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Viktoriia Boichenko

# Acoustic Frequency Response Modelling of Multi-Bubble Compounds

Master thesis in Acoustic Engineering

Supervised: Jens Greinert

Tutor: Christian Kanarski

Academic supervisor: Emilie Franceschini

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# Abstract

Acoustic Frequency Response Modelling of Multi-Bubble Compounds

Viktoriia Boichenko[1]

[1] vik.boichenko@gmail.com

This master's thesis focuses on developing a theoretical model for the acoustic frequency response of multi-bubble compounds, such as methane gas bubble seepage from natural reservoirs. The objective is to simulate and analyze the acoustic behavior of these compounds in water columns and to invert the models to identify individual bubble contributions. The research begins with a review of existing single- and multi-bubble acoustic response models, integrating these with effects like acoustic shadowing and bubble interactions. Initial models will be simple and progressively refined based on empirical data from previous studies and new measurements. Experiments will be conducted using a water tank to generate single and multi-bubble responses, with data used to validate and adjust the theoretical models. The final model will be incorporated into existing simulation software, such as KiRAT, to demonstrate its application as a digital twin for underwater column simulations and for generating SONAR data to train AI systems.

Key words: acoustic frequency response, multi-bubble compound, SONAR

# Dedication

To mum, and dad, and two lovely cats

# Declaration

I hereby declare that this thesis is my own original work and has not been submitted previously for any degree or diploma at any institution. All sources of information and data used have been acknowledged, and I have followed all ethical guidelines in conducting this research.

I confirm that the content of this thesis is the result of my own work, except where specifically stated otherwise in the text. Any assistance received has been duly acknowledged.

# Acknowledgements

I would like to express my sincere gratitude to my co-supervisors, Christian Kanarski and Jens Greinert, for their invaluable guidance, support, and expertise throughout this research project. Your insights and feedback have been crucial to the completion of this thesis.

I am deeply grateful to the team at GEOMAR, especially the DSM-group, for providing the necessary resources and a conducive environment for my research. Special thanks to my colleagues for their collaboration and encouragement.

I also extend my thanks to the technical faculty staff for their assistance with the experimental setups and measurements, and to all those who provided the empirical data used in this study.

Lastly, I would like to thank my family and friends for their unwavering support, patience, and understanding throughout this journey. Your belief in me has been a constant source of motivation and strength.

Thank you all for making this achievement possible.

# Contents

# Chapter 1

# Introduction

The topic of underwater acoustics involving bubbles has sparked significant discussion a long time ago. From cavitation bubbles forming on rotating ship propellers and ultrasonic detection of bubbles in blood, to air-filled fish bladders, gas seepages in deep waters, and bubble curtains used to reduce noise during windmill installation, the applications are diverse.

A fundamental question arises regarding the acoustic frequency response of bubbles in large quantities. By analyzing the backscattering of emitted signals underwater, various properties such as diameter and dynamics can be determined. Consequently, models for the acoustic frequency response of single bubbles have been developed by calculating their backscattering cross-section, with notable contributions from Anderson and Thuraisingham.

## 1.1 Internship Environment

The internship was performed as a part of the project within the hydroacoustic study field of the research instute GEOMAR at the research unit of the Marine Geology, DeepSeaMonitoring group (DSM) with a collaboration of Technical faculty of the Christian-Albert University in Kiel.

This DSM group covers a wide range of scintific research and application-oriented development fields. Main problems are exploring the gas flows from the sea floor, its quantification, detection, and monitoring; seabed resources assessment with the help of habitat mapping, its organisms; and environmental monitoring of munitions dump sites delaboration[1].

Within the technologies used in DSM are hydroacoustic, electromagnetic, and optical in the combination of calibration and post-processing techniques, as well as a geochemical analysis of the sea surface, the water column and atmosphere.

