SIRD Model over Small World, Scale Free Graphs

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1 Introduction

This project seeks to investigate the structural difference between random graphs and small-world, scale-free graphs and in particular Albert-Barabási graphs. Then these properties will be tested by how the affect an SIRD model ran on both types of graph. The motivation for this porject was from a deep interest in graph theory and with the recent COVID-19 pandemic how the way human relations may affect the spread of infectious disease.

2 Literature Review

2.1 Graph Theory

Graph Theory is the study of networks (graphs will be referred to as networks from now on) where vertices (we will also from this point forwards we will be referring to vertices as nodes) are connected by edges, see APPENDIX for a further explanation.

2.2 Small World Graphs

In May 1967 Professor of Psychology at the Graduate School and University Center of the City University of New York, Stanley Milgram ran an experiment to see if a person living in Omaha, Nebraska could get a parcel to a stockbroker in Boston, Massachusetts (Milgram 1967). In his experiment he found the average path length to reach the stockbroker was 5.5, which created the term six degrees of separation (however Milgram's experiment had flaws which puts the exact number into doubt). This idea of having such a small average path length for such numerous nodes is a hallmark of a small world graph.
A Small world graph is formally defined by the following property: $L \propto \log N$ where L is the average shortest path length of the network and N is the total number of nodes (Watts and Strogatz 1998). Several models exist to generate small world graphs such as the Watts-Strogatz Model.

2.3 Scale Free Graphs

In networks that appear in the real world such as the internet and social groups, there exists nodes known as "hubs" (a node that has a higher degree then the average of the graph). This is an important property encapsulated in Scale-Free Graphs.
Scale-free Graphs are formally defined by the following power law: $P(k) \sim k^{-\gamma}$, k is the degree of a vertex, $P(k)$ is the probability of a node having degree k and γ is a parameter determined by the graph typically $2 < \gamma < 3$ (Onnela et al. 2007).

2.4 Barabasi-Albert Model

In 1999, Albert-László Barabási and Réka Albert developed the Albert-Barabási Model which generates small world, scale free graphs by a process of preferential attachment (Barabási and Albert 1999).The model works as such:define two parameters n (The number nodes the graph at the end of the process will have) and e (the number of edges added for each new node) take a seed graph, add one new node to the graph, using preferential attachment add e edges from the new node to nodes on the seed graph, continue till there are n nodes on the graph.
Preferential attachment describes a 'rich get richer effect' that is the higher the degree of the node the more likely it will gain a new edge, the following formula describes it $\prod(k_i) = \frac{k_i}{\sum_j k_j}$ where k_i is the degree of node i

2.5 SIRD Model

TODO

3 Mathematical Methods

TODO

4 Analysis

TODO

5 Evaluation

TODO

References

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