OR : $\cup$
AND : $\cap$

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- Correlation : $\bar{x} = \Sigma x / N = \mu$ $\qquad Cov(x,y) = \dfrac{\Sigma(x-\bar{x})(y-\bar{y})}{N}$

Karl Pearson's Coefficient :-

$$r = \frac{\Sigma d_x d_y - N\bar{d}_x \bar{d}_y}{\sqrt{(\Sigma d_x^2 - N\bar{d}_x^2)(\Sigma d_y^2 - N\bar{d}_y^2)}}$$

$d_x = x - a$ (nearest int to $\bar{x}$) $\qquad d_y = y - b$ (to $\bar{y}$)

for some questions, replace with $x, \bar{x}$ and $y, \bar{y}$

Spearman Rank Correl^n

$$R = 1 - \left(\frac{6\Sigma d_i^2}{N^3 - N}\right) = 1 - 6\,\frac{[\Sigma d_i^2 + 1/12(m_1^3 - m_1) + 1/12(m_2^3 - m_2) + \ldots]}{N^3 - N}$$

for duplicates $\qquad d_i = R_1 - R_2$

$$r = \frac{Cov(x,y)}{\sigma_x \sigma_y}$$

SD

m_1 = no. of similar terms in x
m_2 = no. of similar terms in y

Values	1	1	2	3
Ranks	1.5	1.5	3	4

mean of 1 and 2

- Regression : Normalisation :-

$y = a + bx \qquad x = a + by$

$\Sigma y = aN + b\Sigma x \qquad \Sigma x = aN + b\Sigma y$

$\Sigma xy = a\Sigma x + b\Sigma x^2 \qquad \Sigma xy = a\Sigma y + b\Sigma y^2$

Calculate a, b from the eq^n

b here is b_{yx} as this is y on x $\qquad$ b here is b_{xy} as this is x on y

$$r = \sqrt{b_{xy} b_{yx}}$$

put $x = ?$ (given in question) in $y = a + bx$
put $y = ?$ in $x = a + by$

- Relation :-

$$\tan\theta = \left(\frac{1-r^2}{r}\right)\left(\frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2}\right) \qquad \theta = \tan^{-1} ?$$

$$(y - \bar{y}) = \frac{r\sigma_y}{\sigma_x}(x - \bar{x})$$ when x is given

$\frac{r\sigma_y}{\sigma_x} \rightarrow b_{yx}$

$$(x - \bar{x}) = \frac{r\sigma_x}{\sigma_y}(y - \bar{y})$$ when y is given

$\frac{r\sigma_x}{\sigma_y} \rightarrow b_{xy}$

- Question : $6Y = 5x + 90 \qquad 15x = 8Y + 130 \qquad \sigma_x^2 = 16$

Find : i) $\bar{x}$ and $\bar{y}$ $\quad$ ii) r $\quad$ iii) σ_y^2

→ $10Y = 40 \rightarrow \bar{Y} = 40 \qquad x = 30 \rightarrow \bar{x} = 30$

X on Y : $X = 6/5\,Y - 18 \qquad b_{xy} = 6/5 \qquad r = \sqrt{b_{xy} b_{yx}} = 1.5$

Y on X : $Y = 15/8\,X - 130/8 \qquad b_{yx} = 15/8$

$b_{yx} = \frac{r\sigma_x}{\sigma_y} \longrightarrow \sigma_y = 5 \longrightarrow \sigma_y^2 = 25$

Harmonic Mean $= 1/H = {}_a\int^b \frac{1}{x} f(x)\,dx$

GP sum $= a \frac{1}{1-r}$

- De Morgan's Law, Probability, Addition Rule

at least: $P(A \cup B)$

exactly in 1: $P(A \cup B) - P(A \cap B)$

$P(\overline{A \cup B}) = P(\bar{A} \cap \bar{B})$ $\quad P(\overline{A \cap B}) = P(\bar{A} \cup \bar{B})$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

- Conditional Probability :- $P(A|B) = P(A \cap B) / P(B)$

A and B independent if $P(A \cap B) = P(A)\,P(B)$

- Baye's Theorem :- $P(A_i / B) = \frac{P(A_i)\,P(B/A_i)}{\sum_{i=1}^{n} P(A_i)\,P(B/A_i)}$ $\rightarrow \sum P(A_i \cap B)$ $\rightarrow P(B)$

- Mean $= E(x) = \sum P_i x_i$ $\quad$ check mean $= \sum f_i x_i / \sum f_i$

$\sum P_i = 1$ $\quad$ Variance : $V(x) = E(x^2) - (E(x))^2$

$E(x) = {}_{-\infty}\int^{\infty} x\, f(x)\,dx$ $\quad V(x) = {}_{-\infty}\int^{\infty} x^2 f(x)\,dx$

