- **Binomial Distribution**

success : p failure : q no. of trials ? : n

$p + q = 1$ $P(X = x) = {}^nC_x \, p^x q^{n-x}$

Sum of probabilities :-

$$\sum_{x=0}^{n} p(x) = \sum_{x=0}^{n} {}^nC_x \, p^x q^{n-x} = (p+q)^n = 1$$

Frequency function :-

$$N \times P(X = x)$$

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Mean = np Variance = npq mode = $\lfloor(n+1)p$ or $\approx (n+1)p$

Write the "∴" for long word problems

- Questions:-

1 Mean = 3 Variance = 1.2 of binomial variate

Find n, p and $P(X<4)$

no need to solve for each part

$P(x<4)$ in calc:- $\sum_{x=0}^{3} {}^5C_x (0.6)^x (0.4)^{5-x}$

→ mean = $np = 3$ Variance = $npq = 1.2$

$npq/np = 1.2/3 \longrightarrow q = 0.4$ $p = 1-q = 0.6$

$n \times 0.6 = 3 \rightarrow n = 5$ $P(X=x) = {}^nC_x p^x q^{n-x}$

$P(x<4) = 1 - P(x \geq 4) = 1 - \{P(x=4) + P(x=5)\}$

$= 1 - \{ {}^5C_4 p^4 q^{n-4} + {}^5C_5 p^5 q^0 \} = 0.66304$

2 Mean = 4 Variance = 3. Find binomial distribution & mode

→ $npq/np = 3/4 \longrightarrow q = 0.75$ $p = 1-0.75 = 0.25$

$np = 4 \longrightarrow n = 16$ mode $= (n+1)p = 17 \times (0.25) = 4.25$

mode = floor of $4.25 = \lfloor 4.25 = 4$

3 x: binomial distribution $E(x) = 2$ $var(x) = 4/3$

Find probability distribution of x.

→ $npq/np = 4/3/2 = 2/3 = q$ $p = 1 - 2/3 = 1/3$

$np = 2 \longrightarrow n = 6$

$P(x=x) = {}^nC_x p^x q^{n-x} = {}^6C_x (1/3)^x (2/3)^{6-x} \nearrow \sum_{x=0}^{6} {}^6C_x (1/3)^x (2/3)^{6-x}$ in calc

$= {}^6C_0 (1/3)^0 (2/3)^6 + {}^6C_1 (1/3)^1 (2/3)^5 + \dots + {}^6C_6 (1/3)^6 (2/3)^0$

4 x: Binomial variate satisfies $9P(x=4) = P(x=2)$, $n=6$. Find p

→ $n=6$ $9P(x=4) = P(x=2) \longrightarrow 9\,{}^6C_4 p^4 q^{6-4} = {}^6C_2 p^2 q^{6-2}$

$9 \times 15 p^2 q^{-2} = 15 \longrightarrow 9p^2(1-p)^{-2} = 1 \longrightarrow 9p^2 = 1 + p^2 - 2p$

$8p^2 + 2p - 1 = 0 \rightarrow p = 1/4 = 0.25$ $q = 1-p = 0.75$

5 If 10 fair coins are tossed simultaneously, what's the chance of getting at least 7 heads

→ $n = 10$ trial has 2 outcomes → p (success) $= 1/2$
→ q (failure) $= 1-p = 1/2$

X: no. of heads. $P(X=x) = {}^nC_x p^x q^{n-x}$

$P(x \geq 7) = P(x=7) + P(x=8) + P(x=9) + P(x=10)$

$= {}^{10}C_7 p^7 q^3 + {}^{10}C_8 p^8 q^2 + {}^{10}C_9 p^9 q^1 + {}^{10}C_{10} p^{10} q^0$

$= 0.171$

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6 The odds in favour of X's winning a game against Y are 4:3
Find prob. of Y winning 3 out of 7 games.

