Professor McDonald FNCE 5352 – Financial Programming and Modeling March 13, 2019 Solutions to March 5 Assignment

5.2.4 Exercise 1

Find all flights that

- 1. Had an arrival delay of two or more hours
- 2. Flew to Houston (IAH or HOU)
- 3. Were operated by United, American, or Delta
- 4. Departed in summer (July, August, and September)
- 5. Arrived more than two hours late, but didn't leave late
- 6. Were delayed by at least an hour, but made up over 30 minutes in flight
- 7. Departed between midnight and 6 am (inclusive)

The answer to each part follows.

1. Since the arr_delay variable is measured in minutes, find flights with an arrival delay of 120 or more minutes.

```
filter(flights, arr_delay >= 120)
#> # A tibble: 10,200 x 19
    year month day dep_time sched_dep_time dep_delay arr_time
  <int> <int> <int> <int> <int> <dbl>
                                                 <int>
                     811
#> 1 2013 1 1
                                   630
                                            101
                                                    1047
#> 2 2013
           1
                1
                       848
                                   1835
                                            853
                                                    1001
#> 3 2013
           1
                1
                       957
                                    733
                                             144
           1
#> 4 2013
                1
                     1114
                                    900
                                             134
                                                    1447
#> 5 2013
            1
                 1
                      1505
                                    1310
                                             115
                                                    1638
#> 6 2013
               1
                                   1340
           1
                       1525
                                             105
                                                    1831
#> # ... with 1.019e+04 more rows, and 12 more variables: sched arr time <int>,
      arr_delay <dbl>, carrier <chr>, flight <int>, tailnum <chr>,
      origin <chr>, dest <chr>, air time <dbl>, distance <dbl>, hour <dbl>,
      minute <dbl>, time_hour <dttm>
```

2. The flights that flew to Houston are those flights where the destination (dest) is either "IAH" or "HOU".

```
filter(flights, dest == "IAH" | dest == "HOU")
#> # A tibble: 9,313 x 19
    year month day dep time sched dep time dep delay arr time
    <int> <int> <int> <int> <int> <int> <int>
#> 1 2013 1
                 1
                       517
                                   515
                                             2
                                                     830
#> 2 2013
            1
                 1
                       533
                                    529
                                               4
                                                     850
#> 3 2013
            1
                 1
                                    627
                                                    933
                        623
                                              -4
#> 4 2013
            1
                 1
                                    732
                                              -4
                        728
                                                    1041
#> 5 2013
            1
                 1
                        739
                                    739
                                              0
                                                   1104
#> 6 2013
           1
                 1
                        908
                                    908
                                                    1228
#> # ... with 9,307 more rows, and 12 more variables: sched arr time <int>,
#> # arr_delay <dbl>, carrier <chr>, flight <int>, tailnum <chr>,
```

```
#> # origin <chr>, dest <chr>, air_time <dbl>, distance <dbl>, hour <dbl>,
#> # minute <dbl>, time_hour <dttm>
```

However, using %in% is more compact and would scale to cases where there were more than two airports we were interested in.

```
filter(flights, dest %in% c("IAH", "HOU"))
#> # A tibble: 9,313 x 19
#>
     year month
                 day dep time sched dep time dep delay arr time
#>
    <int> <int> <int>
                       <int>
                                      <int>
                                               <dbL>
                                                          <int>
#> 1 2013 1 1
                           517
                                         515
                                                     2
                                                            830
#> 2 2013
                    1
                           533
                                          529
                                                            850
              1
#> 3 2013
                                          627
                    1
                           623
                                                     -4
                                                            933
              1
#> 4 2013
              1
                    1
                           728
                                          732
                                                     -4
                                                           1041
#> 5 2013
              1
                    1
                           739
                                          739
                                                     0
                                                           1104
#> 6 2013
                           908
                                          908
                                                           1228
              1
                    1
#> # ... with 9,307 more rows, and 12 more variables: sched arr time <int>,
#> # arr_delay <dbl>, carrier <chr>, flight <int>, tailnum <chr>,
      origin <chr>, dest <chr>, air_time <dbl>, distance <dbl>, hour <dbl>,
#> # minute <dbl>, time hour <dttm>
```