$V(ax) = a^2 V(x)$

Mode : $f'(x) = 0$ or $f''(x) < 0$ $\quad V(x+b) = V(x)$

$E(ax+b) = aE(x) + b$ $\quad V(ax+b) = a^2 V(x)$

$V(ax_1 \mp bx_2) = a^2 V(x_1) \mp b^2 V(x_2)$

Median : M : ${}_0\int^M f(x)\,dx = 1/2 = {}_M\int^b f(x)\,dx$

$f(x)$ is pdf if ${}_a\int^b f(x)\,dx = 1$

To find k : ${}_a\int^{\infty} k f(x)\,dx = 1$

- Joint Probability :- $\quad$ Marginal prob x :- $\quad$ x : and P_i :

$P(E_1, E_2) = \underbrace{{}^{n_1}C}_{\text{total for } E_1}\, \underbrace{{}^{n_2}C_2}_{\text{active}} / \underbrace{{}^{n}C_3}_{\text{total active}} = n(A)/n(S)$

Marginal Prob and stuff in continuous :-

Maybe draw graphs.

$${}_{-\infty}\int^{\infty} f_{xy}(x,y)\,dx\,dy = 1 = \int_{x=a}^{b}\int_{y=c}^{d} f_{xy}(x,y)\,dy\,dx$$

$f_x(x) = \int_{y=c}^{d} f_{xy}(x,y)\,dy$ $\quad$ for MPD of x, we use y stuff

$f_y(y) = \int_{x=a}^{b} f_{xy}(x,y)\,dx$ $\quad$ for MPD of Y, we us x stuff

- Binomial Distribution :- $\quad (x+y) \rightarrow (n, n_2, P)$

$p+q = 1$ $\quad q = 1-p$ $\quad P(X=x) = {}^nC_x\, p^x\, q^{n-x} = (p+q)^n = 1$

Frequency : $N\,P(X=x)$ $\quad SD = \sqrt{\sigma^2}$

Mean $= np$ $\quad$ Variance $= npq$ $\quad$ mode $= \lfloor (n+1)p$

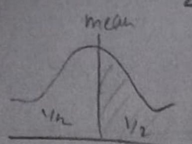
for fair coins : $p = 1/2 = q$ $\quad$: children = boy or girl

- Poisson Distribution :- $\quad P(X=x) = e^{-m} m^x / x!$

$V(\alpha x + \beta y) = \alpha^2 V(x) + \beta^2 V(y)$

$p(m < x < x_1)$

$z_1 = \frac{x_1 - m}{\sigma}$



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- Normal Distribution :- $f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-1/2 (x-m/\sigma)^2}$

mean = median = mode = m $\qquad z = (x-m)/\sigma$

Quartile deviation : $[Q_3 - Q_1]/2 = 2/3\,\sigma$ $\qquad Q_1 = m - 2/3\,\sigma$

Mean deviation : $4/5\,\sigma$ $\qquad Q_3 = m + 2/3\,\sigma$

$\sigma = \sqrt{npq}$ $\qquad Z = (x - np)/\sqrt{npq}$ $\qquad np > 15 \qquad nq > 15$

- Exponential Distribution :-

$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$: pdf of x

Mean $E(x^r) = \underbrace{\Gamma(r+1)}_{\text{gamma fn}} / \lambda^2 = \int_{-\infty}^{\infty} x f(x) dx$

Var $\sigma^2 = E(x^2) - [E(x)]^2 = 1/\lambda^2 = \int_{-\infty}^{\infty} \lambda^2 f(x) dx$

$P(\text{cond}^n) = \int_{\text{cond}}^{\text{cond}} \lambda e^{-\lambda x} dx$

- Uniform distribution : Discrete [similar to simple problems]

$E(x) = \sum P_i x_i$ $\qquad P(X = x_i) = 1/n$ $\qquad E(x) = (n+1)/2$

$Var(x) = (n^2 - 1)/12$

- Uniform distribution : Continuous :-

$f(x) = \begin{cases} 1/(b-a) & a \leq x \leq b \\ 0 & \text{otherwise} \end{cases}$: density

$E(x) = (b+a)/2$ $\qquad Var(x) = (b-a)^2/12$

Keep the units in mind.