→ Y: Y wins the game $p = 3/7$ $q = 1-p = 4/7$ $n = 7$

$P(x=3) = {}^7C_3 (3/7)^3 (4/7)^4 = 0.293$

7 In a binomial distribution consisting of 5 independent trials, prob. of 1 and 2 successes are 0.4096 and 0.2048. Find p.

→ $n = 5$ $P(x=1) = 0.4096$ $P(x=2) = 0.2048$

$P(X=x) = {}^nC_x \, p^x q^{n-x}$

$P(x=1) = {}^5C_1 \, p q^4 = 0.4096$

$P(x=2) = {}^5C_2 \, p^2 q^3 = 0.2048$

$\frac{P(x=1)}{P(x=2)} = \frac{5pq^4}{10p^2q^3} = \frac{0.4096}{0.2048} = 2$

$q/p = 2 \times 2 \rightarrow q = 4(1-q)$

$5q = 4 \rightarrow q = 4/5 = 0.8$ $p = 1-q = 0.2$

8 If hens of a certain breed lay eggs on 5 days a week on average; find on how many days during a season of 100 days, a poultry keeper with 5 hens of this breed, will expect to receive at least 4 eggs.

→ $p = 5/7$ $q = 2/7$ $n = 5$ $N = 100$

$P(x \geq 4) = P(4) + P(5) = {}^5C_4 \, p^4 q^1 + {}^5C_5 \, p^5 q^0 = 0.557$

$NP(x \geq 4) = 100 \times 0.557 = 55.7 \approx 56$

9 A communication system consists of n components, each of which function independently with probability p. The total system will be able to function effectively if at least one-half of its components are functioning. For what value of p is a 5-component system more likely to operate effectively than a 3-component system.

→ $n = 5$ $n = 3$

$P(X \geq 3) \geq P(X \geq 2)$

$P(3) + P(4) + P(5) \geq P(2) + P(3)$

$({}^5C_3 p^3 q^2 + {}^5C_4 p^4 q + {}^5C_5 p^5) \geq ({}^3C_2 p^2 q + {}^3C_3 p^2)$

$10p^2 - 20p^4 + 10p^5 + 5p^4 + p^5 - 5p^5 + 3p^3 + p^3 - 3p^2 \geq 0$

$6p^5 - 15p^4 + 12p^3 - 3p^2 \geq 0 \rightarrow p^2(6p^3 + 12p - 15p^2 - 3) \geq 0$

$p^2(p-1)^2(2p-1)$ if $(2p-1) \geq 0 \rightarrow p \geq 1/2$

∴ For $p \geq 1/2$, a 5-component system is more likely to operate effectively

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10 Out of 800 families with 5 children each, how many would you expect to have i) 3 Boys and 2 Girls ii) 5 girls iii) 5 boys?

→ $n=5$ $N=800$ X: no. of girls $p=1/2$ $q=1/2$

i) $P(x=2) = {}^5C_2 p^2 q^3 = 10(1/2)^2 (1/2)^3$

freq $= N(P(x=2)) = 800 \times 10 \times 1/2^5 = 200$

ii) $P(x=5) = {}^5C_5 (0.5)^5 (0.5)^0 = 1/32$

freq $= 1/32 \times 800 = 25$

iii) $P(x=0) = {}^5C_0 (0.5)^0 (0.5)^5 = 1/32$

freq $= 1/32 \times 800 = 25$

11 MCQs. 20 questions. 1 correct option out of 4. Correct = +4 marks, wrong: -1 mark. Student must secure at least 50% of maximum possible marks to pass the examination. Suppose student hasn't studied at all, so that he answers the questions by guessing only what's the probability that he'll pass the exam.

→ 20 Q × 4 M = 80 M exam

To pass, secure 50% marks = 40M

Borderline: 12 correct + 8 wrong = $12\times4 - 8 = 40$ M

X: correct answer

$P(X \geq 12)$ or $P(Y \leq 8)$

$p = 1/4$, $q = 3/4$ $p = 3/4$, $q = 1/4$

$P(Y \leq 8) = P(Y=0) + P(Y=1) + \dots + P(Y=8)$

$= {}^{20}C_0 (3/4)^0 (1/4)^{20} + \dots + {}^{20}C_8 (1/4)^{12} (3/4)^8$

$= 9.35 \times 10^{-4}$

12 An irregular 6 faced dice is thrown & prob. that in 20 throws it'll give 5 even no.s is twice the prob that it'll give 5 odd no.s. How many times in 10000 sets of 10 throws would you expect it to give no even number.

