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Abstract—This manual is an introduction to control systems based on GATE problems. Links to sample Python codes are available in the text.

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
control/codes
```

1 STABILITY

1.1 Second order System

2 ROUTH HURWITZ CRITERION

3 COMPENSATORS

4 NYQUIST PLOT

- 4.1. Question-The open loop transfer function of a unity feedback system is given by

$$G(s) = \frac{\pi e^{-0.25s}}{s}$$

- 4.2. Find $\text{Re}\{G(j\omega)\}$ and $\text{Im}\{G(j\omega)\}$

Solution: substitute

$$S = j\omega \quad (4.2.1)$$

we get

$$G(j\omega) = -\frac{\pi}{\omega}(\sin 0.25\omega + j \cos 0.25\omega) \quad (4.2.2)$$

$$\text{Re}\{G(j\omega)\} = -\frac{\pi}{\omega}(\sin 0.25\omega) \quad (4.2.3)$$

$$\text{Im}\{G(j\omega)\} = -\frac{\pi}{\omega}(j \cos 0.25\omega) \quad (4.2.4)$$

- 4.3. Sketch the Nyquist plot.

Solution: The Nyquist plot is a graph of $\text{Re}\{G(j\omega)\}$ vs $\text{Im}\{G(j\omega)\}$. The following python code generates the Nyquist plot in Fig. 4.3

```
codes/ee18btech11007/nyquist.py
```

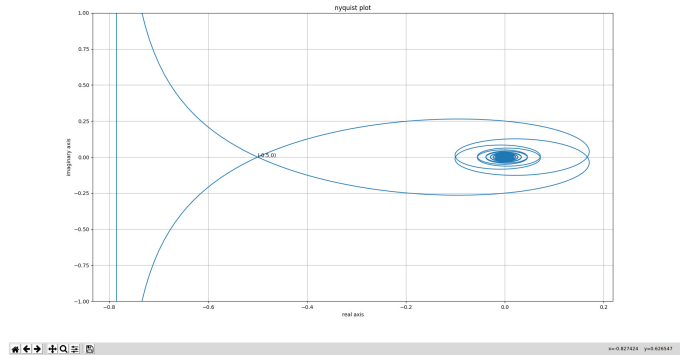


Fig. 4.3

- 4.4. Find the point at which the Nyquist plot of $G(s)$ passes through negative real axis

Solution:

$$G(s) = \frac{\pi e^{-0.25s}}{s} \quad (4.4.1)$$

Nyquist plot cuts the negative real Axis at $\omega =$ phase cross over frequency, at phase cross over frequency the phase of nyquist plot becomes $-\pi$ radians. substitute

$$s = j\omega. \quad (4.4.2)$$

$$G(j\omega) = -\frac{\pi}{\omega}(\sin 0.25\omega + j \cos 0.25\omega) \quad (4.4.3)$$

$$\angle G(j\omega) = -\pi/2 - 0.25\omega. \quad (4.4.4)$$

$$\angle G(j\omega)|_{\omega=\omega_{pc}} = -\pi \quad (4.4.5)$$

by solving for ω we get $\omega_{pc} = 2\pi$.
magnitude at any point is

$$X = |G(j\omega)| = \frac{\pi}{\omega}. \quad (4.4.6)$$

substituting $\omega = 2\pi$ in magnitude equation we get $X=0.5$.

so it intersects at $(-0.5,0j)$

we can verify with the following plot that it intersects at $(-0.5,0j)$

- 4.5. Use Nyquist stability criterion to determine if the system is stable?

Solution: Nyquist Stability Criterion - for the stability of a closed loop transfer function

$G(s)/(1+G(s)*H(s))$,the number of poles of $G(s)*H(s)$ on right half of s-plane must equal the number of encirclement of nyquist plot of $G(s)*H(s)$ about the critical point $-1+0j$

we must find

$$Z = P + N \quad (4.5.1)$$

Variable	Description
Z	no of poles of $G(s)/(1+G(s)*H(s))$ in the right half of s-plane
P	no of poles of $G(s)*H(s)$ in right half of s-plane
N	no of encirclement of nyquist plot of $G(s)*H(s)$ about $-1+j0$ in clockwise direction

NOTE-

- If a clockwise contour does not encircle zeros nor poles, then the plot will not encircle the origin.
- If a clockwise contour encircles a zero, then the plot will encircle the origin clockwise once.
- If a clockwise contour encircles a pole, then the plot will encircle the origin counterclockwise once.
- clockwise encirclement of nyquist plot is taken as positive and counterclockwise encirclement of nyquist plot is taken as negative

from plot we get $N=0$ because there are no encirclements about $-1+0j$,and we already know $P=0$ since our $G(s)$ doesnt have any poles on right half of s-plane

$$Z = 0 + 0 = 0 \quad (4.5.2)$$

∴ the system is stable.