3. In the flights dataset, the column carrier indicates the airline, but it uses two-character carrier codes. We can find the carrier codes for the airlines in the airlines dataset. Since the carrier code dataset only has 16 rows, and the names of the airlines in that dataset are not exactly "United", "American", or "Delta", it is easiest to manually look up their carrier codes in that data.

```
airlines
#> # A tibble: 16 x 2
#>
    carrier name
#>
   <chr> <chr>
#> 1 9E
             Endeavor Air Inc.
#> 2 AA
            American Airlines Inc.
#> 3 AS
             Alaska Airlines Inc.
#> 4 B6
             JetBlue Airways
#> 5 DL
             Delta Air Lines Inc.
             ExpressJet Airlines Inc.
#> # ... with 10 more rows
```

The carrier code for Delta is "DL", for American is "AA", and for United is "UA". Using these carriers codes, we check whether carrier is one of those.

```
filter(flights, carrier %in% c("AA", "DL", "UA"))
#> # A tibble: 139,504 x 19
#>
     year month day dep_time sched_dep_time dep_delay arr_time
     <int> <int> <int>
#>
                       <int>
                                       <int>
                                                 <dbl>
                                                           <int>
#> 1 2013
              1
                    1
                           517
                                          515
                                                      2
                                                              830
                                          529
#> 2 2013
              1
                    1
                           533
                                                      4
                                                              850
#> 3 2013
              1
                    1
                            542
                                          540
                                                      2
                                                              923
#> 4 2013
                           554
                                          600
              1
                    1
                                                     -6
                                                              812
#> 5 2013
              1
                    1
                            554
                                          558
                                                      -4
                                                              740
#> 6 2013
              1
                    1
                           558
                                          600
                                                     -2
                                                              753
#> # ... with 1.395e+05 more rows, and 12 more variables: sched arr time <int>,
      arr_delay <dbl>, carrier <chr>, flight <int>, tailnum <chr>,
       origin <chr>, dest <chr>, air_time <dbl>, distance <dbl>, hour <dbl>,
      minute <dbl>, time_hour <dttm>
```

4. The variable month has the month, and it is numeric. So, the summer flights are those that departed in months 7 (July), 8 (August), and 9 (September).

```
filter(flights, month >= 7, month <= 9)</pre>
```

```
#> # A tibble: 86,326 x 19
     year month
                  day dep_time sched_dep_time dep_delay arr_time
                                                   <dbL>
     <int> <int> <int> <int>
                                         <int>
                                                            <int>
#> 1 2013
              7
                    1
                             1
                                          2029
                                                     212
                                                               236
#> 2 2013
               7
                     1
                             2
                                          2359
                                                       3
                                                              344
#> 3 2013
               7
                     1
                             29
                                          2245
                                                              151
                                                      104
               7
#> 4 2013
                     1
                             43
                                          2130
                                                      193
                                                               322
#> 5 2013
               7
                     1
                             44
                                                     174
                                                              300
                                          2150
#> 6 2013
              7
                     1
                             46
                                          2051
                                                     235
                                                              304
#> # ... with 8.632e+04 more rows, and 12 more variables: sched arr time <int>,
       arr_delay <dbl>, carrier <chr>, flight <int>, tailnum <chr>,
       origin <chr>, dest <chr>, air_time <dbl>, distance <dbl>, hour <dbl>,
       minute <dbl>, time_hour <dttm>
```