→ $n = 20$ X: getting even number

$P(X=5) = 2P(X=15)$

find p, then $n=10$, $N=10000$, $q=1-p$

$p = 1/4$ $q = 3/4$ $E(X) = np = 10 \times 1/4 = 2.5$

Freq $= 2.5 \times 10000 = 25000$

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13 Let X, Y be two independent binomial variates with parameters $(n_1 = 6, p = 1/2)$ and $(n_2 = 4, p = 1/2)$ respectively.
Find $P(X+Y=3)$, $P(X+Y \geq 3)$

→ $X : (n_1, p)$ $\quad$ $X_2 : (n_2, p)$ $\quad$ ${}^nC_x \, p^x q^{n-x}$

$X+Y \to (n_1+n_2, p)$ ∵ Additive property

$X+Y \to (10, 1/2)$ $\quad$ $p = 1/2 \quad q = 1-p = 1/2$

$P(X+Y=3) = {}^{10}C_3 \cdot (1/2)^3 (1/2)^7$

$P(X+Y \geq 3) = 1 - P(X+Y \leq 3) = 1 - \{P(0) + P(1) + P(2)\}$

$= 1 - \{{}^{10}C_0 (1/2)^0 (1/2)^{10} + {}^{10}C_1 (1/2)^1 (1/2)^9 + {}^{10}C_2 (1/2)^2 (1/2)^8\}$

$=$

14 Five dice are thrown together 96 times. The no of times 4, 5 or 6 was obtained is given below

No. of times :	0	1	2	3	4	5
Frequency :	1	10	24	35	18	8

Fit in a Binomial distribution if :-
i) dice are unbiased
ii) the nature of the dice is not known

→ $n = 5 \quad p = 1/2 \quad q = 1/2 \quad N = 96$

i) $P(X=0) = {}^5C_0 (1/2)^0 (1/2)^5 = 1/2^5$
Freq $= N \times P(X=0) = 96 \times 1/2^5 = 3$
} Do the same for other numbers.

X :	0	1	2	3	4	5
$P(X=x)$:	$1/2^5$	$5/2^5$	$10/2^5$	$10/2^5$	$5/2^5$	$1/2^5$
$N \times P(X=x)$:	$96 \times 1/2^5 = 3$	$3 \times 5 = 15$	$10 \times 3 = 30$	30	15	3

ii) check mean : $\bar{x} = \Sigma f_i x_i / \Sigma f_i = 275/96 = 2.8645$

Binomial : $\bar{x} = np \to p = 2.8645/5$

$p = 0.5729 \approx 0.57 \quad q = 0.43$

Calculate the table for this too

15 A biased coin is tossed 5 times and the whole exp. is repeated 230 times. The following frequencies of 0, 1, 2 heads were obtained.

No. of heads :	0	1	2	3	4	5	Total
Frequency :	12	56	74	69	18	1	230

Fit a binomial distribution and find the theoretical frequencies. Also find the mean and variance of the fitted distribution

→ $\bar{x} = \sum f_i x_i / \sum f_i = 488/230 = 2.12$

$\bar{x} = np \longrightarrow p = 2.12/5 = 0.424 \approx 0.42$

$q = 1-p = 0.58$ Make the table (similar to 15)

- Poisson Distribution :-

$P(X=x) = \dfrac{e^{-m} m^x}{x!}$ $x = 0, 1, 2 \ldots$

variance = mean = m

$$\left.\begin{aligned} P(x+1) &= \frac{n-x}{x+1} \frac{p}{q} P(x) \\ F(x+1) &= \frac{n-x}{x+1} \frac{p}{q} F(x) \end{aligned}\right\} \text{Binomial}$$