The internship was aimed to build a theoretical model for the acoustic frequency response of multi-bubble compounds, such as gas bubble flares from natural reservoirs. The main objective is to validate and adapt this theoretical model using emperical data and new measurements, integrating it into existing simulation software for applications such as underwater column simulations and SONAR data generation for AI training purposes.

## 1.2 Literature Overview

In 1950s the paper of Foldy the mathematical background behind backscattering cross-section and the multiple scattering of the sphere has given a push to the further development of the bubbles research.

After that the Lax model was provided [2] for the generalised view of multiple scattering of waves.

In the following years, some papers provided an extended view of previous papers, such as Kaczer and Gemperle paper [3, p. .] where they highlighted a mathematical point of view at the multiple scattering with a justification of the Foldy-Lax model.

An extensive and fundamental knowledge about the bubble acoustics has been provided with the book of Leighton [4]

(Manasseh et al. 2004[5]) Bubbles produce an acoustic signal owing to compression of the gas in the bubble. The 'spring' of the compressible gas and the mass of liquid around the bubble create a natural oscillator, sending a pressure oscillation through the liquid and interacting with the neighbouring bubbles

(Zhang et al. 2022[6]) This paper provides a volume-scattering strength optimization model. This model allows to estimate the bubble size distribution. It provides a thorough explanation with the help of the case study experiment with a multibeam sonar to identify a bubble leakage in the sea.

It identifies three parameters: two in probability density function of gas leakage bubble sizes and the total number of bubbles inside the sample volume N0. Direct method was used for obtaining parameters.

(Li et al. 2020[7]) Mentions theory regarding the bubble size distribution and provides the data for the backscattering cross-section of a single bubble. The model of the bubble plume's acoustic backscattering consists of the model of the single bubble, distribution of size of the bubbles and computation of the volume scattering strength.

Regarding the signal processing an important part has played the book of Ainslie [8]. It has provided fundamentals for the sonar modelling which were essential for understanding while working with a sonar simulation.

## 1.3 Outline

The thesis is structured in the following way:

- Chapter 1 gives the current state of art of the problem and available literature overview.
- Chapter 2 overviews the basic theoretical concepts necessary for accomplishing goals of this internship.
- Chapter 3 presents simulations of the acoustic frequency response of bubbles in different settings of the motion, quantity, and between bubble interaction.
- Chapter 4 shows the setup and the process of experiment of the bubble flare in the water tank. Also, it provides results of the comparison of the emperical and experimental data.

- Chapter 5 concludes the overall results of the thesis and further steps in this research field direction.
- Appendix contains static and animated plots of the results obtained from the simulations and experiment measurements.

# Chapter 2

# Theoretical framework

Chapter 2 is devoted to the theoretical part of this paper of undewater acoustics and signal processing essentials. It presents main concepts, basic definitions of bubble acoustics and signal processing techniques. An explanation of the chosen terms and approaches is provided below.

## 2.1 Sound Propagation Concepts

The sound represents the transmission of the mechanical energy that propagates through the medium and relies on its properties [4, p.1]. The understanding presented below underwater acoustics basic terminology is essential for deeper comprehension of the processes that were involved in this research problem.

### Wavelength

It defines the length of the wave over a unit period. It is an inverse of the wavenumber. The higher frequency, the shorter wavelength.

$$\lambda = \frac{c}{f} \quad \frac{[\mathrm{m/s}]}{[1/\mathrm{s}]} = \mathrm{m} \tag{2.1}$$

### Speed of sound

Generally, it is the speed of sound dependent on other parameter such as a temperature, a salinity and a depth, which make the speed of sound vary and refraction is appearing [9, p. 28]. In order to simplify the simulation, the value of the speed of sound in a water was set at 1500 m/s. The equation 2.2 shows a relation of the speed of the sound ($c$) to frequency ($\omega = 2\pi f$) and a wavelength ($\lambda$)

$$c_{water} = f\lambda = \omega/k \tag{2.2}$$

where $\omega$ is an angular frequency and $k = 2\pi/\lambda$ is a wavenumber.

