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PSOT Mod I

- Normal Distribution

Continuous rv X is said to follow normal distribution with parameter m and σ^2 if its probability density function is given by

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2} \qquad -\infty < x < \infty,$$
$$-\infty < m < \infty, \quad \sigma^2 > 0$$

$X \sim N(m, \sigma)$: continuous random variate X

$Z = \frac{X-m}{\sigma}$: normal variate

$$\text{Mean} = E(X) = \int_{-\infty}^{\infty} x f(x)\,dx = \int_{-\infty}^{\infty} [(x-m)+m] f(x)\,dx$$
$$= \int_{-\infty}^{\infty} (x-m) f(x)\,dx + m\int_{-\infty}^{\infty} f(x)\,dx$$

$$Var(x) = E(X-m)^2 = \int_{-\infty}^{\infty} (x-m)^2 f(x)\,dx = \int_{-\infty}^{\infty} (x-m)^2 \frac{1}{\sqrt{2m}\,\sigma} e^{\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2} dx$$

put $(x-m)/\sigma = t$ $\quad \therefore dx/\sigma = dt$

$$Var(x) = \frac{2\sigma^2}{\sqrt{2\pi}} \int_0^{\infty} t^2 e^{-\frac{1}{2}t^2}\,dt$$

extra

For mode, $f'(x) = 0$

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{m-x}{\sigma}\right)^2} \qquad \text{diff}^n\ f(x) \text{ wrt } x$$

$$f'(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2} \cdot \left(-\frac{1}{2}\right) \cdot 2\left(\frac{x-m}{\sigma}\right)\frac{1}{\sigma} = -f(x)\frac{(x-m)}{\sigma^2}$$

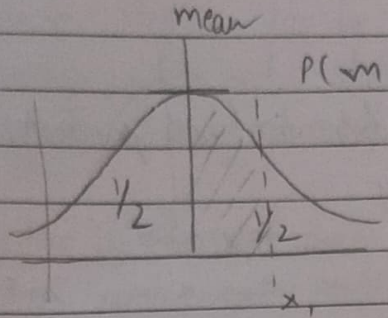
$$f''(x) = 1/\sigma^4 \quad f(x)\left[(x-m)^2 - \sigma^2\right]$$

where $x = m$ $\quad f'(x) = 0$ $\quad \boxed{\text{Mode} = m}$

Median = m = M

Mean = Median = Mode = m

Area b/w curve of $X = m$ and $X = x_1$ is equal to area under SN curve of Z b/w $Z = 0$ to $Z = z_1$



$P(m < x < x_1)$

$Z_1 = \frac{x_1 - m}{\sigma}$

Quartile Deviation :-

$Q_1 = m - \frac{2}{3}\sigma$

$Q_2 = m + \frac{2}{3}\sigma$

Quartile deviation $= \frac{Q_3 - Q_1}{2} = \frac{2}{3}\sigma$

Mean Deviation $= 4/5\ \sigma$

$\sigma = \sqrt{npq}$ when $q = 1-p$ then $Z = \frac{X - np}{\sqrt{npq}}$

$np > 15$ $nq > 15$

ND can also be obtained from Poisson distribution when the $m \to \infty$

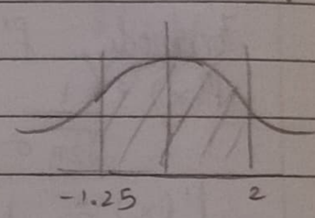
• Questions

1 X is a normal variate $X \sim N(10, 4)$ with mean 10 and SD = 4, then find i) $P(5 \le x \le 18)$ ii) $P(x \le 12)$ iii) $P(|x - 14| < 1)$

