中国矿业大学 2019～2020 学年第 1 学期

《常微分方程》试卷（A）卷

考试时间：120 分钟　　　　考试方式：闭卷

学院____________班级______________姓名____________学号____________

题号	1	2	3	4	5	6	7	8	总分
得分									
阅卷人									

1.（30 points）Find the general solutions of the differential equations in problem (1) through (6).

(1) $\dfrac{dy}{dx}=e^{2x+y}$.

(2) $(3x^2y^2+e^x)dx+(2x^3y+\cos y)dy=0$.

(3) $y'+2y=e^x$.

(4) $\dfrac{dy}{dx}=\tan(x+y)-x.$

(5) $y=xy'+\dfrac{1}{2}(y')^2.$

(6) $\dfrac{dy}{dx}=\dfrac{x+y}{x-y}.$

2. (10 points) Find the general solution of the following first-order system

$$\begin{cases} \dfrac{dx}{dt} = 2x - 2y \\ \dfrac{dy}{dt} = 2x + 2y \end{cases}.$$

3. (10 points) Find the general solution of

$$\frac{d^3x}{dt^3} + 3\frac{d^2x}{dt^2} + 3\frac{dx}{dt} + x(t) = e^{2t}.$$

4. (10 points) If $\varphi(x)$ is a particular solution of Riccati's equation

y'

$$\cancel{y'(x)} = p(x)y^2 + q(x)y + r(x),$$

then we can solve the general solution of this equation. Try to prove it.

5. (10 points)

(1) Write the content of Existence and Uniqueness Theorem.

(2) Compute the first two Picard's sequence of the following initial value problem.

$$\begin{cases} \dfrac{dy}{dx} = 2x^2 + y \\ y(0) = 1 \end{cases}.$$

6. （10 points）If $\Phi(t)$ is a fundamental solution matrix of

$$\frac{d\vec{x}}{dt}=A(t)\vec{x}(t)$$

Try to prove that

(1) $\Phi(t)$ satisfies the equation $\frac{d\vec{x}}{dt}=A(t)\vec{x}(t)$, that is $\frac{d\Phi(t)}{dt}=A(t)\Phi(t)$.

(2) $\Phi(t)C$ is another fundamental solution matrix, where C is a inverse $n\times n$ matrix.

7. （10 points）Assume that $\vec{\varphi}(t)$ is the solution of $\frac{d\vec{x}}{dt}=A\vec{x}(t)$, which satisfies the initial condition $\vec{\varphi}(t_0)=\eta$，prove that $\vec{\varphi}(t)=e^{A(t-t_0)}\eta$.

8. (10 points) If $\varphi(x)$ is any non-zero solution of the second-order linear differential equation

$$y''(x)+p(x)y'+q(x)y=0$$

where $p(x),q(x)$ are continuous in $(-\infty,+\infty)$, then prove that $\varphi(x)$ is not tangent to x axis in xoy plane.