### 2.1.1 Radiation of sound

#### Intensity

It is how much energy gets through the area perpendicular to the direction of the emitted wave [4, p.18]. The formula of acoustic intensity for a plane wave can be written in the following form:

$$I = \frac{P_A^2}{2\rho c} = \frac{W}{4\pi s^2} \tag{2.3}$$

Where $P_A$ is an acoustic pressure amplitude of the wave, $\rho$ is a density of the medium, $c$ is a speed of wave.

Where W is an energy, s is a distance from the source, $4\pi s^2$ is a sphere surface.

#### Intensity level

Intensity level is a measure over some area, which is a ratio of the sound intensity $I$ to some reference intensity $I_{ref}$ [4, p.19]:

$$IL = 10 \log_{10}(\frac{I}{I_{ref}}) \tag{2.4}$$

$I_{ref} = 10^{-12}\text{W m}^{-2}$ in air. As the pressure amplitude is proportional to the square root of intensity, we can write down sound pressure level in the form:

$$SPL = 20 \log_{10}(\frac{P}{P_{ref}}) \tag{2.5}$$

#### Spherical spreading

The wave propagation with a $1/s^2$ dependance is defined as a spherical spreading.

#### Attenuation of sound

The process of absorbtion and scattering of the sound can be defined as an attenuation.

It is an amount of the transmitted sound signal received back by a receiver.

## 2.2 Signal Processing Concepts

### 2.2.1 Beamforming

A beamforming is a way of processing the received or transmitted signal, so that we can direct the directed and amplified signal from teh projector with multiple receivers or transmitters, therefore improving the SNR and eliminating unwanted interfering signals. Also, it can be referred as a spatial filtering.

Withing the scope of the work we have used beamforming in the sonar environment. A conventional beamformer in the form of the delay-and-sum was used.

### 2.2.2 SONAR

#### SIMO-sonar

The general principle of the projector SIMO-sonar is in the following way: the signal is emitted with a single transducer and receiving a signal back with several receivers, which in our case were hydrophones. Also, it can be considered with another interpretation. For example, we provide a single output, and obtain multiple input data.

The sonar type is the SIMO-sonar, as we needed to start with a simple model of the current research. The experiments with a MIMO-sonar can be implemented for further research therefore expanding the complexity and variability of usages of the developed model, as [this type provides an improved resolution and enhances the signal-to-noise ratio of the received signal] (http://jset.sasapublications.com/wp-content/uploads/2017/09/6702529.pdf).

### 2.2.3 Sonar Equation

The Sonar equation is a fundamental tool in underwater acoustics used to predict the performance of sonar systems. The basic form of the Sonar equation is:

$$SNR = SL - TL + TS - NL \tag{2.6}$$

Where SNR is the signal-to-noise ratio, SL is the source level, TL is the transmission loss, TS is the target strength, and NL is the noise level.

To implement the Sonar equation, one needs to calculate or estimate each of these parameters. Source level (SL) represents the acoustic power radiated by the sonar system, which can be determined based on the characteristics of the sonar transducer and the electrical input power. Transmission loss (TL) accounts for the attenuation of sound energy as it propagates through the water, considering factors such as spreading loss, absorption, and bottom and surface reflections. Target strength (TS) quantifies the amount of sound energy reflected or scattered by the target, which depends on its size, shape, and composition. Noise level (NL) encompasses all sources of ambient noise in the environment, including thermal noise, wind-generated noise, biological noise, and anthropogenic noise.

#### Near- and far-field

The phase difference that comes from the path difference is proportional to the ration of the path to the wavelength [4, p.30].

$$D_{\text{Near to far field}} = L_s^2/\lambda$$

where $L_s$ is the spacing between receivers In the near field the delay-and-sum equation for an array would look like:

$$\tag{2.7}$$

## 2.3 Reconstruction of the bubble frequency response

It will allow demonstrating the ability to invert the model to find individual bubble contributions from compund analysis. The following equation2.8 was the base of the calculations:

$$R = T \times H + N \tag{2.8}$$

where R is a received signal, T is a transmitted signal by sonar, H is frequency response of the bubble, N is an added background noise.