→ $P(x_1 \le x \le x_2)$ $Z_1 = \frac{(x_1 - m)}{\sigma}$ $Z_2 = \frac{(x_2 - m)}{\sigma}$: To convert to SNV

i) When $x_1 = 5$, $Z_1 = (5-10)/4 = -1.25$

$x_2 = 18$, $Z_2 = (18-10)/4 = 2$

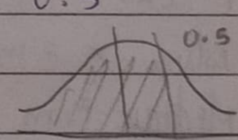


$P(5 \le x \le 18) = P(-1.25 < z < 2)$

$= P(-1.25 \le z \le 0) + P(0 \le z \le 2)$

$= P(0 \le z \le 1.25) + P(0 \le z \le 2)$ $= 0.3944$ to 0.4772 from the table

ii) $P(x \le 12)$ $Z = (12-10)/\sigma = 2/4 = 0.5$



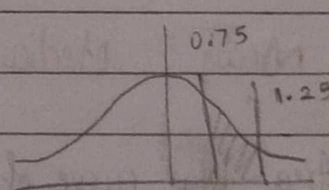
$P(x \le 12) = P(Z \le 0.5)$

$= 0.5 + P(0 < z < 0.5)$

$= 0.5 + 0.1915$

iii) $P(|x - 14| < 1) \to P(-1 < x - 14 < 1)$

$= P(13 < x < 15)$



$x_1 : 13$ $Z_1 = (13-10)/4 = 0.75$

$x_2 : 15$ $Z_2 = (15-10)/4 = 1.25$

$\therefore P(0.75 \le$ $Z \le 1.25) = 0.2794$ to 0.3944

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2 The 1^{st} and 3^{rd} quartile from ND is 36 and 44. Then find its mean, standard deviation and mean deviation

→ $Q_1 = 36 \quad Q_3 = 44$

$Q_1 = m - 2/3\,\sigma \qquad Q_2 = m + 2/3\,\sigma$

$36 = m - 2/3\,\sigma \qquad 44 = m + 2/3\,\sigma$

Add them : $80 = 2m \rightarrow m = 40$

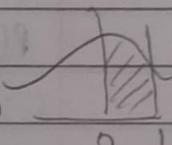
$2/3\,\sigma = 40 - 36 \rightarrow \sigma = 4 \times 3/2 = 6$

mean deviation $= 4/5 \times \sigma = 24/5 = 4.8$

3 For a normal variate x, the mean is 25, $\sigma = 10$, then find the area b/w i) $x = 25$ and $x = 35$ ii) $x = 15$ and $x = 35$ iii) area greater than 15, and ~~less than 35~~ area greater than 35

→ $X \sim N(m, \sigma) \quad : \quad X \sim N(25, 10)$

SNV is given by $Z = (x - m)/\sigma = (Z - 25)/10$



i) $P(\underset{x_1}{25} < x < \underset{x_2}{35})$ $\qquad Z_1 = (x_1 - m)/\sigma = (25 - 25)/10 = 0$

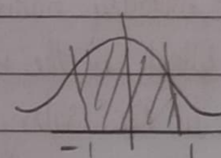
$Z_2 = (x_2 - m)/\sigma = (35 - 25)/10 = 1$

$= P(0 \le Z \le 1) = 0.3413$

ii) $P(15 < x < 35)$

$Z_1 = (x_1 - m)/\sigma = (15 - 25)/10 = -1$

$Z_2 = (x_2 - m)/\sigma = (35 - 25)/10 = 1$

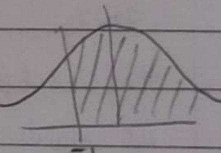


$= P(-1 < Z < 1) = 2P(0 < Z < 1)$

$= 2 \times 0.3413 = 0.6826$

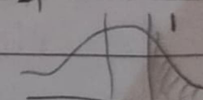
iii) $P(X > 15) = P(Z > -1) = 0.5 + P(0 < Z < 1)$

$= 0.5 + 0.3413 = 0.8413$



$P(x > 35) = P(Z > 1) = 0.5 - P(0 < Z < 1)$

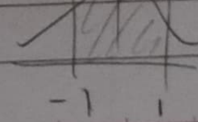
$= 0.5 - 0.3141 = 0.1858$



4 A manufacturer knows that the resistance of a resistor is normal with $\mu = 100\,\Omega$ and $\sigma = 2\,\Omega$. Then what % of resistors will have resistance b/w 98 to 102 Ω