- Integration :- $x^n \rightarrow x^{n+1}/n+1$ $\qquad 1/x \rightarrow \log x$

$a^x \rightarrow a^x / \log a$ $\qquad e^{ax} \rightarrow e^{ax}/a$

$\int_0^\infty e^{-x} x^4 dx = \Gamma n+1 = \Gamma 4+1$

- Gamma Function :- $\Gamma 1 = 1$ $\qquad \Gamma 0 = \infty$ $\qquad \Gamma 1/2 = \sqrt{\pi}$

$\Gamma n = (n-1)\Gamma n-1$ $\qquad \Gamma p\,\Gamma 1-p = \pi / \sin p\pi$

- Large : z-test $n \geq 30$ $\qquad$ Small : t-test $n < 30$

Null H_0 : μ = sumn $\qquad$ Alt H_a : $\mu \neq$ sumn

los : rejection $\qquad$ loc : acceptance $\qquad$ if 95% confidence then 5% significance

- $|z| = \frac{\bar{x} - \mu}{s/\sqrt{n}} < z_\alpha$ then H_0 accepted : insignificant

If alt H_a is accepted, it confirms the steps in the question

Interval = Confidence limit :- $-z_\alpha < z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} < z_\alpha$

$\mu - \sigma/\sqrt{n}\ Z_\alpha < \bar{x} < \mu + \sigma/\sqrt{n}\ Z_\alpha$

los normally 5% → 0.05

	1% (0.01)	5% (0.05)	10% (0.1)
= 2 tailed	$\lvert z\rvert = 2.58$	1.96	1.645
> right	2.33	1.645	1.28
< left	-2.33	-1.645	-1.28

- Diff b/w means :-

known
1. σ_1, σ_2 known $Z = (\bar{x}_1 - \bar{x}_2)/S$ [not equal]
2. $\sigma_1 = \sigma_2 = \sigma$ same population known $Z = \dfrac{\bar{x}_1 - \bar{x}_2}{\sigma\sqrt{1/n_1 + 1/n_2}}$

not known
3. $Z = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{s_1^2/n_1 + s_2^2/n_2}}$ $\sigma_1 \neq \sigma_2$ not known
4. $Z = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{s_1^2/n_2 + s_2^2/n_1}}$ $\sigma_1 = \sigma_2 = \sigma$ not known same population

- Interval $(\bar{x}_1 - \bar{x}_2) - Z_\alpha SE < (\mu_1 - \mu_2) < (\bar{x}_1 - \bar{x}_2) + Z_\alpha SE$
- T-test :- $t = \dfrac{\bar{x} - \mu}{s/\sqrt{n-1}}$ $s^2 = \dfrac{\Sigma(x_i - \bar{x})^2}{n}$ $\sigma = \sqrt{s^2}$ dof = n - 1

  $\bar{x} - t_\alpha (s/\sqrt{n-1}) < \mu < \bar{x} + t_\alpha (s/\sqrt{n-1})$
- Testing diff b/w means dof : $n_1 + n_2 - 2$

  $t = \dfrac{\bar{x}_1 - \bar{x}_2}{SE}$ $SE = S_p \times \sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}$ $S_p = \sqrt{\dfrac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$
- Chi - square (χ^2)

  $\chi^2 = \Sigma\left(\dfrac{(O-E)^2}{E}\right)$ N > 50 dof : n - 1

  $\chi^2_{cal} < \chi^2_{table}$: H_0 accepted

  Calculate O, E, O-E, $(O-E)^2$ if E is diff for each O, calculate $(O-E)^2/E$

  E can be mean
- $E_{11} = R_1 C_1 / \text{Total}$ $E_{21} = R_2 \times C_1 / \text{Total}$
- If cell freq < 10, group them.
- Yate's Correction :- $\chi^2 = \Sigma\left[\dfrac{(\lvert O-E\rvert - 0.5)^2}{E}\right]$

  when freq of any cell < 5

  dof = (r-1)(c-1)

  $E_{11} = \dfrac{R_1 \times C_1}{\text{Total}}$ (Total Total)

  $E_{21} = C_1 - E_{11}$

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- $\{(M/M/1) : (\infty / FCFS)\}$

$\rho = \lambda / \mu$ (arrival, service) : busy   idle: $1-\rho$ $P_w = 1 - P_0 = \lambda/\mu$

$P(W_s > t) = e^{-(\mu-\lambda)t}$

$P(W_q > t) = \lambda/\mu \; e^{-(\mu-\lambda)t}$

$P_0 = 1 - \lambda/\mu = 1-\rho$ nobody in system

$P_n = (\lambda/\mu)^2 (1 - \lambda/\mu) = \rho^n (1-\rho)$

$L_s = \dfrac{\rho}{1-\rho} = \dfrac{\lambda}{\mu-\lambda}$ customer in line + customer being served

$L_q = \dfrac{\lambda^2}{(\mu-\lambda)\mu}$ customers waiting in queue (queue length)

