

Baye's Theorem

$$P(A_i/B) = \frac{P(A_i) P(B/A_i)}{\sum_{i=1}^n P(A_i) P(B/A_i)} \quad i = 1, 2, 3, \dots, n$$

Questions

1. The chances of X, Y, Z becoming managers of a certain company are 4:2:3. The probabilities that the bonus scheme will be introduced if X, Y, Z become managers are 0.3, 0.5, and 0.8 respectively. If bonus scheme has been introduced, what is probability that X is appointed as the manager

→ Events: X: X becomes manager $P(X) = 4/9$

Y: Y becomes manager $P(Y) = 2/9$

Z: Z becomes manager $P(Z) = 3/9 = 1/3$

They form a partition: Event B: Bonus is introduced

$$P(B/X) = 0.3 \quad P(B/Y) = 0.5 \quad P(B/Z) = 0.8$$

$$P(X/B) = ? \quad P(B \cap X) = ?$$

By Baye's Theorem, $P(X/B) = \frac{P(B \cap X)}{P(X \cap B) + P(Y \cap B) + P(Z \cap B)}$

$$= \frac{P(X) \cdot P(B/X)}{P(X) P(B/X) + P(Y) P(B/Y) + P(Z) P(B/Z)}$$

$$= \frac{4/9 \times 0.3}{0.3 \times 4/9 + 0.5 \times 2/9 + 0.8 \times 1/3} = \frac{0.4 \times 3}{1.2 + 1 + 2.4} = \frac{1.2}{4.6} = 0.260$$

$$P(B/X) = \frac{P(B \cap X)}{P(X)}$$

$$0.260 \times 4/9 = P(B \cap X) = 0.133$$

2. Members of consulting firm hired cars from three rental agencies, 60% from agency 1, 30% from agency 2 and 10% from agency 3. 9% of the cars from agency 1 need repairs and 20% from agency 2 and 6% from agency 3. If a rental car delivered to consulting firm's need need repairs, what is the probability that it came from rental agency 2.

Given: E_1 : car hired from agency 1 $P(E_1) = 0.6$

E_2 : car hired from agency 2 $P(E_2) = 0.3$

E_3 : car hired from agency 3 $P(E_3) = 0.1$

R : car selected needs repairs

$$P(R|E_1) = 0.09 \quad P(R|E_2) = 0.2 \quad P(R|E_3) = 0.06$$

Using Baye's theorem,

$$P(E_2|R) = \frac{P(E_2 \cap R)}{P(E_1 \cap R) + P(E_2 \cap R) + P(E_3 \cap R)}$$

$$= \frac{P(E_2) P(R|E_2)}{P(E_1) P(R|E_1) + P(E_2) P(R|E_2) + P(E_3) P(R|E_3)}$$

$$= \frac{0.3 \times 0.2}{0.3 \times 0.09 + 0.6 \times 0.2 + 0.1 \times 0.06} = \frac{0.06}{0.06 + 0.12 + 0.006} = \frac{0.06}{0.186} \approx 0.3226$$

- 3 In certain area 25% of women were black, 75% were white. The literacy rates for black women were 48% and for white women was 83%. What proportion of the literate women was black.

B : black women W : white women

racism :-

L : literate

$$P(B) = 0.25 \quad P(W) = 0.75 \quad P(L|B) = 0.48 \quad P(L|W) = 0.83$$

$$P(B/L) = \frac{P(B) P(L|B)}{P(B) P(L|B) + P(W) P(L|W)} = \frac{0.25 \times 0.48}{(0.25 \times 0.48) + (0.75 \times 0.83)}$$

$$= \frac{0.12}{0.12 + 0.6225} = \frac{0.12}{0.7425} \approx 0.162$$

- 4 Product is produced in factories X, Y, Z. X produces twice as many items as Y and Y, Z produce the same number of items. 3% of items produced by X, Z are defective while 5% of Y are defective. All items produced in the 3 factories are stored and an item is selected at random.

i) What is the probability that the item is defective.

ii) If an item selected at random is defective, what's the prob. that it was produced by factory X, Y and Z respectively.

X : product produced in factory X $P(X) = 3/5 = 0.6$

Y : product produced in factory Y $P(Y) = 1/5 = 0.2$

Z : product produced in factory Z $P(Z) = 1/5 = 0.2$

D: product is defective

$$P(D/X) = 0.03 \quad P(D/Y) = 0.03 \quad P(D/Z) = 0.05$$

$$P(D) = P(D \cap X) + P(D \cap Y) + P(D \cap Z) \\ = P(X) \cdot P(D/X) + P(Y) \cdot P(D/Y) + P(Z) \cdot P(D/Z) \\ = 0.018 + 0.010 + 0.006 = 0.034$$

$$P(X/D) = \frac{P(X \cap D)}{P(D)} = \frac{0.018}{0.034} \rightarrow P(X) \cdot P(D/X) = 0.052$$

$$P(Y/D) = \frac{P(Y \cap D)}{P(D)} = \frac{P(Y) \cdot P(D/Y)}{P(D)} = \frac{0.2 \times 0.05}{0.034} = 0.294$$

$$P(Z/D) = \frac{P(Z \cap D)}{P(D)} = \frac{P(Z) \cdot P(D/Z)}{P(D)} = \frac{0.2 \times 0.03}{0.034} = 0.176$$

5 3 candidates for Principal's posⁿ : X, Y, Z

chances of getting appointment = 4:2:3

Probability that X is selected who would introduce co-ed is 0.3

Prob. of Y and Z doing the same are 0.5 and 0.8

i) What's the prob. that there will be co-ed in college

ii) If we ~~know~~ know that there is co-ed in college, find probability of each X, Y, Z become the principal.

→ X: chance of getting appointment by X

$$P(X) = 4/9 = 0.44$$

Y: chance of getting appt. by Y

$$P(Y) = 2/9 = 0.22$$

Z: chance of getting appt. by Z

$$P(Z) = 3/9 = 0.33$$

C: introducing co-ed

$$P(C/X) = 0.3$$

$$P(C/Y) = 0.5$$

$$P(C/Z) = 0.8$$

$$P(C) = P(C \cap X) + P(C \cap Y) + P(C \cap Z)$$

$$= P(X) \cdot P(C/X) + P(Y) \cdot P(C/Y) + P(Z) \cdot P(C/Z)$$

$$= 0.44 \times 0.3 + 0.22 \times 0.5 + 0.33 \times 0.8 = 0.506$$

$$P(X/C) = \frac{P(X) \cdot P(X \cap C)}{P(C)} = \frac{0.44 \times 0.3}{0.506} = 0.26$$

$$P(Y/C) = \frac{P(Y) \cdot P(Y \cap C)}{P(C)} = \frac{0.22 \times 0.5}{0.506} = 0.22$$

$$P(Z/C) = \frac{P(Z) \cdot P(Z \cap C)}{P(C)} = \frac{0.33 \times 0.8}{0.506} = 0.52$$