

Control Systems

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CONTENTS

1 Feedback Circuits

1

Abstract—This manual is an introduction to control systems based on GATE problems. Links to sample Python codes are available in the text.

Download python codes using

svn co <https://github.com/gadepall/school/trunk/control/codes>

1 FEEDBACK CIRCUITS

1.0.1. draw the block diagram of a Feedback Transconductance Amplifier(series-series)

Solution: below figure gives us the required block diagram

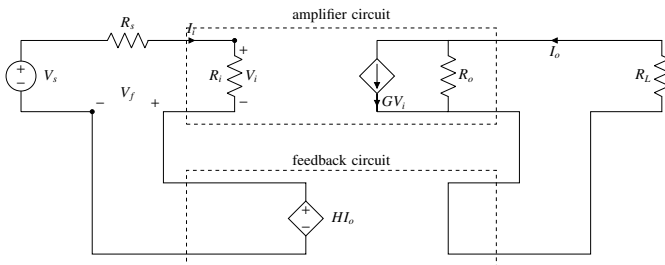


Fig. 1.0.1: Transconductance Amplifier

1.0.2. draw the equivalent amplifier circuit block diagram of fig.1.0.1

Solution:

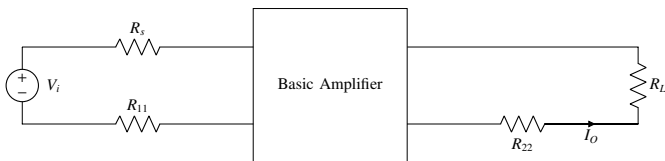


Fig. 1.0.2: G block diagram

$$G = \frac{I_o}{V_i} \quad (1.0.2.1)$$

R_{11} and R_{22} are obtained from fig.1.0.2

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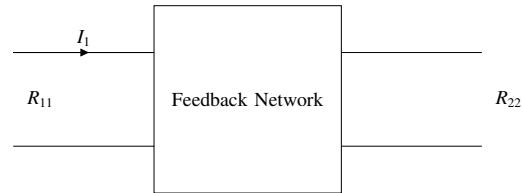


Fig. 1.0.2: feedback network

1.0.3. draw the equivalent feedback circuit block diagram of fig.1.0.1

Solution:

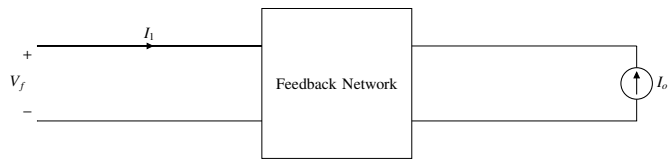


Fig. 1.0.3: H block diagram

$$H = \frac{V_f}{I_o} \bigg|_{I_i=0} \quad (1.0.3.1)$$

1.0.4. Part of the circuit of the MC1553 Amplifier is shown in circuit1 in fig.1.0.4 ,Answer below questions using equivalent block diagrams in fig.1.0.1 and values from Table 1.0.4

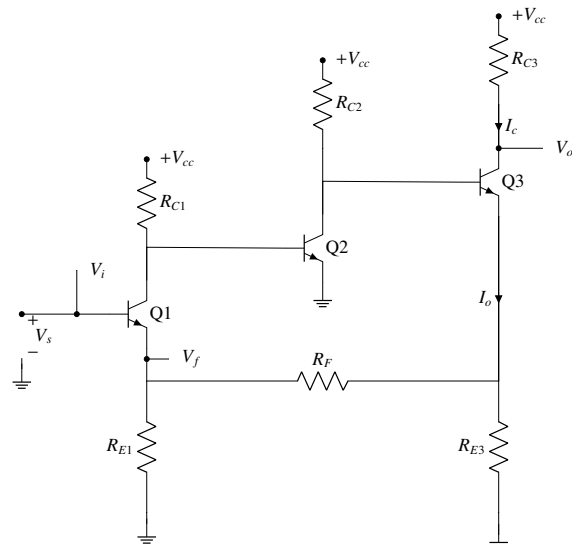


Fig. 1.0.4: circuit1

Parameter	Value
R_{C1}	$9k\Omega$
R_{E1}	100Ω
R_{C2}	$5k\Omega$
R_F	640Ω
R_{E2}	100Ω
R_{C3}	600Ω
h_{fe}	100
r_o	$\infty\Omega$
I_{C1}	0.6mA
I_{C2}	1mA
I_{C3}	4mA
r_{e1}	41.7Ω
$r_{\pi2}$	$2.5k\Omega$
α_1	0.99
g_{m2}	40mA/V
r_{e3}	6.25Ω
r_{o3}	$25k\Omega$
$r_{\pi3}$	625Ω