$W_q = \dfrac{\lambda}{(\mu-\lambda)\mu}$ $W_s = \dfrac{1}{\mu-\lambda}$ waiting time of queue & service

$P(n>1) = 1 - P_0 - P_1 = (\lambda/\mu)^2$

$P(n \geq k) = (\lambda/\mu)^k$ $P(n > k) = (\lambda/\mu)^{k+1}$

- $\{(M/M/1) : (N, FCFS)\}$

$P_0 = \begin{cases} \left(\dfrac{1-\rho}{1-\rho^{N+1}}\right) \\ \dfrac{1}{N+1} \end{cases}$ $P_n = \begin{cases} \rho^n \left(\dfrac{1-\rho}{1-\rho^{N+1}}\right) \\ \dfrac{1}{N+1} \end{cases}$

$L_s = \sum_{n=0}^{n} n P_n = \begin{cases} \dfrac{\rho}{1-\rho} - \dfrac{(N+1)\rho^{N+1}}{1-\rho^{N+1}} \\ \dfrac{N}{2} \end{cases}$

$L_q = L_s - (1-P_0)$

$W_s = \dfrac{L_s + 1}{\mu}$

$W_q = W_s - \dfrac{1}{\mu} = \dfrac{L_s}{\mu}$

↳ $P_N = P_0 \rho^N$ Potential customers lost

effective arrival rate, $\lambda_{eff} = \lambda(1-P_N)$

effective traffic intensity, $\rho_{eff} = \lambda_e / \mu$

- m: no. of equations (subject to)   n: no. of variables $(x_1, x_2, x_3 \dots)$

No. of non basic variables : $n-m$

When NBV like $x_3 = 0$, $x_2 = 0$, $x_1 = 0$,

Basic variables respectively be x_1, x_2 and x_1, x_3 and x_3, x_2 (the other variables except NBV) 5

And equations column will have the equations with $x_3 = 0$,

eg $x_1 + 2x_2 + 3x_3 = 7$
$3x_1 + 4x_2 + 6x_3 = 15$

Equation with NBV $x_3 = 0$
$x_1 + 2x_2 = 7$
$3x_1 + 4x_2 = 15$
Calculate x_1 and x_2

Max : $\leq$
Min : $\geq$

If there's an error in solving eq^n, then its unbounded
Feasible Solution? $x_i \geq 0$ Yes else No,
Unbounded ' - '
Degenerate : bv = 0 $\boxed{\text{Non Degenerate}}$: bv > 0 ✓ Yes
Optimal : if ~~[illegible]~~ value of z is max, then its a Yes, others will be No. or min depends on question
Find value of z too, wrt the given values.

- Standard Form :- i) obj fn : Maximize [Convert Min to Max]
ii) Every contraint in '=' iii) RHS constant +ve

the same shit we do for simplex

$z = 3x_1 + 5x_2 + 7x_3$ $6x_1 - 4x_2 + s_1 = 5$ $3x_1 + 2x$

- Big M :- $z - (c_1 + Ma_{11} + Ma_{21})x_1 - (c_2 + Ma_{12} + Ma_{22})x_2$
$--- + Ms_2 + Ms_1 = -Mb_1 - Mb_2$
for $Z = c_1x_1 + c_2x_2 + c_3x_3 - MA_1 - MA_2$
$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 - s_1 + A_1 = b_1$
$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 - s_2 + A_2 = b_2$
$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + s_3 = b_3$

- Find the dual of the following problems

i) Max $z = 40x_1 + 35x_2$
Sub to $2x_1 + 3x_2 \leq 60$, $4x_1 + 3x_2 \leq 96$
$x_1, x_2 \geq 0$ convert it to min

→ Dual is given by
Min $W = 60y_1 + 96y_2$
Sub to $2y_1 + 4y_2 \geq 40$
$3y_1 + 3y_2 \geq 35$
$y_1, y_2 \geq 0$

ii) Min $Z = 10x_1 + 20x_2$
$3x_1 + 2x_2 \geq 18$, $x_1 + 3x_2 \geq 8$
$2x_1 - x_2 \leq 6$, $x_1, x_2 \geq 0$

convert everything to min

→ Primal is given by
Min $Z = 10x_1 + 20x_2$
Sub to $3x_1 + 2x_2 \geq 18$
$x_1 + 3x_2 \geq 8$, $-2x_1 + x_2 \geq -6$
$x_1, x_2 \geq 0$
Dual is given by
Max $W = 18y_1 + 8y_2 - 6y_3$
Sub to $3y_1 + y_2 - 2y_3 \leq 10$
$2y_1 + 3y_2 + y_3 \leq 20$
$y_1, y_2, y_3 \geq 0$