1. Calculat entropy of X: $-\sum_{x}^{n} (P_x)\log(P_x)$ $a^x = b$ Domini Cherry

$$X = \begin{cases} a \text{ w/ probability } \frac{1}{2} \\ b \text{ w/ prob. } \frac{1}{4} \\ c \text{ w/ prob. } \frac{1}{8} \\ d \text{ w/ prob. } \frac{1}{8} \end{cases}$$

a.) $-\sum \frac{1}{2}\log\frac{1}{2} = -(\frac{1}{2}\log\frac{1}{2} + \frac{1}{2}\log\frac{1}{2})$

$= -(\frac{1}{2}(-1) + \frac{1}{2}(-1))$

$= \boxed{1}$

---

b.) $-\sum \frac{1}{4}\log\frac{1}{4} = -(\frac{1}{4}\log\frac{1}{4} + \frac{3}{4}\log\frac{3}{4})$

$= -(\frac{1}{4}(-2) + \frac{3}{4}(-.42))$

$= -(-.5 + -.31)$

$= \boxed{.81}$

C(&D) $-\sum \frac{1}{8}\log\frac{1}{8} = -(\frac{1}{8}\log\frac{1}{8} + \frac{7}{8}\log\frac{7}{8})$

$= -(\frac{1}{8}(-3) + \frac{7}{8}(-.19))$

$= -((-\frac{3}{8}) + (-.17))$

$= \boxed{.54}$

---

2. $X = \{0, 1\}$ for two distributions P and Q on X $P(0) = 1-a$ $Q(0) = 1-b$ $a = \frac{1}{2}$

$P(1) = a$ $Q(1) = b$ $b = \frac{1}{4}$

$D_{KL}(P\|Q) = \sum_{i \in X} P(x)\log_2\left(\frac{P(i)}{Q(i)}\right)$

$= \sum_{x \in X} 1-a\log_2\left(\frac{1-a}{1-b}\right) + a\log\left(\frac{a}{b}\right)$

$= (1-(\frac{1}{2}))\log_2\left(\frac{1-(\frac{1}{2})}{1-(\frac{1}{4})}\right) + \frac{1}{2}\log_2\left(\frac{(\frac{1}{2})}{(\frac{1}{4})}\right)$

$= .5\log\left(\frac{.5}{.75}\right) + .5\log(2)$

$= -.29 + .5 = \boxed{.21}$

$D_{KL}(Q\|P) = \sum_i Q(x)\log_2\left(\frac{Q(x)}{P(x)}\right)$

$= \sum_i 1-b\log_2\left(\frac{1-b}{1-a}\right) + b\log_2\left(\frac{b}{a}\right)$

$= (1-\frac{1}{4})\log_2\left(\frac{1-(\frac{1}{4})}{1-(\frac{1}{2})}\right) + \frac{1}{4}\log_2\left(\frac{\frac{1}{4}}{\frac{1}{2}}\right)$

$= .75\log_2\left(\frac{.75}{.5}\right) + .25\log_2(.5)$

$= .44 + (-.25) = \boxed{.19}$

---

3. Cross-Entropy: $H(p(x), q(x)) = -\sum_x P(x)\log_2 q(x)$

$X = \{0, 1\}$ for two distributions P and Q on X, let $P(0) = 1-a$ $Q(0) = 1-b$ $a = \frac{1}{2}$

$P(1) = a$ $Q(1) = b$ $b = \frac{1}{4}$

$H(p(x), q(x)) = -\sum_x P(x)\log_2 Q(x)$

$= -(1-a\log_2 1-b + a\log b)$

$= -(1-\frac{1}{2}\log_2 1-\frac{1}{4} + \frac{1}{2}\log_2\frac{1}{4})$

$= -(\frac{1}{2}\log\frac{3}{4} + \frac{1}{2}\log_2\frac{1}{4})$

$= -(\frac{1}{2}\cdot(-.42) + \frac{1}{2}(-2))$

$= \boxed{1.21}$

4.) $JSD(P,Q) = \frac{1}{2} D(P\|M) + \frac{1}{2} D(Q\|M)$

$X = \{0,1\}$ for two distributions $P + Q$ on X

$P(0) = 1-a \quad Q(0) = 1-b$

$P(1) = a \quad Q(1) = b$

$m = \frac{1}{2}(P+Q) \quad a = \frac{1}{2} \quad b = \frac{1}{4}$

$M(0) = \frac{1}{2}((1-\frac{1}{2}) + (1-\frac{1}{4})) = .625$

$M(1) = \frac{1}{2}(\frac{1}{2} + \frac{1}{4}) = .375$

$JSD(P,Q) = \frac{1}{2} D \log(\frac{P}{M}) + \frac{1}{2} Q \log(\frac{Q}{M})$

$= \frac{1}{2}(1-\frac{1}{2}) \log_2 (\frac{1-\frac{1}{2}}{.625}) + \frac{1}{2}(1-\frac{1}{4}) \log_2 (\frac{1-\frac{1}{4}}{.625})$