# Chapter 3

# Bubble models & Simulations

This chapter describes the model of the bubble simulation problem and its formulation. Bubble's acoustic frequency response can depend on multiple factors. Among them are a bubble radius, water pressure, ensonification frequency.

For the simplicity, in most studies the bubble shape is assumed to be spherical, as the optical equipment at long distances can't provide an accurate estimation of the bubble's shape and size [6, p.2].

### 3.0.1 Bubble flare

A continuous column of bubbles with the different radii moving with a different speed within the same direction (???). In laboratory conditions simulation can be performed with the use of the bubble generator connected to the air-compressor.

### 3.0.2 Target strength

It is the measure of the single object's (e.g. bubble) reflection of the transmitted signal.

### 3.0.3 Scattering cross-section

The amount of energy transmitted sound wave being reflected by the small object in the direction of the receiving system is evaluated with a scattering cross-section, the ratio of the total scattered power from the object to incident plane wave intencity [9, p. 40]:

$$\sigma_{bs} = \frac{W}{I_{inc}} \tag{3.1}$$

We often want to determine exactly how much of an incident acoustic beam's energy is scattered by a bubble, rather than being lost through thermal and viscous dissipation.

### 3.0.4 Resonance frequency of the bubble

Due to the mass-spring system which appears from the interaction of the incoming wave with a bubble. The gas inside and the surface tension in the bubble with the liquid.

Often this concept is related to the Minaert frequency and the natural frequency of the bubble. The paper of Ainslie and Leighton [10] provides an overview on different usages of those terms within multiple papers. In this work the next definition will apply to these terms.

The natural frequency is referred as $f_{res} = \omega_{res}/2\pi$, where:

$$\omega_{res} = \frac{1}{R_0}\sqrt{\frac{3\gamma P_{liq}}{\rho_{liq}}} \tag{3.2}$$

($\gamma$ is a specific heat ratio, $R_0$ is radius of the bubble)

The larger bubble radius, the lower frequency resonance

## 3.1 Single bubble model

Single bubbles can be used as comparison to the fish in the water as they have bludders filled with an air, and this is usually a typical reflective surface recorded with a sonar.

- Anderson Anderson model provides a modal solution and allows to model the backscattering cross-section of a single bubble for $ka > 1$ [11].
- Thuraisingham

  In this paper of Zhang et al. 2022 [6] modal solution is used when ka > 1, and Thuraisingham solution is preferred for the ka<1, as the first doesn't consider the bubble damping effect. Such approach can be used for our multiple compund bubble simulation.
- Church, Medwin,

  Andreeva is for fish.

### 3.1.1 Anderson model

The process of establishing the model for the acoustic frequency response of the bubble dates back to the paper of Anderson in 1950 [11]. Initially, the theory of Rayleigh scattering has provided a big input into the scatterers which are comparable to a wavelength. While this work presented spheres with medium-like acoustic properties, and dimensions are as a couple wavelengths. Using provided calculation we can obtain the pressure and the total energy in the scattered wave.

The following limitation of this model which should be considered is when ka approaches 0. That which means that the radius of sphere becomes less than a wavelength. For the cases when ka <1 or >1 there are no limitations.

This paper results can be used for comparison with other models available nowadays. The implementation in Matlab has allowed us to see how the peak at resonance frequency corresponds to the one of Thuraisingham model. However, at higher frequencies we can observes additional peaks, which indicate other modes of the fluid sphere, when $ka$ is closer or over 1.

