

(A) Imagine a furniture company that makes tables and chairs. A table requires 40 board feet of wood and a chair requires 30 board feet of wood. Wood costs \$1 per board foot and 40,000 board feet of wood are available. It takes 2 hours of skilled labor to make an unfinished table or an unfinished chair. Three more hours of labor will turn an unfinished table into a finished table; two more hours of skilled labor will turn an unfinished chair into a

finished chair. There are 6000 hours of skilled labor available. (Assume that you do not need to pay for this labor.)

The prices of output are given in the table below:

Product	Price
Unfinished table	\$70
Finished table	\$140
Unfinished chair	\$60
Finished chair	\$110

Answers:

1.) Formulate an LP that describes the production plans that the firm can use to maximize its profits.

A.)  $x_1$  = number of unfurnished tables

$x_2$  = number of furnished tables

$x_3$  = number of unfurnished chairs

$x_4$  = number of furnished chairs

Maximize  $z = 70x_1 + 140x_2 + 60x_3 + 110x_4$

subject to

$40x_1 + 40x_2 + 30x_3 + 30x_4 \leq 40000$ .

$2x_1 + 5x_2 + 2x_3 + 4x_4 \leq 6000$ .

Objective	$x_1$	$x_2$	$x_3$	$x_4$			
	70	140	60	110			
z	0	0	0	1333.333	146666.7		
c1	40	40	30	30	40000	<=	40000
c2	2	5	2	4	5333.333	<=	6000

## Sensitivity Analysis Report:

### Variable Cells

Cell Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
\$G\$3 z x1	0	-76.66666667	70	76.66666667	1E+30
\$H\$3 z x2	0	-6.666666667	140	6.666666667	1E+30
\$I\$3 z x3	0	-50	60	50	1E+30
\$J\$3 z x4	1333.333333	0	110	1E+30	5

### Constraints

Cell Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$K\$5 c1	40000	3.666666667	40000	5000	40000
\$K\$6 c2	5333.333333	0	6000	1E+30	666.6666667

2.) What would happen if the price of unfinished chairs went up?

A.) Currently they are selling for \$60. Allowable increase in the coefficient is \$50, so it is not profitable to produce them even if they sold for the same amount as finished chairs.

3.) What would happen if the price of unfinished tables went up?

A.) The allowable increase is greater than \$70(70+76.6667). At current prices we have zero production of finished tables. There won't be any profit.

4.) What if the price of finished chairs fell to \$100?

A.) If the price of finished chairs fell to \$100. Since the allowable decrease of this is \$105. So, there exist a new optimal solution

5.) How would profit change if lumber supplies changed?

A.) The shadow price of the lumber constraint is \$3.6667. The range of values for which the basis remains unchanged is 0 to 45,000.

6.) How much would you be willing to pay an additional carpenter?

A.) Out of 6000 working hours, we are utilizing only 5333.333 hours. Hence, the allowable decrease is 666.666.

**7.)** Suppose that industrial regulations complicate the finishing process, so that it takes one extra hour per chair or table to turn an unfinished product into a finished one. How would this change your plans?

**A.)** If we change the finishing process by 1 unit each. The new constraint will be  $2x_1 + 6x_2 + 2x_3 + 5x_4 \leq 6000$ . So, the maximum profit will change to \$140000. It is comparatively less. So, spending extra time for finishing process is reducing the total profit and now we started producing 1000 finished tables from 0 tables and stopped producing 1333 finished chairs.

**8.)** The owner of the firm comes up with a design for a beautiful hand-crafted cabinet. Each cabinet requires 250 hours of labor (this is 6 weeks of full time work) and uses 50 board feet of lumber. Suppose that the company can sell a cabinet for \$200, would it be worthwhile?

**A.)**

For making hand crafted-cabinet								
	x1	x2	x3	x4	x5			
	70	140	60	110	200			
z	0	0	0	1328.767	2.739726	146712.3		
c1	40	40	30	30	50	40000	<=	40000
c2	2	5	2	4	250	6000	<=	6000

Therefore, after adding 200 additional profit and 50 foot woods and 250 hours. We got \$146712.3 profit. There is an increase in profit. So, making the hand-crafted cabinet is a good-idea.