- **Random Variable :-**
A variable used to denote the numerical value of the outcome of an experiment is called random variable (r.v)
It's denoted by capital letters X, Y, Z . and its values are denoted by $x_1, x_2 \ldots, y_1, y_2 \ldots, z_1, z_2 \ldots$ etc.
If it takes discrete (distinct) values $x_1, x_2, \ldots x_n$, its called discrete random variable.
If it takes all values b/w interval (a, b), its continuous random variable. Example: age, height, weight.

- **Probability Distribution of Discrete r.v**
Let X = r.v (discrete)
Let $x_1, x_2, \ldots, x_n$ be possible values of X. With each possible outcome x_i, we associate a number $p(x_i) = p(X = x_i) = p_i$ called the probability of x_i.
The no.s $p(x_i)$ must satisfy :-
$p(x_i) \geq 0$ for all i $\qquad \sum p(x_i) = 1$
It is probability mass function (pmf) or prob. density function (p.d.f) of random variable X and set of pairs (x_i, p_i)

X :	x_1	x_2	x_3	...	x_n
$P(X = x_i)$:	p_1	p_2	p_3	...	p_n

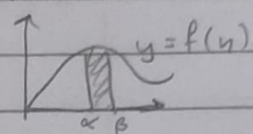
- **Distribution Function of a discrete r.v. X :-**
Consider F defined by $F(x_i) = P(X \leq x_i)$, $i = 1, 2, 3 \ldots$
i.e. $F(x_i) = P(x_1) + P(x_2) + \ldots P(x_i)$, then the F is cumulative or simply distribution function.
The set of pairs $(x_i, F(x_i))$ is called the cumulative probability distribution.

X	x_1	x_2	x_3	...	x_n
$F(x_i) = F(X = x_i)$	$P(x_1)$	$\sum_{i=1}^{2} P(x_i)$	$\sum_{i=1}^{3} P(x_i)$	...	$\sum_{i=1}^{n} P(x_i)$

The function F is called function distribution.

- Probability Density Function of a Continuous r.v

A continuous fn. $y = f(x)$ such that :- i) $f(x)$ is integrable
ii) $f(x) \geq 0$ iii) $\int_a^b f(x)\, dx = 1$ if x lies in $[a, b]$ and
iv) $\int_\alpha^\beta f(x)\, dx = P(\alpha \leq X \leq \beta)$ where $a < \alpha < \beta < b$
is called prob. density function of a continuous r.v. X.
The curve given by $y = f(x)$ is called prob. density curve.
The expression $f(x)\, dx$ is denoted by $d f(x)$ and is known as prob. differential



For discrete r.v the probability at $x = c$ may not be zero.
For continuous r.v. the prob at $x = c$ is always zero because $P(X = c) = \int_c^c f(x)\, dx$
For continuous r.v. X :-
$P(\alpha \leq X \leq \beta) = P(\alpha < X < \beta) = P(\alpha < X \leq \beta) = P(\alpha \leq X \leq \beta)$

- Continuous Distribution Function :-

If X is a continuous random variable X, having the probability density function $f(x)$ then the function
$F(x) = P(X \leq x) = \int_{-\infty}^{\infty} f(t)\, dt$, $-\infty < x < \infty$
is called distribution function or cumulative distribution function of r.v. X.

- Expectation of a Random Variable :-

$E(X) = P_1 X_1 + P_2 X_2 + \dots + P_n X_n = \boxed{\sum P_i X_i}$
mean of X where $\sum P_i = 1$
Let X be continuous, then $\boxed{E(X) = \int_{-\infty}^{\infty} x \cdot f(x)\, dx}$
where $\int_{-\infty}^{\infty} f(x)\, dx = 1$

- Laws of Expectation

X : discrete rv where $x_i \geq 0$, then $E(X) \geq 0$
X : discrete or continuous, a & b are constants, then $E(aX + b) = aE(X) + b$
$E(b) = b$ when $a = 0$, $b = 0$ then $E(aX) = aE(X)$
$a = 1$, $b = -\bar{x}$ then $E(X - \bar{x}) = 0$

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Theorem of Add^n : $E(x \pm y) = E(x) \pm E(y)$

Theorem of Multiplication : $E(xy) = E(x) \cdot E(y)$

- Variance :-

$Var(x) = E(x-\bar{x})^2 = E[x - E(x)]^2$

$= E[x^2 - 2x E(x) + \{E(x)\}^2]$

$= E[x^2 - 2E(x) \cdot E(x) + [E(x)]^2]$

$\boxed{Var(x) = E(x^2) + [E(x)]^2}$

Properties :-

Variance of ~~conts~~ constant $V(c) = 0$

~~$V(ax) = a^2$~~

If x is a random variable and a, b are constants

then, $V(ax+b) = a^2 V(x)$

$V(ax) = a^2 V(x)$

$V(x+b) = V(x)$

Note: $E(ax+b) = aE(x) + b$

$V(ax+b) = a^2 V(x)$

$V(a_1 x_1 + a_2 x_2) = a_1^2 V(x_1) + a_2^2 V(x_2)$

where x_1 and x_2 are independent r.v

If $a_1 = 1, a_2 = 1$, $V(x_1 + x_2) = V(x_1) + V(x_2)$

If $a_1 = 1, a_2 = -1$, $V(x_1 - x_2) = V(x_1) + V(x_2)$