Control Systems

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Contents

***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```

1 Feedback Circuits

1.0.1. Part of the circuit of the MC1553 Amplifier is shown in circuit1 in fig.1.0.1 ,Answer below questions using values from Table 1.0.1

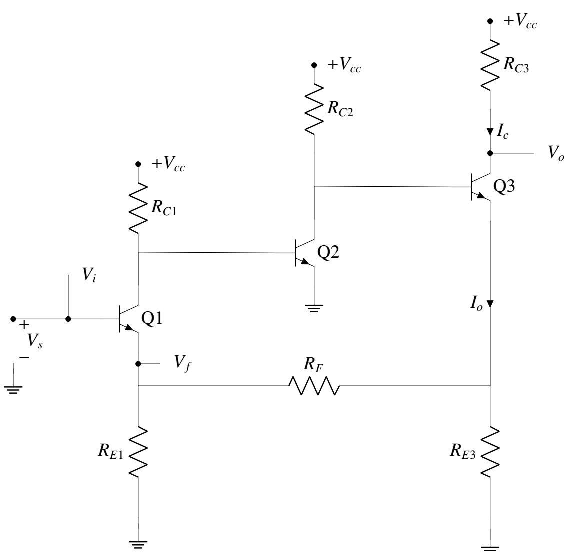

Fig. 1.0.1: circuit1

1.0.2. use feedback analysis to find open loop gain G

Solution: employing loading rules in fig.1.0.1,we obtain circuit3 given in fig.1.0.2 to find $G = \frac{I_0}{V_i}$ we determine the gain of first stage,this is written by inspection as-

$$\frac{V_{c1}}{V_i} = \frac{-\alpha(R_{c1} \| r_{\pi 2})}{r_{e1} + (R_{E1} \| (R_F + R_{E2}))} \tag{1.0.2.1}$$

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Parameter	Value
R_{C1}	$9k\Omega$
R_{E1}	100Ω
R_{C2}	$5k\Omega$
R_F	640Ω
R_{E2}	100Ω
R_{C3}	600Ω
h_{fe}	**100**
r_o	$\infty\Omega$
I_{C1}	**0.6mA**
I_{C2}	**1mA**
I_{C3}	**4mA**
r_{e1}	41.7Ω
$r_{\pi 2}$	$2.5k\Omega$
$\alpha 1$	**0.99**
g_{m2}	**40mA/V**
r_{e3}	6.25Ω
r_{o3}	$25k\Omega$
$r_{\pi 3}$	625Ω

TABLE 1.0.1: parameters

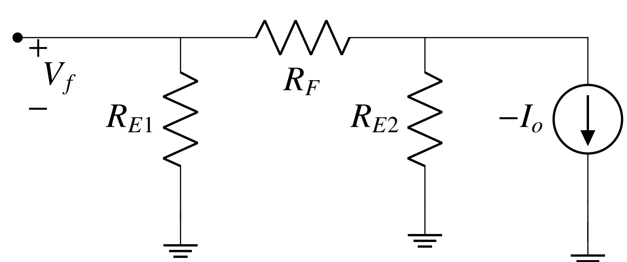

Fig. 1.0.1: circuit2

using values from 1.0.1

$$\frac{V_{c1}}{V_i} = -14.92V/V \tag{1.0.2.2}$$

Next, we determine the gain of the second stage,which

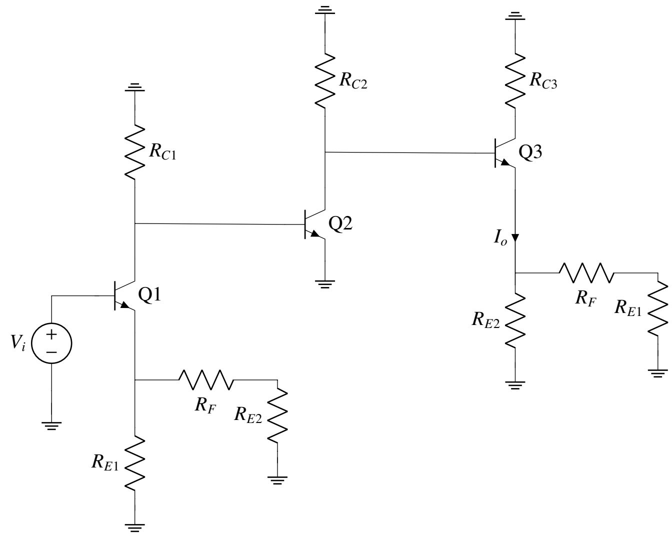

Fig. 1.0.2: circuit3

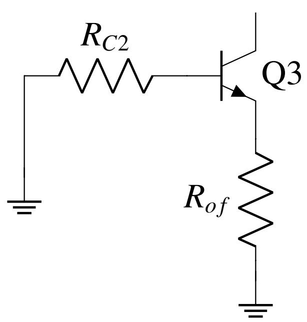

Fig. 1.0.2: circuit4

can be written by inspection(noting that $V_{b2} = V_{c1}$)as

$$\frac{V_{c2}}{V_{c1}} = -g_{m2}R_{c2}||(h_{fe} + 1)[r_{e3} + (R_{E2}||(R_F + R_{E1}))] \tag{1.0.2.3}$$

substituting ,results in

$$\frac{V_{c2}}{V_{c1}} = -131.2V/V \tag{1.0.2.4}$$

Finally,for the third stage we can write by inspection

$$\frac{I_0}{V_{c2}} = \frac{I_{e3}}{V_{b3}} = \frac{1}{r_{e3} + (R_{E2}||(R_F + R_{E1}))} \tag{1.0.2.5}$$

substituing values from 1.0.1 gives

$$\frac{I_0}{V_{c2}} = 10.6mA/V \tag{1.0.2.6}$$

combining the gains of the three stags results in

$$G = \frac{I_0}{V_i} = -14.92 \times -131.2 \times 10.6 \times 10^{-3} = 20.7A/V \tag{1.0.2.7}$$