→ $\mu = 100\,\Omega \quad \sigma = 2\,\Omega \qquad X \sim N(100, 2)$

SNV $Z = \dfrac{x - 100}{2}$ $\qquad P(\underset{x_1}{98} < x < \underset{x_2}{102})$



$P(-1 < Z < 1)$ as $x_1 = 98$ $\quad z_1 = (98-100)/2 = -1$

and $x_2 = 102$ $\quad z_2 = (102-100)/2 = 1$

$P(-1 < Z < 1) = 2P(0 < Z < 1) = 2 \times 0.3413 = 0.6826$

5 Marks obtained by students in college with normally distributed mean is 65 and variance = 25. If 3 students are selected at random, what is the probability that at least one of them has scored more than 75 marks.

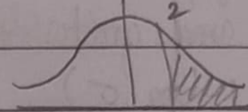
→ $\mu = 65 \quad \sigma^2 = 25 \quad \sigma = 5$

SNV $\quad Z = (x - 65)/5$

$P(x > 75)$ $\quad x = 75 \quad Z = (75-65)/5 = 2$

$P(Z > 2) = 0.5 - P(0 < Z < 2)$

$= 0.5 - 0.4772 = 0.0228$



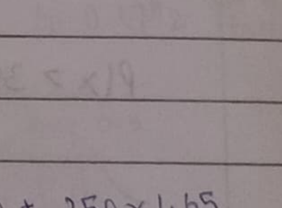
P(at least one student out of 3 get ≥ 75 marks)

$= 1 - P(\text{all } 3 < 75)$ $\qquad P(x < 75) = 1 - 0.0228 = 0.9772$

$= 1 - [P(x < 75) \times P(x < 75) \times P(x < 75)]$

$= 1 - [0.9772]^3 = 0.07$

6 A monthly salary normally distributed with mean = 3000 and σ = 250. What is the minimum salary of worker who belongs to top 5% in the salary register.



→ $\mu = 3000 \quad \sigma = 250$

SNV : $Z = (x - 3000)/250$

Since we are to find top 5% as per diagram, we search for $Z = Z_1$

$P(0 < Z < Z_1) = 0.45$

Using table $\quad Z_1 = 1.65$

then, $Z_1 = (x - 3000)/250 \longrightarrow x_1 = 3000 + 250 \times 1.65$

$= 3412.5$

7 In a factory, it was found out of 100 plates, 16 plates are defective then find the σ and can this distribution be approximated by normal distribution. If yes, write its equation.

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$n = 100$ $\quad p = 16/100 = 0.16$ $\quad q = 1 - p = 0.84$

$\sigma = \sqrt{npq} = \sqrt{100 \times 0.16 \times 0.84} = \sqrt{13.44} = 3.66$

check np : $100 \times 0.16 = 16$ $\quad nq = 100 \times 0.84 = 84$

Both are greater than 15, it can be approx by normal distribution

$$Z = \frac{x - np}{\sqrt{npq}} = \frac{x - 16}{3.67}$$

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-1/2\,(x - m/\sigma)^2} = \frac{1}{\sqrt{2\pi}\,3.67} \times e^{-1/2 \times \left(\frac{x-16}{3.67}\right)^2}$$

leave it like this

8 2 independent random variates X and Y are normally distributed with mean and σ : $X \sim N(52, 3)$ $\quad Y \sim N(50, 2)$

Find the random chosen pair X and Y differ by 1.7 or more

$P(|X - Y| \geq 1.7)$

$X - Y$: $m = m_1 - m_2$

$\sigma^2 = \sigma_1^2 + \sigma_2^2 = 3^2 + 2^2 = 13$ $\quad \sigma = \sqrt{13}$

$X - Y \sim N(2, \sqrt{3})$

$P(|X - Y| \geq 1.7)$:- $\quad z = (x - 2)/\sqrt{13}$ $\quad Z_1 = (1.7 - 2)/\sqrt{13} = 0.083$

$P(-1.7 < \mu < 1.7) = 1 - P(-1.7 < \mu < 1.7)$

$= 1 - P(-0.083 < z < 0.083)$

$= 1 - 2P(z < 0.083)$

$= 1 - 2 \times 0.0319 = 0.9362$