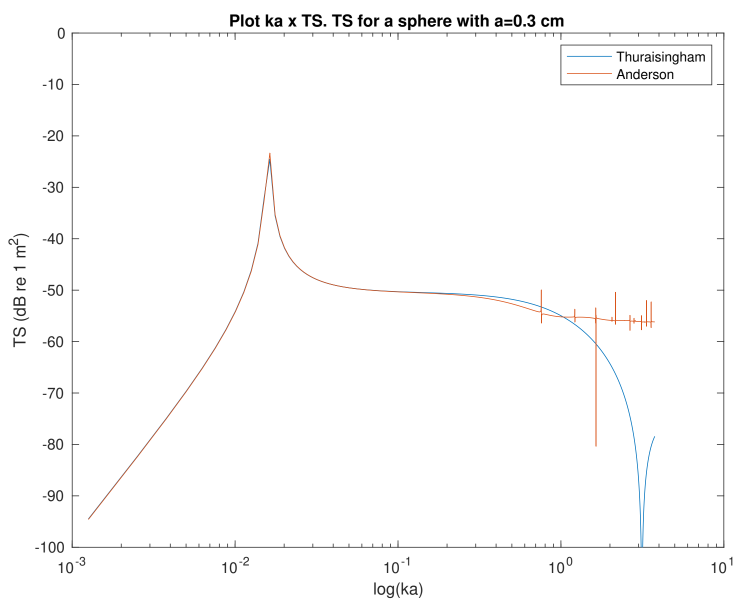
### 3.1.2 Thuraisingham model

The scattering cross-section $\sigma_s$ of a single bubble is a ratio of the energy loss averaged over time while being scattered from the bubble to the intensity of incident signal [12, p.408]. Thuraisingham formula for the scattering cross-section with the condition of a spherical pulsation of the bubble was presented in this form:

$$\sigma_s = \frac{4\pi a^2}{\delta^2 + (\frac{\omega_r^2}{\omega^2} - 1)^2} \frac{(\frac{\sin ka}{ka})^2}{1 + (ka)^2} \tag{3.3}$$

Where $a$ is a bubble radius, $k$ is a wavenumber in the water, $\delta$ is a damping constant ,$\omega_r$ is the resonance frequency, $\omega$ is the angular frequency.

This model applies over all values of $ka$. After that a model was optimised by Li et al. [7] for the omnidirectional breathing mode. As the impact of the last factor decreases with the increase of the $ka$-value, we require more adaptable model to different modes. In this paper Li et al. have used for $ka << 1$ a Thuraisingham formula and for $ka > 1$ was used an adjusted modal solution.




Target strength over $ka$-values for a Thuraisingham and Anderson solutions of a single bubble of radius a = 0.3 cm

Figure **??** shows a plot of the single bubble target strength against $ka << 1$, and significantly larger values $ka > 1$. The highes peak corresponds to the resonance frequency of the bubble, following smaller peaks of the modal solution might correspond to the normal modes of the bubble, which occur as a resulting oscillations. Yet it doesn't take into account the damping effect present in a bubble.

## 3.2 Multiple bubble model

For more than one bubble we can add various scenarious for bubbles' location in space, distance between each other, their dimensions, dynamics, and interactions. Those new properties pose more challenges in constructing a valid model for multiple bubbles.

Among the possible real-world applications are gas seapages underwater, bubble curtains which play role of sound attenuators.

### 3.2.1 Volume-scattering strength

In case we have more than one bubble, where we were describing it with a target strength, the volume backscattering strength should be used instead.