1.0.3. Find Feedback Factor H
Solution: feedback factor H can be found from feedback network,The feedback network consists of resistors R_{E1}, R_F, R_{E2} using circuit2 in fig.1.0.1 we get

$$H = \frac{V_f}{I_0} = \frac{R_{E2}}{R_{E2} + R_F + R_{E1}} \times R_{E1} \tag{1.0.3.1}$$

$$H = \frac{100}{100 + 640 + 100} \times 100 = 11.9\Omega \tag{1.0.3.2}$$

1.0.4. Find closed loop gain T and Voltage Gain V_0/V_s
Solution:

$$T = \frac{I_0}{V_s} = \frac{G}{1 + GH} = \frac{20.7}{1 + 20.7 \times 11.9} = 83.7mA/V \tag{1.0.4.1}$$

the voltage gain is found from

$$\frac{V_0}{V_s} = \frac{-I_c R_{c3}}{V_s} \approx \frac{-I_0 R_{C3}}{V_s} = -TR_{C3} \tag{1.0.4.2}$$

$$= -83.7 \times 10^{-3} \times 600 = -50.2V/V \tag{1.0.4.3}$$

1.0.5. Now assume Loop gain is large and find approximate expression for closed loop gain $T = \frac{I_o}{V_s}$
Solution: When GH >>1,

$$T = \frac{I_0}{V_s} \approx \frac{1}{H} \tag{1.0.5.1}$$

as

$$H = \frac{V_f}{I_0} = \frac{R_{E2}}{R_{E2} + R_F + R_{E1}} \times R_{E1} \tag{1.0.5.2}$$

$$= \frac{100}{100 + 640 + 100} \times 100 = 11.9\Omega \tag{1.0.5.3}$$

thus,

$$T = \frac{1}{11.9} = 84mA/V \tag{1.0.5.4}$$

$$\frac{I_c}{V_s} \approx \frac{I_0}{V_s} = 84mA/V \tag{1.0.5.5}$$

which we note is very close to the approximate value found in (1.0.4.1)

1.0.6. Find Voltage gain $\frac{V_0}{V_s}$ for above approximation
Solution:

$$\frac{V_0}{V_s} = \frac{-I_c R_{C3}}{V_s} = -84 \times 0.6 = -50.4V/V \tag{1.0.6.1}$$

1.0.7. Find R_{in} and R_{out} for circuit in fig.1.0.1
Solution:

$$R_{in} = R_{if} = R_i(1 + GH) \tag{1.0.7.1}$$

where R_i is the input resistance of the G circuit.The value of R_i can be found from the circuit in fig.1.0.2 as follows:

$$R_i = (h_{fe} + 1)(r_{e1} + (R_{E1}||(R_F + R_{E2}))) = 13.65K\Omega \tag{1.0.7.2}$$

$$R_{if} = 13.65(1 + 20.7 \times 11.9) = 3.38M\Omega \quad (1.0.7.3)$$

$$R_{of} = R_o(1 + GH) \quad (1.0.7.4)$$

where R_o can be determined to be

$$R_o = (R_{E2} \| (R_F + R_{E1})) + r_{e3} + \frac{R_{C2}}{h_{fe} + 1} \quad (1.0.7.5)$$

from values in Table 1.0.1, yields $R_o = 143.9\Omega$. The output resistance R_{of} of the feedback amplifier can now be found as

$$R_{of} = R_o(1 + GH) = 143.9(1 + 20.7 \times 11.9) = 35.6K\Omega \quad (1.0.7.6)$$

R_{out} is found by using circuit4 in fig.1.0.2

$$R_{out} = r_{o3} + [R_{of} \| (r_{\pi 3} + R_{C2})](1 + g_{m3} r_{o3} \frac{r_{\pi 3}}{r_{\pi 3} + R_{C2}}) \quad (1.0.7.7)$$

$$= 25 + [35.6 \| (5.625)][1 + 160 \times 25 \frac{0.625}{5.625}] = 2.19M\Omega \quad (1.0.7.8)$$

thus R_{out} is increased (from r_{o3}) but not by (1+GH)

1.0.8. put the obtained parameters in a table
Solution:

Parameter	**Value**
G	**20.7A/V**
H	11.9Ω
T	**83.7mA/V**
V_o/V_s	**-50.2V/V**
R_{in}	$3.38M\Omega$
R_{out}	$2.19M\Omega$
R_{of}	$35.6k\Omega$

TABLE 1.0.8: parameters

1.0.9. Represent this amplifier in a control system Block Diagram
Solution: figure in fig.1.0.9 represents our control system

1.0.10. write a code for doing calculations and verify the values obtained in 1.0.8
Solution: following code does all the calculations of above equations to give parameters in 1.0.8

```
codes/ee18btech11007/circuit_calc.py
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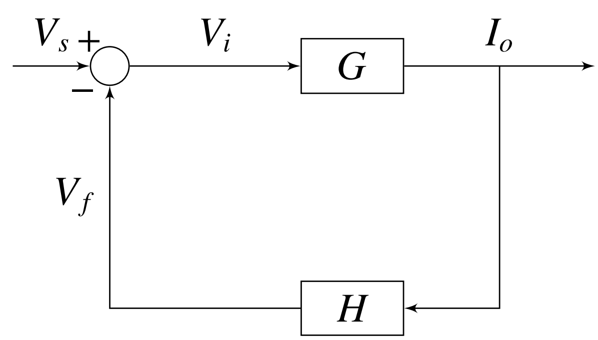

Fig. 1.0.9: block diagram