The **%in%** operator is an alternative. If the : operator is used to specify the integer range, the expression is readable and compact.

```
filter(flights, month %in% 7:9)
#> # A tibble: 86,326 x 19
     year month day dep time sched dep time dep delay arr time
#>
    <int> <int> <int>
                       <int>
                                         <int>
                                                   <dbl>
#>
                                                              236
#> 1 2013
              7
                                          2029
                                                     212
                    1
                              1
#> 2 2013
               7
                              2
                                          2359
                     1
                                                       3
                                                              344
#> 3 2013
               7
                     1
                             29
                                          2245
                                                     104
                                                              151
#> 4 2013
               7
                             43
                                          2130
                                                     193
                     1
                                                              322
#> 5 2013
               7
                     1
                             44
                                          2150
                                                     174
                                                              300
              7
#> 6 2013
                     1
                             46
                                          2051
                                                     235
                                                              304
#> # ... with 8.632e+04 more rows, and 12 more variables: sched_arr_time <int>,
#> # arr_delay <dbl>, carrier <chr>, flight <int>, tailnum <chr>,
       origin <chr>, dest <chr>, air time <dbl>, distance <dbl>, hour <dbl>,
      minute <dbl>, time_hour <dttm>
```

We could also use the | operator. However, the | does not scale to many choices. Even with only three choices, it is quite verbose.

```
filter(flights, month == 7 | month == 8 | month == 9)
#> # A tibble: 86,326 x 19
     year month day dep_time sched_dep_time dep_delay arr_time
#>
     <int> <int> <int>
                       <int>
                                         <int>
                                                   <dbl>
                                                            <int>
#>
#> 1 2013
                                          2029
                                                     212
                                                               236
                     1
                              1
                                                               344
               7
                              2
                                          2359
#> 2 2013
                     1
                                                       3
#> 3 2013
               7
                             29
                     1
                                          2245
                                                     104
                                                               151
#> 4 2013
               7
                     1
                             43
                                          2130
                                                     193
                                                               322
#> 5 2013
                     1
                             44
                                          2150
                                                     174
                                                               300
#> 6 2013
               7
                     1
                             46
                                          2051
                                                     235
                                                               304
#> # ... with 8.632e+04 more rows, and 12 more variables: sched_arr_time <int>,
      arr_delay <dbl>, carrier <chr>, flight <int>, tailnum <chr>,
       origin <chr>, dest <chr>, air time <dbl>, distance <dbl>, hour <dbl>,
       minute <dbl>, time_hour <dttm>
```

We can also use the between() function as shown in Exercise 5.2.2.

5. Flights that arrived more than two hours late, but didn't leave late will have an arrival delay of more than 120 minutes (arr_delay > 120) and a non-positive departure delay (dep_delay <= 0).

```
filter(flights, arr_delay > 120, dep_delay <= 0)
#> # A tibble: 29 x 19
#> year month day dep_time sched_dep_time dep_delay arr_time
#> <int> <int> <int> <int> <int><</pre>
```

```
#> 1 2013 1
                    27
                           1419
                                          1420
                                                             1754
#> 2 2013
              10
                     7
                           1350
                                          1350
                                                       0
                                                             1736
#> 3 2013
              10
                    7
                           1357
                                          1359
                                                      -2
                                                             1858
              10
                            657
#> 4 2013
                    16
                                           700
                                                      -3
                                                             1258
#> 5 2013
             11
                    1
                            658
                                           700
                                                      -2
                                                             1329
#> 6 2013
              3
                    18
                           1844
                                          1847
                                                      -3
                                                               39
#> # ... with 23 more rows, and 12 more variables: sched_arr_time <int>,
       arr_delay <dbl>, carrier <chr>, flight <int>, tailnum <chr>,
       origin <chr>, dest <chr>, air time <dbl>, distance <dbl>, hour <dbl>,
#> #
       minute <dbl>, time_hour <dttm>
```