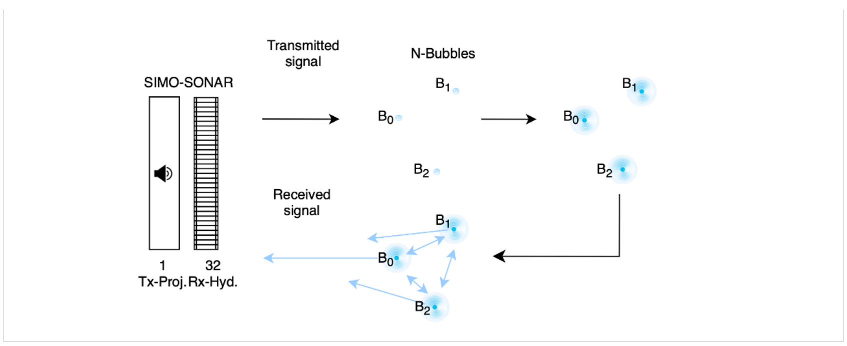
$$s_v = \int \sigma_{bs} n(a)$$

$$S_v = 10 \log_{10}(s_v)(dB\ re\ 1m^2)$$

## 3.3 Multiple scattering

Leblond et al. paper states that for multiple discrete bubbles, we can neglect the effect of the multiple scattering [13]. Foldy paper equation explanation and concept of the multiple scattering

When the signal is emitted, the scatterer removes from the wave a specific amount of flux which is equal to the its extintion cross section multiplied by flux per unit area. This amount is partially absorbed and scattered. The next step lies in adding up all the scattering from the received transmitted signal scatterer to other bubbles, then repeating the procedure in the regard of other bubbles, till all scatterers in the close enough vicinity of each other got an interaction with each other.




The scheme of the multiple scattering

But this concept is applicable when amount of scatterers is sufficiently small, otherwise they start interacting with each other and no longer act as a scatter independently.

Hazard and Cassier paper provides a mathematical point of view at the multiple scattering with a justification of the Foldy-Lax model. This model help to get an approximation of scattered waves in a deterministic as well as in a random media. Also, local error estimates for circular objects in 2D problem were obtained in the scope of results of this paper.

- Foldy, Leslie L. 'The Multiple Scattering of Waves. I. General Theory of Isotropic Scattering by Randomly Distributed Scatterers'. Physical Review 67, no. 3–4 (1 February 1945): 107–19. https://doi.org/10.1103/PhysRev.67.107.

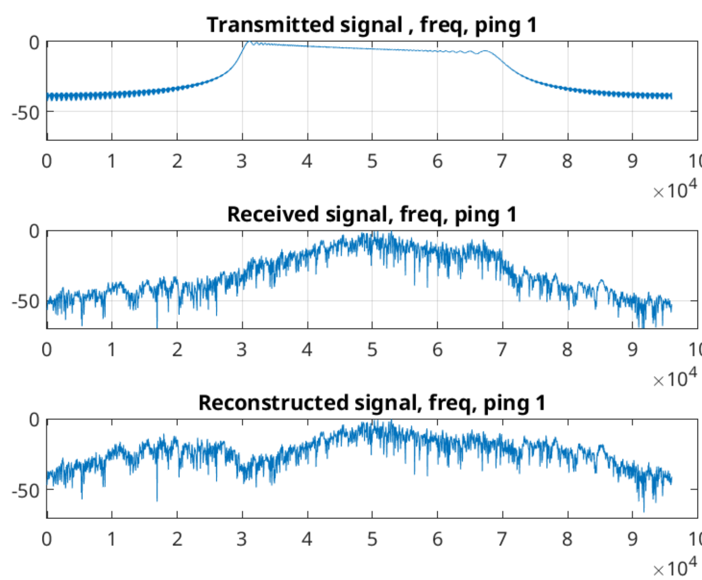
- Multiple Scattering of Acoustic Waves by Small Sound-Soft Obstacles in Two Dimensions: Mathematical Justification of the Foldy–Lax Model, Hazard and Cassier

# Chapter 4

# Experiment

This section will include the simulation and programming part of the work, experimental part

## 4.1 Simulation of bubble response




some simulation

## 4.2 Experiment of measuring bubbles

| | |
|---|---|
| Date Performed: | May 17, 2024 |
| Partners: | Viktoriia BOICHENKO |
| Instructor: | Christian KANARSKI |

### 4.2.1 Objective

This laboratory project aims to connect theoretical concepts with empirical observations through the use of volume strength backscattering. The primary objective is to illustrate the practical applications of theoretical knowledge acquired during the internship and master's thesis research.