TABLE 1.0.4: parameters

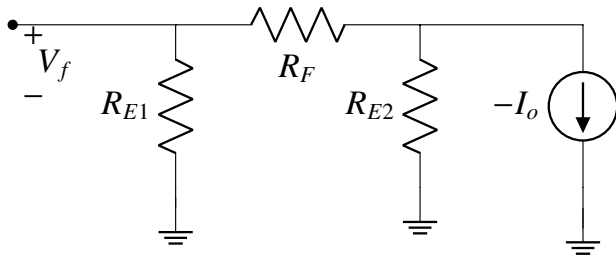


Fig. 1.0.4: circuit2

1.0.5. use feedback analysis to find open loop gain G

Solution: employing equivalent amplifier block diagram fig.1.0.2 into circuit in fig.1.0.4, we obtain circuit3 given in fig.1.0.5 to find $G = \frac{I_o}{V_i}$ we determine the gain of first stage, this is written by inspection as-

$$\frac{V_{c1}}{V_i} = \frac{-\alpha(R_{c1} \parallel r_{\pi2})}{r_{e1} + (R_{E1} \parallel (R_F + R_{E2}))} \quad (1.0.5.1)$$

using values from 1.0.4

$$\frac{V_{c1}}{V_i} = -14.92V/V \quad (1.0.5.2)$$

Next, we determine the gain of the second stage, which

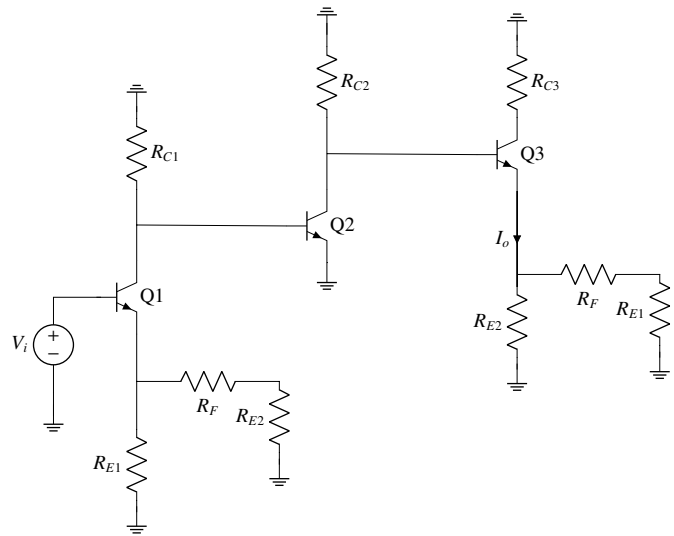


Fig. 1.0.5: circuit3

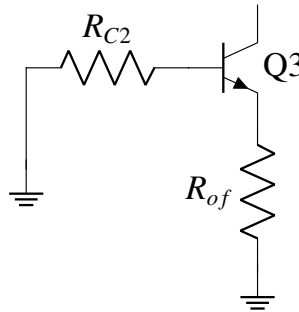


Fig. 1.0.5: circuit4

can be written by inspection (noting that $V_{b2} = V_{c1}$) as

$$\frac{V_{c2}}{V_{c1}} = -g_{m2}R_{c2} \parallel (h_{fe} + 1)[r_{e3} + (R_{E2} \parallel (R_F + R_{E1}))] \quad (1.0.5.3)$$

substituting, results in

$$\frac{V_{c2}}{V_{c1}} = -131.2V/V \quad (1.0.5.4)$$

Finally, for the third stage we can write by inspection

$$\frac{I_o}{V_{c2}} = \frac{I_{e3}}{V_{b3}} = \frac{1}{r_{e3} + (R_{E2} \parallel (R_F + R_{E1}))} \quad (1.0.5.5)$$

substituting values from 1.0.4 gives

$$\frac{I_o}{V_{c2}} = 10.6mA/V \quad (1.0.5.6)$$

combining the gains of the three stages results in

$$G = \frac{I_o}{V_i} = -14.92 \times -131.2 \times 10.6 \times 10^{-3} = 20.7A/V \quad (1.0.5.7)$$