6. Were delayed by at least an hour, but made up over 30 minutes in flight. If a flight was delayed by at least an hour, then dep_delay >= 60. If the flight didn't make up any time in the air, then its arrival would be delayed by the same amount as its departure, meaning dep_delay == arr_delay, or alternatively, dep_delay - arr_delay == 0. If it makes up over 30 minutes in the air, then the arrival delay must be at least 30 minutes less than the departure delay, which is stated as dep_delay - arr_delay > 30.

```
filter(flights, dep_delay >= 60, dep_delay - arr_delay > 30)
#> # A tibble: 1,844 x 19
#>
     year month day dep_time sched_dep_time dep_delay arr_time
#>
     <int> <int> <int>
                                         <int>
                                                   <dbl>
                         <int>
#> 1 2013
             1
                                         1720
                                                     285
                                                               46
                   1
                           2205
#> 2 2013
                           2326
                                          2130
                                                              131
              1
                    1
                                                     116
#> 3 2013
              1
                    3
                           1503
                                          1221
                                                     162
                                                             1803
#> 4 2013
              1
                     3
                           1839
                                          1700
                                                      99
                                                             2056
#> 5 2013
                    3
                           1850
              1
                                          1745
                                                      65
                                                             2148
                          1941
                                          1759
#> 6 2013
              1
                    3
                                                     102
                                                             2246
#> # ... with 1,838 more rows, and 12 more variables: sched arr time <int>,
      arr_delay <dbl>, carrier <chr>, flight <int>, tailnum <chr>,
      origin <chr>, dest <chr>, air_time <dbl>, distance <dbl>, hour <dbl>,
      minute <dbl>, time_hour <dttm>
```

7. Finding flights that departed between midnight and 6 a.m. is complicated by the way in which times are represented in the data. In dep_time, midnight is represented by 2400, not 0. This means we cannot simply check that dep_time < 600, because we also have to consider the special case of midnight.

```
filter(flights, dep_time <= 600 | dep_time == 2400)</pre>
#> # A tibble: 9,373 x 19
     year month day dep_time sched_dep_time dep_delay arr_time
#>
#>
     <int> <int> <int>
                        <int>
                                         <int>
                                                   <dbl>
                                                             <int>
#> 1 2013
              1
                     1
                            517
                                            515
                                                        2
                                                               830
#> 2 2013
               1
                            533
                                            529
                                                               850
                     1
                                                        4
                            542
                                            540
                                                        2
                                                               923
#> 3 2013
               1
                     1
#> 4 2013
               1
                     1
                            544
                                            545
                                                              1004
                                                       -1
                            554
#> 5 2013
               1
                     1
                                            600
                                                               812
                                                       -6
                     1
                            554
#> 6 2013
               1
                                            558
                                                       -4
#> # ... with 9,367 more rows, and 12 more variables: sched_arr_time <int>,
       arr_delay <dbl>, carrier <chr>, flight <int>, tailnum <chr>,
       origin <chr>, dest <chr>, air_time <dbl>, distance <dbl>, hour <dbl>,
#> #
      minute <dbl>, time_hour <dttm>
```

Alternatively, we could use the <u>modulo operator</u>, **%%**. The modulo operator returns the remainder of division. Let's see how how this affects our times.

```
c(600, 1200, 2400) %% 2400
#> [1] 600 1200 0
```

Since 2400 %% 2400 == 0 and all other times are left unchanged, we can compare the result of the modulo operation to 600,

```
filter(flights, dep time %% 2400 <= 600)</pre>
#> # A tibble: 9,373 x 19
    year month day dep_time sched_dep_time dep delay arr time
                              <int> <dbl>
    <int> <int> <int> <int>
                                                   <int>
#>
#> 1 2013 1 1
                       517
                                    515
                                             2
                                                     830
#> 2 2013
            1
                  1
                        533
                                    529
                                                     850
           1
                                    540
                                               2
#> 3 2013
                 1
                       542
                                                     923
#> 4 2013
           1
                 1
                       544
                                    545
                                               -1
                                                    1004
#> 5 2013
            1
                 1
                        554
                                    600
                                                    812
                                               -6
#> 6 2013
            1
                 1
                        554
                                     558
                                               -4
                                                      740
#> # ... with 9,367 more rows, and 12 more variables: sched arr time <int>,
#> # arr delay <dbl>, carrier <chr>, flight <int>, tailnum <chr>,
      origin <chr>, dest <chr>, air_time <dbl>, distance <dbl>, hour <dbl>,
      minute <dbl>, time_hour <dttm>
```

This filter expression is more compact, but its readability will depend on the familiarity of the reader with modular arithmetic.

5.2.4 Exercise 3

How many flights have a missing dep_time? What other variables are missing? What might these rows represent?