Poisson :- $P(x+1) = m/(x+1) \cdot P(x)$

$F(x+1) = \dfrac{m}{x+1} F(x)$

- Questions

1 The mean and variance of p.d. is 2. Write Probability Distribution

→ $m = 2$ $P(X) = \dfrac{e^{-m} m^x}{x!}$

$P(X=k) = e^{-2} 2^k / k!$

2 In a Poisson distribution, the prob. $P(X=3)$ is 2/3 of $P(X=4)$. Find mean and standard deviation

→ mean = variance = m

$P(X=3) = 2/3 P(X=4) \longrightarrow \dfrac{e^{-m} m^3}{3!} = \dfrac{2}{3} \dfrac{e^{-m} m^4}{4!}$

$m = 36/6 = 6 = \sigma^2$ $S.D = \sqrt{\sigma^2} = \sqrt{6}$

3 Mean of Poisson distribution is 4. Find $P(m-2\sigma < x < m+2\sigma)$

→ $m = 4 = var \qquad \sigma = \sqrt{var} = 2$

$P(m-2\sigma < x < m+2\sigma) \rightarrow P(0 < x < 8)$

$= P(1) + P(2) + P(3) \ldots P(8)$

$= \frac{e^{-4}4^1}{1!} + \frac{e^{-4}4^2}{2!} + \frac{e^{-4}4^3}{3!} + \ldots + \frac{e^{-4}4^7}{7!} = e^{-4} \times 9a8063$

$= 0.930$

4 Poisson distribution : $P(x=2) = 9P(x=4) + 90P(x=6)$

Find mean and variance of x.

→ $\frac{e^{-m}m^x}{x!} = P(x=x) \rightarrow \frac{e^{-m}m^2}{2!} = 9\frac{e^{-m}m^4}{4!} + 90\frac{e^{-m}m^6}{6!}$

$m = 0, 1.34$

5 In a Poisson distribution, the prob $p(x)$ for $x=0$ is 20%. Find mean.

→ $P(x=0) = 20\% = 0.2$

$e^{-m}m^0/0! = 0.2 \rightarrow e^{-m} = 0.2$

$\log_e 0.2 = -m = 1.609$

6 If x is a Poisson variable and $P(x=0) = 6P(x=3)$ Find $P(x=2)$

→ $\frac{e^{-m}m^0}{0!} = 6 \times \frac{e^{-m}m^3}{3!} \rightarrow \frac{3\times 2}{6} = m^3 \rightarrow m = 1$

$P(x=2) = \frac{e^{-m}1^2}{2!} = 0.367$

7 If X and Y are independent Poisson variates such that $P(x=1) = P(x=2)$ and $P(Y=2) = P(Y=3)$, find variance of $3X - 4Y$.

→ for X :- $P(x=1) = P(x=2)$

$\frac{e^{-m_1}m_1^1}{1!} = \frac{e^{-m_1}m_1^2}{2!} \rightarrow m_1 = 2$

for Y :- $P(Y=2) = P(Y=3)$

$\frac{e^{-m_2}m_2^2}{2!} = \frac{e^{-m_2}m_2^3}{3!} \rightarrow m_2 = 3$

$\boxed{V(\alpha X + \beta Y) = \alpha^2 V(X) + \beta^2 V(Y)}$ Formula

$V(3X - 4Y) = (3)^2 \times 2 + (-4)^2(3)$

$= 66$

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8 If X_1, X_2, X_3 are three independent Poisson variates with parameters $m_1 = 1, m_2 = 2, m_3 = 3$ respectively.
find i) $P[(X_1 + X_2 + X_3) \geq 3]$ ii) $P[(X_1 + X_2 + X_3) \leq 3]$
iii) $P[X_1 = 1 / (X_1 + X_2 + X_3) = 3]$

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→ $X_1 : m_1 = 1 \quad X_2 : m_2 = 2 \quad X_3 : m_3 = 3$