By leveraging volume strength backscattering as a key analytical tool, we will examine experimental data, providing insights into how theoretical abstractions translate into concrete experimental methodologies. This integration is designed to promote a comprehensive understanding of the reciprocal relationship between theory and practice, underscoring the significance of a multidimensional approach in scientific inquiry and engineering practice.

### 4.2.2 Experimental model

The experimental setup entails an experiment of bubbles flares detection with a sonar. They were emitted in a water with a bubble generator and observed with the help of the SIMO-sonar with an ultrasound in a laboratory conditions of the water tank.

Preliminary tests of the water tank were performed in order to identify the impulse response of the environment, and verify the correctness of the equipment configuration as well as whether it is in a usable condition.

The primary focus of the observation was the volume strength back scattering (Vs), chosen as the key parameter for assessing the acoustic bubble response with acoustic radiation in the high-frequency range.

### 4.2.3 The setup

Our bubble experiment was conducted using a predesigned setup aimed at exploring the acoustic frequency response of the bubbles, which can be seen at Figure 4.1. The experimental arrangement included the following:

- **Bubble generator**: a set of the equipment which included
- **Sonar** it is a projector with 32 hydrophones in an array. The distance between elements is 0.0139 m.
- **Laptop with a processing software KiRAT, contains an inbuilt wave generator**
- **Soundcards** for the signal conversion from the analog to digital and vice versa.
- **Water tank** is a $5 \times 5 \times 5$ m$^3$ concrete open box-like construction filled with water. The filtration system was turned off during experiment conduction to avoid the creation of additional bubbles and noise of the system.
- **Videocamera**: a mobilephone's camera was used for the experiment recordings

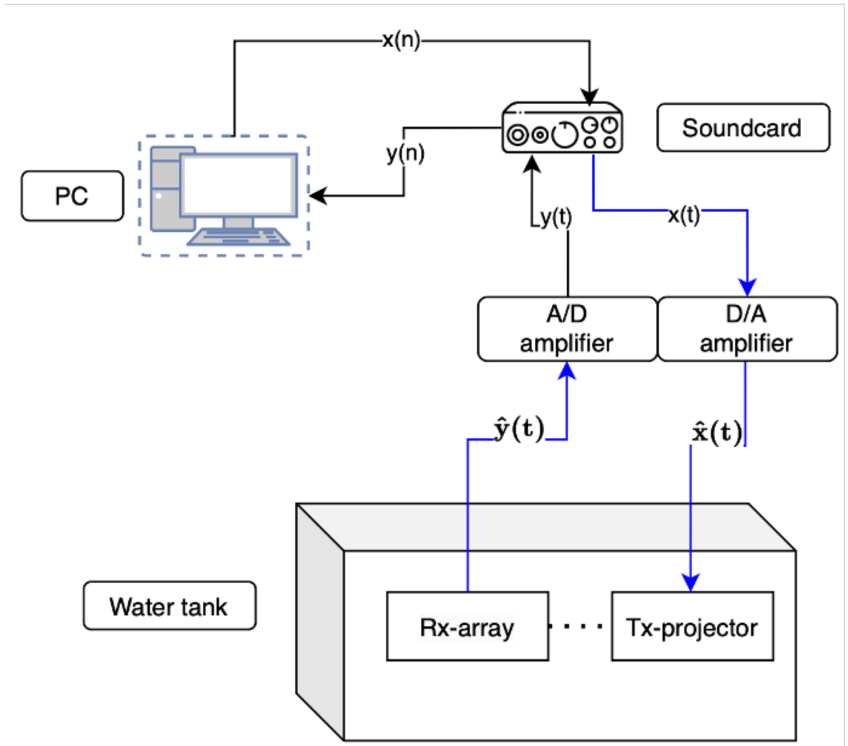



Figure 4.1: The scheme of the setup

During the experiment different signal types (noise and chirp) and length (2 ms and 10 ms) were used for the analysis of the bubble flares. The emitted central frequency was 50 kHz with a bandwidth of 40 kHz. The sampling frequency was 192 kHz.

