ITC Mod 4

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- **Single error correcting Hamming Codes**

  $[2^{n-k} - 1]$ no. of distinct rows.

  $H = [P^T \,\vdots\, I_{n-k}]$ $\qquad H^T = \begin{bmatrix} P \\ \hdashline I_{n-k} \end{bmatrix}$

  $2^{n-k} - 1 \geq n$ or $n-k \geq \log_2 (n+1)$

  or $k \leq n - \log_2 (n+1)$

  Code length : $n \leq 2^{n-k} - 1$

  No. of Msg bits : $k \leq n - \log_2 (n+1)$

  No. of Parity Check bits : $(n-k)$

  For correcting capability : $t = (d_{min} - 1)/2$

- **Hamming Bound**

  total no. of syndromes $2^{n-k} \geq \sum_{i=0}^{t} {}^nC_i$ ← (n,k) LBC can correct upto 't' errors, then

  no. of possible error patterns.

  Perfect code : Binary code for which Hamming bound is an '='

- **Questions**

1. Consider code vectors $C_1 = 10010$ $C_2 = 01101$ $C_3 = 11001$

   Find i) $d(C_1, C_2)$ ii) $d(C_1, C_3)$ iii) $d(C_2, C_3)$

   and $d(C_1, C_2) + d(C_2, C_3) \geq d(C_1, C_3)$

   → i) $d(C_1, C_2) = 5$ ii) $d(C_1, C_3) = 3$ iii) $d(C_2, C_3) = 2$

   $d(C_1, C_2) + d(C_2, C_3) \geq d(C_1, C_3)$

   $5 + 2 \geq 3 = \boxed{7 \geq 3}$

2. For a (6,3) LBC, 3 parity check bits are formed using

   $C_4 = d_1 \oplus d_3$, $C_5 = d_1 \oplus d_2 \oplus d_3$, $C_6 = d_2 \oplus d_3$

   Write G, received code word = 111010

   Decode code word by finding location of error & transmitted data bits.

→ $P = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ $\qquad G = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{array}\right]$

$S = R H^T$

$= [111010]\, H^T \rightarrow \begin{bmatrix} P \\ \hline I_{n-k} \end{bmatrix}$

$= [000]$ no error as syndrome is zero

~~The transmitted~~

The transmitted and received bits are same

3 (6,3) LBC i) Find all code words

ii) Min^m weight of code

iii) Hamming distance

$P = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

→ $G = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$

2^k codes $= 2^3 = 8$

$C = [d_1 \quad d_2 \quad d_3 \quad d_1 + d_3 \quad d_2 + d_3 \quad d_1 + d_2]$

Message	Codewords	Min^m weight	Hamming distance
000	000000	0	3
001	001110	3	4
010	010011	3	3
011	011101	4	3
100	100101	3	3
101	101011	4	4
110	110110	4	3
111	111000	3	3

4 Parity check bits for (8, 4) LBC is

$C_5 = d_2 \oplus d_3 \oplus d_4$

$C_6 = d_1 \oplus d_2 \oplus d_4$

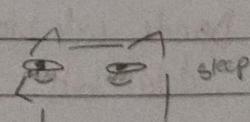
$C_7 = d_1 \oplus d_2 \oplus d_3$

$C_8 = d_2 \oplus d_3 \oplus d_4$

Find :- i) G } ez

ii) H

iii) Min^n weight



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→ $C = DG$ $\quad 2^4 = 16$ codewords

$C = [d_1, d_2, d_3, d_4, d_4 + d_2 + d_3 \ldots$ from question$]$

Message	Codewords	Min^m weight
0000	00000000	0
0001	00011101	4
0010	00101011	4
⋮	⋮	⋮
1111	11111111	8

$d_{min} = 4$

5 Design (n, k) hamming code with $d_{min} = 3$, message length $= 4$ bits ($\to k$). Find all code vectors, show that it can correct single errors.

→ given: $k = 4$ $\quad d_{min} = 3$

$n \le 2^{n-k} - 1$

if $n = 5$ $\quad 5 \le 2^1 - 1$ ✗

if $n = 6$ $\quad 6 \le 2^2 - 1$ ✗

if $n = 7$ $\quad 7 \le 2^3 - 1$ ✓

$n = 7$ ∴ min^m value $\quad (7,4)$ LBC

$H^T = \begin{bmatrix} [P] \\ \hline 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

2^{n-k} ∴ 8 combinations

000 ∴ cannot be used.

001, 010, 100 cannot be used as they are present in I

$P = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

$[C] = [D][G]$

$= [d_1, d_2, d_3, d_4, d_2 + d_3 + d_4, d_1 + d_3 + d_4, d_1 + d_2 + d_4]$

Calculate $[D]$ and $[C]$ for 0 − 15

message vector ↙ $\quad$ code vector ↙

$t = \dfrac{d_{min} - 1}{2} = \dfrac{3 - 1}{2} = 1$ ∴ to show that it can correct single error d_{min}

take $R\,[1111001]$, $\quad S = [110]$ $\quad$ error is in 3rd row, so we need to correct it

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2 Consider (7, 4) Hamming code with $d_{min} = 3$, show that its a perfect code.

→ $2^{n-k} \geq \sum_{i=0}^{t} {}^nC_i$ where $t = \frac{d_{min}-1}{2} = 1$

$2^{7-4} \geq {}^7C_0 + {}^7C_1$

$2^3 \geq 1 + 7$

$8 \geq 8 \longrightarrow 8 = 8$

(7,4) is a perfect code

3 Given $M(x) = 10111011$ → polynomial can be given, $G(x) = 1001$

Determine the message to sent, or check if it has errors & correct it, using CRC

Cyclic Redundancy Check

→

```
        1 1 1              extra bits for error detection &
                                       correction
1001 | 10111 011 000 |
       1001      110
        1010
        1001
         1111
         1001
          1100
          1001
           1010
           1001
           0110   CRC bit
```

Divide 10111011110 by 1001, if remainder is 0, then its correct? idk