Find the rows of flights with a missing departure time (dep_time) using the is.na() function.

```
filter(flights, is.na(dep_time))
#> # A tibble: 8,255 x 19
    year month day dep_time sched_dep_time dep_delay arr time
#>
    <int> <int> <int> <int> <int> <int> <int> <int>
#>
#> 1 2013 1 1
                                     1630
                                              NA
                                                        NA
                        NA
             1
                  1
                                    1935
                                                NA
                                                        NΔ
#> 2 2013
#> 3 2013
            1
                 1
                        NA
                                     1500
                                                NA
#> 4 2013
            1
                 1
                        NA
                                      600
                                                NA
                                                        NA
#> 5 2013
#> 6 2013
            1
                  2
                         NA
                                      1540
                                                NA
                                                        NA
#> 6 2013
             1
                  2
                          NA
                                     1620
                                                NA
#> # ... with 8,249 more rows, and 12 more variables: sched arr time <int>,
#> # arr delay <dbl>, carrier <chr>, flight <int>, tailnum <chr>,
      origin <chr>, dest <chr>, air_time <dbl>, distance <dbl>, hour <dbl>,
      minute <dbl>, time_hour <dttm>
```

Notably, the arrival time (arr_time) is also missing for these rows. These seem to be cancelled flights.

5.3.1 Exercise 1

How could you use <code>arrange()</code> to sort all missing values to the start? (Hint: use <code>is.na()</code>).

The arrange() function puts NA values last.

```
arrange(flights, dep time) %>%
 tail()
#> # A tibble: 6 x 19
     year month day dep time sched dep time dep delay arr time
    <int> <int> <int>
                      <int>
                                      <int>
                                                <dbL>
            9
                                        1842
                                                            NA
#> 1 2013
                  30
                           NA
                                                   NA
             9
#> 2 2013
                  30
                           NA
                                        1455
                                                   NA
                                                            NA
            9
#> 3 2013
                                        2200
                 30
                           NA
                                                   NA
                                                            NΔ
#> 4 2013
            9
                 30
                           NA
                                        1210
                                                   NA
                                                            NA
#> 5 2013
            9
                  30
                           NA
                                        1159
                                                   NA
                                                            NA
#> 6 2013 9
                  30
                           NA
                                         840
                                                   NA
#> # ... with 12 more variables: sched_arr_time <int>, arr_delay <dbl>,
#> # carrier <chr>, flight <int>, tailnum <chr>, origin <chr>, dest <chr>,
      air_time <dbl>, distance <dbl>, hour <dbl>, minute <dbl>,
#> # time_hour <dttm>
```

Using desc() does not change that.

```
arrange(flights, desc(dep_time))
#> # A tibble: 336,776 x 19
                day dep_time sched_dep_time dep_delay arr_time
#>
     year month
    <int> <int> <int> <int> <int> <dbl>
#>
#> 1 2013
            10
                  30
                        2400
                                       2359
                                                   1
                                                          327
#> 2 2013
            11
                  27
                         2400
                                       2359
                                                          515
                                                    1
#> 3 2013
            12
                  5
                         2400
                                       2359
                                                    1
                                                          427
           12
#> 4 2013
                   9
                         2400
                                       2359
                                                   1
                                                          432
                  9
#> 5 2013
            12
                         2400
                                       2250
                                                   70
                                                           59
#> 6 2013
          12
                  13
                         2400
                                       2359
                                                   1
                                                          432
#> # ... with 3.368e+05 more rows, and 12 more variables: sched_arr_time <int>,
      arr_delay <dbl>, carrier <chr>, flight <int>, tailnum <chr>,
      origin <chr>, dest <chr>, air_time <dbl>, distance <dbl>, hour <dbl>,
#> # minute <dbl>, time_hour <dttm>
```

