
$c = 1/8$

ii) $f_x(x) = \int_{-x}^{x} f_{xy}(x,y)\,dy = \int_{-x}^{x} \frac{1}{8} x(x-y)\,dy$

$= x^3/4$ $0 < x < 2$

iii) $f_{y/x}(y/x) = \dfrac{f_{xy}(x,y)}{f_x(x)} = \dfrac{1/8\, x(x-y)}{x^3/4} = \dfrac{x-y}{2x^2}$

$0 < x < 2$, $-x < y < x$ Graph →



3 $f_{xy}(x,y) = xy^2 + x^2/8$, $0 \le x \le 2$, $0 \le y \le 1$

Compute i) $P(x>1)$ ii) $P(y<1/2)$ iii) $P(x>1, y<1/2)$

iv) $P(y<1/2 \,/\, x>1)$ v) $P(x<y)$ vi) $P(x+y \le 1)$

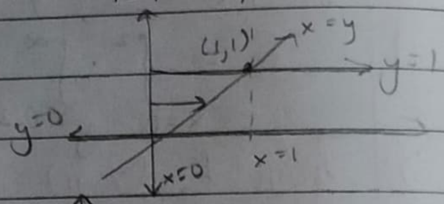
→ i) $P(x>1) = \iint f_{xy}(x,y)\,dx\,dy = \int_{y=0}^{1}\int_{x=1}^{2}\left(xy^2 + \frac{x^2}{8}\right)dx\,dy = \frac{19}{24}$

ii) $P(y<1/2) = \iint f_{xy}(x,y)\,dx\,dy = \int_{y=0}^{1/2}\int_{x=0}^{2}\left(xy^2 + \frac{x^2}{8}\right)dx\,dy = \frac{1}{4}$

iii) $P(x>1, y<1/2) = \int_{y=0}^{1/2}\int_{x=1}^{2}\left(xy^2 + \frac{x^2}{8}\right)dx\,dy = \frac{5}{24}$

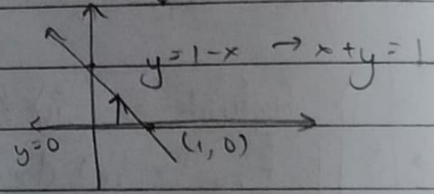
iv) $P(y<1/2 \,/\, x>1) = \frac{P(x>1, y<1/2)}{P(x>1)} = \frac{5/24}{19/24} = \frac{5}{19}$

v) $P(x<y) = \int_{x=0}^{1}\int_{y=x}^{1}\left(xy^2 + \frac{x^2}{8}\right)dx\,dy$



$= 53/480$

vi) $P(x+y \le 1) = \int_{x=0}^{1}\int_{y=0}^{1-x}(xy^2 + x^2/8)\,dx\,dy$



$= 13/480$

- Binomial Distribution

success : p failure : q no. of trials : n

$p + q = 1$ $P(X=x) = {}^nC_x\, p^x q^{n-x}$

Sum of probabilities :-

$$\sum_{x=0}^{n} P(x) = \sum_{x=0}^{n} {}^nC_x\, p^x q^{n-x} = (p+q)^n = 1$$

Frequency function :-

$N \times P(X = x)$