1.0.6. Find Feedback Factor H

Solution: employing the equivalent feedback circuit

block diagram we get circuit2 in fig.1.0.4

$$H = \frac{V_f}{I_0} = \frac{R_{E2}}{R_{E2} + R_F + R_{E1}} \times R_{E1} \quad (1.0.6.1)$$

$$H = \frac{100}{100 + 640 + 100} \times 100 = 11.9\Omega \quad (1.0.6.2)$$

1.0.7. Find closed loop gain T and Voltage Gain V_0/V_s

Solution:

$$T = \frac{I_0}{V_s} = \frac{G}{1 + GH} = \frac{20.7}{1 + 20.7 \times 11.9} = 83.7mA/V \quad (1.0.7.1)$$

the voltage gain is found from

$$\frac{V_0}{V_s} = \frac{-I_c R_{C3}}{V_s} \approx \frac{-I_0 R_{C3}}{V_s} = -T R_{C3} \quad (1.0.7.2)$$

$$= -83.7 \times 10^{-3} \times 600 = -50.2V/V \quad (1.0.7.3)$$

1.0.8. Now assume Loop gain is large and find approximate expression for closed loop gain $T = \frac{I_0}{V_s}$

Solution: When $GH \gg 1$,

$$T = \frac{I_0}{V_s} \approx \frac{1}{H} \quad (1.0.8.1)$$

as

$$H = \frac{V_f}{I_0} = \frac{R_{E2}}{R_{E2} + R_F + R_{E1}} \times R_{E1} \quad (1.0.8.2)$$

$$= \frac{100}{100 + 640 + 100} \times 100 = 11.9\Omega \quad (1.0.8.3)$$

thus,

$$T = \frac{1}{11.9} = 84mA/V \quad (1.0.8.4)$$

$$\frac{I_c}{V_s} \approx \frac{I_0}{V_s} = 84mA/V \quad (1.0.8.5)$$

which we note is very close to the approximate value found in (1.0.7.1)

1.0.9. Find R_{in} and R_{out} for circuit in fig.1.0.4

Solution:

$$R_{in} = R_{if} = R_i(1 + GH) \quad (1.0.9.1)$$

where R_i is the input resistance of the G circuit. The value of R_i can be found from the circuit in fig.1.0.5 as follows:

$$R_i = (h_{fe} + 1)(r_{e1} + (R_{E1} \parallel (R_F + R_{E2}))) = 13.65K\Omega \quad (1.0.9.2)$$

$$R_{if} = 13.65(1 + 20.7 \times 11.9) = 3.38M\Omega \quad (1.0.9.3)$$

$$R_{of} = R_o(1 + GH) \quad (1.0.9.4)$$

where R_o can be determined to be

$$R_o = (R_{E2} \parallel (R_F + R_{E1})) + r_{e3} + \frac{R_{C2}}{h_{fe} + 1} \quad (1.0.9.5)$$

from values in Table 1.0.4, yields $R_o = 143.9\Omega$. The output resistance R_{of} of the feedback amplifier can now be found as

$$R_{of} = R_o(1 + GH) = 143.9(1 + 20.7 \times 11.9) = 35.6K\Omega \quad (1.0.9.6)$$

R_{out} is found by using circuit4 in fig.1.0.5

$$R_{out} = r_{o3} + [R_{of} \parallel (r_{\pi3} + R_{C2})](1 + g_{m3}r_{o3} \frac{r_{\pi3}}{r_{\pi3} + R_{C2}}) \quad (1.0.9.7)$$

$$= 25 + [35.6 \parallel (5.625)][1 + 160 \times 25 \frac{0.625}{5.625}] = 2.19M\Omega \quad (1.0.9.8)$$

thus R_{out} is increased (from r_{o3}) but not by $(1+GH)$

1.0.10. put the obtained parameters in a table

Solution:

Parameter	Value
G	20.7A/V
H	11.9Ω
T	83.7mA/V
V_o/V_s	-50.2V/V
R_{in}	3.38MΩ
R_{out}	2.19MΩ
R_{of}	35.6kΩ

TABLE 1.0.10: parameters

1.0.11. Represent this amplifier in a control system Block Diagram

Solution: figure in fig.1.0.11 represents our control system

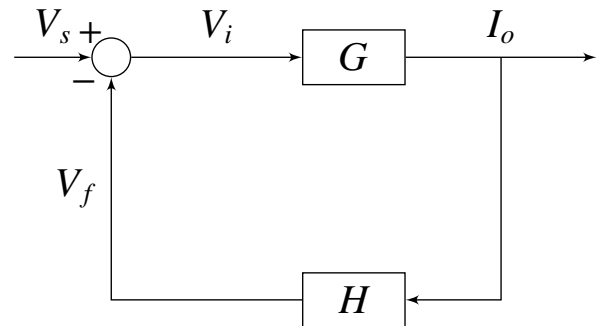


Fig. 1.0.11: block diagram

1.0.12. write a code for doing calculations and verify the values obtained in 1.0.10

Solution: following code does all the calculations of above equations to give parameters in 1.0.10

```
codes/ee18btech11007/circuit_calc.py
```