To put NA values first, we can add an indicator of whether the column has a missing value. Then we sort by the missing indicator column and the the column of interest. For example, to sort the data frame by departure time (dep_time) in ascending order but NA values first, run the following.

```
arrange(flights, desc(is.na(dep_time)), dep_time)
#> # A tibble: 336,776 x 19
#>
                 day dep_time sched_dep_time dep_delay arr_time
     year month
                       <int>
                                                  <dbl>
                                                           <int>
#>
     <int> <int> <int>
                                        <int>
             1
                    1
                                                     NA
                                                              NA
#> 1 2013
                                         1630
#> 2 2013
              1
                    1
                            NA
                                         1935
                                                     NA
                                                              NΔ
#> 3 2013
              1
                    1
                            NA
                                         1500
                                                     NA
#> 4 2013
              1
                    1
                            NA
                                          600
                                                     NA
                                                              NA
#> 5 2013
                    2
              1
                            NA
                                         1540
                                                     NA
                                                              NA
#> 6 2013
              1
                    2
                            NA
                                         1620
                                                     NA
#> # ... with 3.368e+05 more rows, and 12 more variables: sched arr time <int>,
#> # arr_delay <dbl>, carrier <chr>, flight <int>, tailnum <chr>,
      origin <chr>, dest <chr>, air_time <dbl>, distance <dbl>, hour <dbl>,
      minute <dbl>, time_hour <dttm>
```

The flights will first be sorted by desc(is.na(dep_time)).

Since desc(is.na(dep_time)) is either TRUE when dep_time is missing, or FALSE, when it is not, the rows with missing values of dep time will come first, since TRUE > FALSE.

5.5.2 Exercise 2

Compare air_time with arr_time - dep_time. What do you expect to see? What do you see? What do you need to do to fix it?

I expect that air_time is the difference between the arrival (arr_time) and departure times (dep_time). In other words, air_time = arr_time - dep_time.

To check that this relationship, I'll first need to convert the times to a form more amenable to arithmetic operations using the same calculations as the previous exercise.

```
flights_airtime <-
   mutate(flights,
   dep_time = (dep_time %/% 100 * 60 + dep_time %% 100) %% 1440,
   arr_time = (arr_time %/% 100 * 60 + arr_time %% 100) %% 1440,
   air_time_diff = air_time - arr_time + dep_time
)</pre>
```

So, does air_time = arr_time - dep_time? If so, there should be no flights with non-zero values of air time diff.

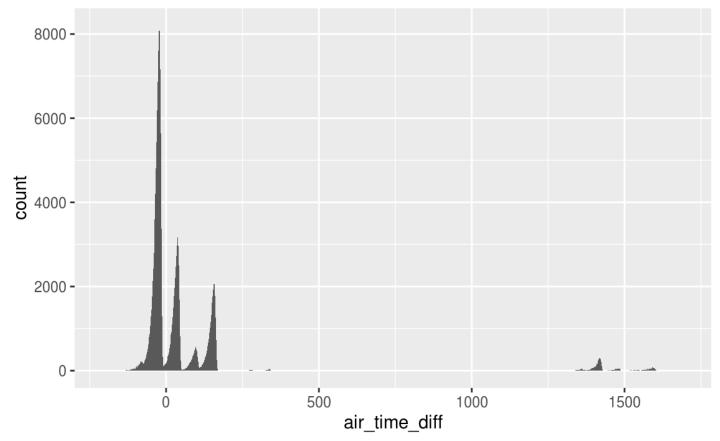
```
nrow(filter(flights_airtime, air_time_diff != 0))
#> [1] 327150
```

It turns out that there are many flights for which air_time != arr_time - dep_time. Other than data errors, I can think of two reasons why air_time would not equal arr_time - dep_time.

- 1. The flight passes midnight, so arr_time < dep_time. In these cases, the difference in airtime should be by 24 hours (1,440 minutes).
- 2. The flight crosses time zones, and the total air time will be off by hours (multiples of 60). All flights in flights departed from New York City and are domestic flights in the US. This means that flights will all be to the same or more westerly time zones. Given the time-zones in the US, the differences due to time-zone should be 60 minutes (Central) 120 minutes (Mountain), 180 minutes (Pacific), 240 minutes (Alaska), or 300 minutes (Hawaii).