$X_1 + X_2 + X_3$ is a Poisson variate with parameter $m_1 + m_2 + m_3$

$Y : X_1 + X_2 + X_3 \rightarrow m = 1+2+3 = 6$

i) $P(X_1 + X_2 + X_3 \geq 3) = 1 - P(X < 3) = 1 - \{P(0) + P(1) + P(2)\}$

$= 1 - \left\{ \frac{e^{-6} 6^0}{0!} + \frac{e^{-6} 6^1}{1!} + \frac{e^{-6} 6^2}{2!} \right\} = 0.938$

ii) $P(X_1 + X_2 + X_3 < 3) = P(X < 3) + P(X = 3)$ Calculate on own

iii) $P(A/B) = \frac{P(A \cap B)}{P(B)}$ $\quad A : (X_1 = 1)$ $\quad B : (Y = 3)$

$P(X_1 = 1 / Y = 3) = \frac{P(X_1 = 1) \cap P(X_1 + X_2 + X_3)}{P(Y = 3)} = \frac{25}{72} = 0.347$

9 A car hire firm has 2 cars which it hires out day by day. The number of demands for a car on each day is distributed as Poisson variate with mean 1.5
Calculate the proportion of days on which:-
i) neither car is used ii) some demand is refused.

→ X : demands $\quad m = 1.5$

i) No demands : $P(X=0) = e^{-m} m^x / x! = e^{-1.5} (1.5)^0 / 0! = 0.223$

ii) $P(X > 2) = 1 - P(X \leq 2) = 1 - \{P(0) + P(1) + P(2)\}$

$= 1 - \left\{ \frac{e^{-1.5} 1.5^0}{0!} + \frac{e^{-1.5} 1.5}{1!} + \frac{e^{-1.5} 1.5^2}{2!} \right\} =$ Calculate

10 A hospital switch bard receives an average of 4 emergency calls in a 10 minutes interval. What is the probability that:-
i) there are at least 2 emergency calls
ii) there are exactly 3 emergency calls in interval of 10 minutes

→ X : Calls $\quad m = 4$ $\quad$ i) $P(X \geq 2) = 1 - P(X < 2)$

$= 1 - \{P(0) + P(1)\}$

$= 1 - \left\{ \frac{e^{-4} 4^0}{0!} + \frac{e^{-4} 4^1}{1!} \right\} = 0.908$

ii) $P(X=3) = e^{-4}\, 4^3/3! =$ Calculate //

11 An insurance company found that only 0.01% of the populat^n is involved in a certain type of incident each year. If its ~~certain that~~ 1000 policy holders were randomly selected from the population, what is the prob. that no more than 2 of its clients are involved in such accident next year?

→ accident : $p = 0.01/100 = 0.0001$ $n = 1000$

average $m = np = 1000 \times (0.0001) = 0.1$

$P(X \le 2) = P(0) + P(1) + P(2)$

$$= \frac{e^{-0.1}\, 0.1^0}{0!} + \frac{e^{-0.1}\, 0.1^1}{1!} + \frac{e^{-0.1}\, 0.1^2}{2!} =$$

12 Find prob that at most 4 defective bulbs will be found in a box of 200 bulbs if it is known that 2% of the bulbs are defective

→ $p = 2\% = 0.02$ $n = 200$

$P(X \le 4) = P(0) + P(1) + P(2) + P(3) + P(4)$

13 In sampling a large no. of parts manufactured by a machine, the mean members of defectives in a sample of 20 is 2. Out of 1000 such samples, how many would you expect to contain 3 defectives using i) Binomial distribution ii) Poisson distribution

→ $m = 2$ $n = 20$ $N = 1000$

$m = np = 2 \rightarrow p = 2/n = 0.1$ $q = 1-p = 0.9$

i) $P(X=3) = {}^nC_x\, p^x\, q^{n-x} = {}^{20}C_3\, (0.4)^3\, (0.9)^{17} =$

Frequency $= 1000 \times P(X=3) =$

ii) $$P(X=3) = \frac{e^{-m}\, m^x}{x!} = \frac{e^{-2}\, 2^3}{3!} =$$