中国矿业大学 2021-2022 学年第 1 学期

《常微分方程》试卷（B）卷

考试时间：120 分钟　　考试方式：闭 卷

学院 ________ 班级 ________ 姓名 ________ 学号 ________

题号	一	二	三	四	五	六	七	八	总分
得分									

1. (30 points) Find the general solutions of the differential equations in problem (1) through (6).

(1) $x\dfrac{\mathrm{d}y}{\mathrm{d}x}+2\sqrt{xy}=y \quad (x>0)$.

(2) $y=xy'+(y')^4$.

(3) $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{y}{2x - y^3}$.

(4) $\dfrac{\mathrm{d}y}{\mathrm{d}x} + \tan(x + y) + 1 = 0$.

(5) $(y-3x^2)\mathrm{d}x+(x-4y)\mathrm{d}y=0.$

(6) $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{x-y}{x+y}.$

2. (10 points) Find the general solution of the following first-order system

$$\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}t} = x + 3y \\ \dfrac{\mathrm{d}y}{\mathrm{d}t} = -2x + y \end{cases}.$$

3. (10 point) Find the general solution of

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} - 2\frac{\mathrm{d}x}{\mathrm{d}t} - x(t) = (2t+1)\mathrm{e}^{-t}.$$

4. (10 point) Find all solutions of the following equation

$$xy' + 2y = xy^2 e^{\frac{1}{x}}.$$

5. (10 point) If $R : |x| \le 1, |y| \le 1$, compute the first two Picard's sequence of the following initial value problem

$$\begin{cases} \dfrac{\mathrm{d}y}{\mathrm{d}x} = x^2 + 2y^2 \\ y(0) = 0 \end{cases},$$

and find the existence interval of the solution on R.

6. (10 point) Suppose that $\mu = \mu(x, y)$ is an integrating factor of

$$M(x, y)\,\mathrm{d}x + N(x, y)\,\mathrm{d}y = 0,$$

such that

$$\mathrm{d}\Phi(x, y) = \mu(x, y)M(x, y)\,\mathrm{d}x + \mu(x, y)N(x, y)\,\mathrm{d}y.$$

Show that $\mu(x, y)g(\Phi(x, y))$ is also an integrating factor of $M(x, y)\,\mathrm{d}x + N(x, y)\,\mathrm{d}y = 0$, where $g(t)$ is any derivative function.

7. (10 point) Let $y=\varphi(x)$ be a solution of $y''+p(x)y'+q(x)y=0$ where $p(x), q(x)$ is continuous, show that the general solution of the equation is

$$y=\varphi(x)\left[C_1+C_2\int_{x_0}^{x}\frac{1}{\varphi^2(s)}\mathrm{e}^{-\int_{x_0}^{s}p(t)\,\mathrm{d}t}\,\mathrm{d}s\right].$$

8. (10 point) If $\Phi(t)$ is a fundamental solution matrix of

$$\frac{\mathrm{d}\vec{x}}{\mathrm{d}t} = A\vec{x}(t)$$

Try to prove that

(1) the solution of initial value problem

$$\begin{cases} \dfrac{\mathrm{d}\vec{x}}{\mathrm{d}t} = A\vec{x}(t) \\ \vec{x}(t_0) = \vec{x}^0 \end{cases}$$

is

$$\Phi(t)\Phi^{-1}(t_0)\vec{x}^0.$$

(2) the relation $\Phi(t)\Phi^{-1}(t_0) = \mathrm{e}^{A(t-t_0)}$ is hold.