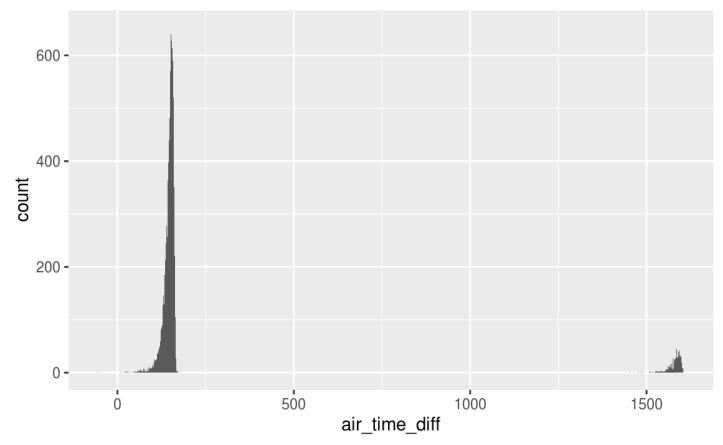
Both of these explanations have clear patterns that I would expect to see if they were true. In particular, in both cases, since time-zones and crossing midnight only affects the hour part of the time, all values of <code>air_time_diff</code> should be divisible by 60. I'll visually check this hypothesis by plotting the distribution of <code>air_time_diff</code>. If those two explanations are correct, distribution of <code>air_time_diff</code>should comprise only spikes at multiples of 60.

```
ggplot(flights_airtime, aes(x = air_time_diff)) +
   geom_histogram(binwidth = 1)
#> Warning: Removed 9430 rows containing non-finite values (stat_bin).
```



This is not the case. While, the distribution of <code>air_time_diff</code> has modes at multiples of 60 as hypothesized, it shows that there are many flights in which the difference between air time and local arrival and departure times is not divisible by 60. Let's also look at flights with Los Angeles as a destination. The discrepancy should be 180 minutes.

```
ggplot(filter(flights_airtime, dest == "LAX"), aes(x = air_time_diff)) +
   geom_histogram(binwidth = 1)
#> Warning: Removed 148 rows containing non-finite values (stat_bin).
```



To fix these time-zone issues, I would want to convert all the times to a date-time to handle overnight flights, and from local time to a common time zone, most likely UTC, to handle flights crossing time-zones. The tzone column of nycflights13::airports gives the time-zone of each airport. See the "Dates and Times" for an introduction on working with date and time data. But that still leaves the other differences unexplained. So what else might be going on? There seem to be too many problems for this to be data entry problems, so I'm probably missing something. So, I'll reread the documentation to make sure that I understand the definitions of arr_time, dep_time, andair_time. The documentation contains a link to the source of the flights data, https://www.transtats.bts.gov/DL_SelectFields.asp?Table_ID=236. This documentation shows that the flights data does not contain the variables TaxiIn, TaxiOff, WheelsIn, and WheelsOff. It appears that the air_time variable refers to flight time, which is defined as the time between wheels-off (take-off) and wheels-in (landing). But the flight time does not include time

spent on the runway taxiing to and from gates. With this new understanding of the

and dep time is air time <= arr time - dep time, supposing that the time zones

data, I now know that the relationship between air time, arr time,

of arr time and dep time are in the same time zone.

5.5.2 Exercise 5

What does 1:3 + 1:10 return? Why?

The code given in the question returns the following.

```
1:3 + 1:10

#> Warning in 1:3 + 1:10: longer object length is not a multiple of shorter

#> object length

#> [1] 2 4 6 5 7 9 8 10 12 11
```

This is equivalent to the following.

```
c(1 + 1, 2 + 2, 3 + 3, 1 + 4, 2 + 5, 3 + 6, 1 + 7, 2 + 8, 3 + 9, 1 + 10)
#> [1] 2 4 6 5 7 9 8 10 12 11
```

When adding two vectors recycles the shorter **vector's values to get vectors of the** same length.

The code also produces a warning that the shorter vector is not a multiple of the longer vector. A warning is provided since often, but not always, this indicates a bug in the code.