$= \frac{1}{2}(1-\frac{1}{2}) \log_2 (\frac{1-\frac{1}{2}}{.625}) + \frac{1}{2}(1-\frac{1}{4}) \log_2 (\frac{1-\frac{1}{4}}{.625})$

$+ \frac{1}{2}(\frac{1}{2}) \log_2 (\frac{\frac{1}{2}}{.375}) + \frac{1}{2}(1-\frac{1}{4}) \log_2 (\frac{\frac{1}{4}}{.375})$

$= \frac{1}{2}(.5) \log_2 (\frac{.5}{.625}) + \frac{1}{2}(.75) \log_2 (\frac{.75}{.625})$

$+ \frac{1}{2}(.5) \log_2 (\frac{.5}{.375}) + \frac{1}{2}(.25) \log_2 (\frac{.25}{.375})$

$= \boxed{.049}$

5.) $JSD(P,Q,R) = \frac{1}{3} D(P\|M) + \frac{1}{3} D(Q\|M) + \frac{1}{2} D(R\|M) \quad m = \frac{1}{3}(P+Q+R)$

$X = \{0,1\}$ for 3 distributions P, Q, R on X

$P(0) = 1-a \quad Q(0) = 1-b \quad R(0) = 1-c$

$P(1) = a \quad Q(1) = b \quad R(1) = c$

$a = \frac{1}{2} \quad b = \frac{1}{3} \quad c = \frac{1}{4}$

$m(0) = \frac{1}{2}(.5 + .66 + .75) = .64$

$m(1) = \frac{1}{3}(.5 + .33 + .25) = .36$

$JSD(P,Q,R) = \frac{1}{3}(.5 \log_2 (\frac{.5}{.64})) + \frac{1}{3}(.66 \log_2 (\frac{.66}{.64})) +$

$+ (\frac{1}{3}) .75 \log_2 (\frac{.75}{.64}) + (\frac{1}{3}) .5 \log_2 (\frac{.5}{.36})$

$+ (\frac{1}{3}) .33 \log_2 (\frac{.33}{.36}) + (\frac{1}{3}) .25 \log_2 (\frac{.25}{.36})$

$= -.059 + .013 + .057 + .079 + (-.012)$

$+ (-.0438)$

$= \boxed{.0342}$

6.)

Y \ X	1	2	3	4
1	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{32}$	$\frac{1}{32}$
2	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{32}$	$\frac{1}{32}$
3	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$
4	$\frac{1}{4}$	0	0	0

→

Y \ X	1	2	3	4
1	$\frac{4}{32}$	$\frac{2}{32}$	$\frac{2}{32}$	$\frac{1}{32}$
2	$\frac{2}{32}$	$\frac{4}{32}$	$\frac{1}{32}$	$\frac{1}{32}$
3	$\frac{2}{32}$	$\frac{2}{32}$	$\frac{2}{32}$	$\frac{2}{32}$
4	$\frac{8}{32}$	0	0	0

X	1	2	3	4
P(X=x)	$\frac{16}{32}$	$\frac{8}{32}$	$\frac{4}{32}$	$\frac{4}{32}$

$E(X) = 1(\frac{16}{32}) + 2(\frac{8}{32}) + 3(\frac{4}{32}) + 4(\frac{4}{32})$

$= \frac{60}{32} = \boxed{\frac{15}{8}}$

Y	1	2	3	4
P(Y=y)	$\frac{8}{32}$	$\frac{8}{32}$	$\frac{8}{32}$	$\frac{8}{32}$

$E(X) = 1(\frac{8}{32}) + 2(\frac{8}{32}) + 3(\frac{8}{32}) + 4(\frac{8}{32})$

$= \frac{80}{32} = \boxed{\frac{5}{2}}$