- **Noise**: a white noise whose frequency is not dependent on the power spectral density[8];
- **Chirp**: a hyperbolic frequency modulation upward signal
- Sound signal: narrow band pulse, chirp, noise
- Sonar position: from top, vertical, horizontal
- Bubbles location: in the center of the experimental pool
- Bubble characteristics: emitting a single bubble of the specified radius; a row of bubbles; creating a bubble flare;

Other things which are important for taking into account are:

- Response of the transducer can influence the received signal
- Near field radiation implementation for measurements with a spherical radiation against the plane wave in a far field

Further set of things which are required to perform the experiment are the setup of the equipment required for performing our measurements. Essentially, it will include the sonar, a bubble generator, processing unit as a laptop/computer.

There different papers which have conducted similar experiments on bubble investigation.

The Zhang et al. 2021[14] paper contains description of the experiment for investigation bubble oscillations. In order to produce cavitaion bubbles the setup used a deionised water.

In order to detect the acoustic radiation released by cavitation bubbles a fiber optic hydrophone was employed, which was calibrated and attached to oscilloscope. Among possible things that could affect results were the cleanliness of sensor connector, water quality and wall reflection of the shock wave.

Leblond paper provides a scheme of the vertical and horizontal observation of the acoustic bubble release[13]. Vertical will allow to detect bubbles all along the acoustic beam, while for horizontal we will see only the crossing of the stream.

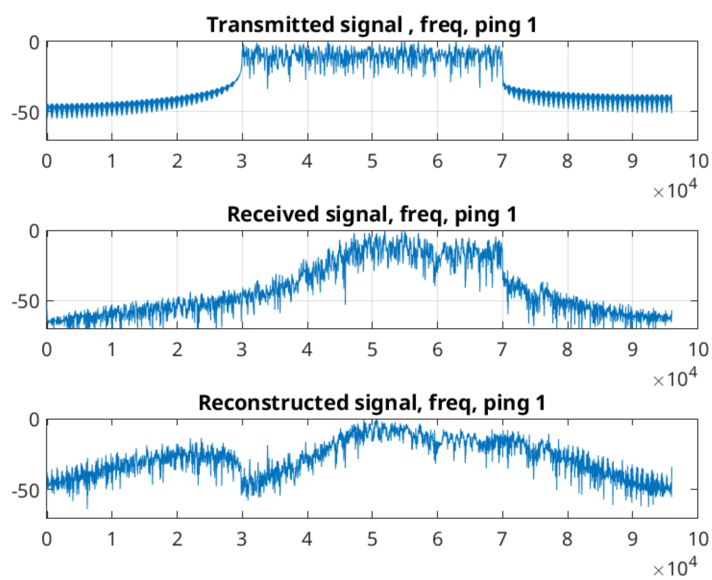
## 4.3 Experimental Data

After the pocessing the date of 23 recorded measurements, a few results as well as comparisons can be presented below.

At the horizontal orientation samples 8 (noise, 10ms) and 11 (chirp, 10ms).

At the vertical orientation sample 19 (noise) and 23 (chirp, 10ms) are seem to be good for comparison.

$H_{wall}$ can be extracted from the initial measurement, calculating an impulse response of the water tank and identifying the location of the walls with the sonar.




'Horizontal orientation of the sonar, 10ms noise, sample 8'

## 4.4 Calculations

- Vs from a single ping
- spectrogram in Audacity for different samples

- bubble localization?
- obtaining $H_{bubble}$, $H_{wall}$

### 4.4.1 Results and Conclusions

### 4.4.2 Discussion of Experimental Uncertainty

# Chapter 5

# Results and Analysis

This section covers the comparison between simulation and experimental results. Their constraints and drawbacks are highlighted.

Results of extracting the bubble response are described.

The simulation of the response from the experiment is trialed.

## Chapter 6

# Conclusion

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