

Control Systems

G V V Sharma*

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Abstract—This manual is an introduction to control systems based on GATE problems. Links to sample Python codes are available in the text.

Download python codes using

svn co <https://github.com/gadepall/school/trunk/control/codes>

1 MASON'S GAIN FORMULA

1.1. The Block diagram of a system is illustrated in the figure shown, where $X(s)$ is the input and $Y(s)$ is the output. Draw the equivalent signal flow graph.

*The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India e-mail: gadepall@iith.ac.in. All content in this manual is released under GNU GPL. Free and open source.

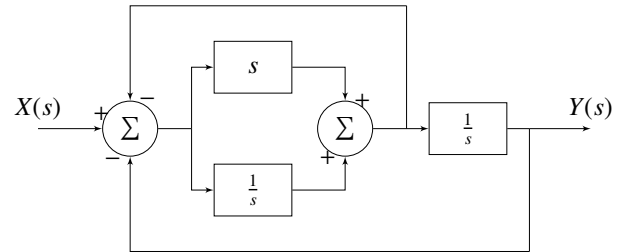


Fig. 1.1.1: Block Diagram

Solution: The signal flow graph of the block diagram in Fig. 1.1.1 is available in Fig. 1.1.2

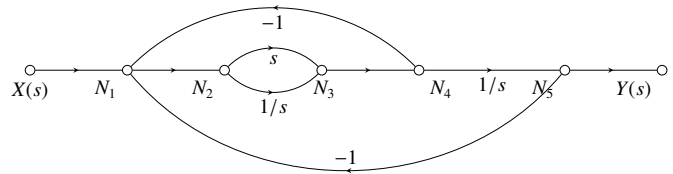


Fig. 1.1.2: Signal Flow Graph

1.2. Draw all the forward paths in Fig. 1.1.2 and compute the respective gains.

Solution: The forward paths are available in Figs. 1.2.3 and 1.2.4. The respective gains are

$$P_1 = s \left(\frac{1}{s} \right) = 1 \quad (1.2.1)$$

$$P_2 = (1/s)(1/s) = 1/s^2 \quad (1.2.2)$$

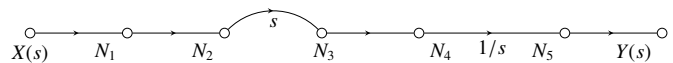


Fig. 1.2.3: P_1

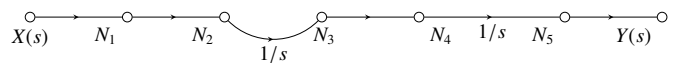


Fig. 1.2.4: P_2

1.3. Draw all the loops in Fig. 1.1.2 and calculate the respective gains.

Solution: The loops are available in Figs. 1.3.5-1.3.8 and the corresponding gains are

$$L_1 = (-1)(s) = -s \quad (1.3.1)$$

$$L_2 = s\left(\frac{1}{s}\right)(-1) = -1 \quad (1.3.2)$$

$$L_3 = \left(\frac{1}{s}\right)(-1) = -\frac{1}{s} \quad (1.3.3)$$

$$L_4 = \left(\frac{1}{s}\right)\left(\frac{1}{s}\right)(-1) = -\frac{1}{s^2} \quad (1.3.4)$$

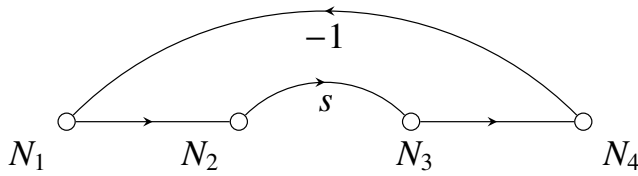


Fig. 1.3.5: L_1

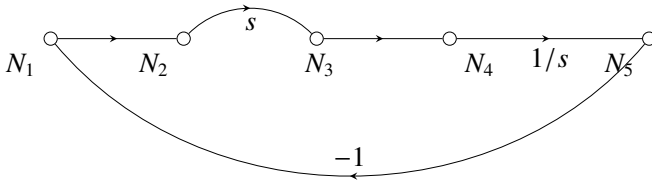


Fig. 1.3.6: L_2

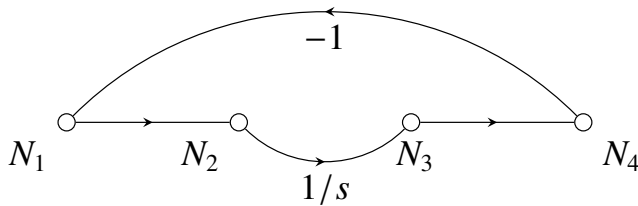


Fig. 1.3.7: L_3

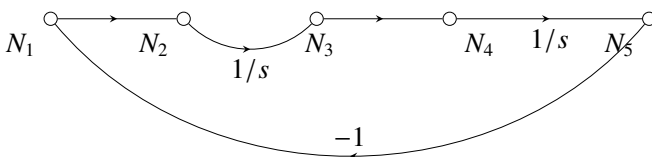


Fig. 1.3.8: L_4

1.4. State Mason's Gain formula and explain the parameters through a table.

Solution: According to Mason's Gain Formula,

$$T = \frac{Y(s)}{X(s)} \quad (1.4.1)$$

$$= \frac{\sum_{i=1}^N P_i \Delta_i}{\Delta} \quad (1.4.2)$$

where the parameters are described in Table 1.4

Variable	Description
P_i	i th forward path
L_j	j th loop
Δ	$1 - \sum L_i + \sum_{L_i \cap L_j = \phi} L_i L_j - \sum_{L_i \cap L_j \cap L_k = \phi} L_i L_j L_k + \dots$
Δ_i	$1 - \sum_{L_k \cap P_i = \phi} L_k + \sum_{L_k \cap L_j \cap P_i = \phi} L_k L_j - \dots$

TABLE 1.4

1.5. List the parameters in Table 1.4 for Fig. 1.1.2.

Solution: The parameters are available in Table 1.5

Path	Value	Parameter	Value	Remarks
P_1	1	Δ_1	1	All loops intersect with P_1
P_2	$\frac{1}{s^2}$	Δ_2	1	All loops intersect with P_2
L_1	$-s$	Δ	$1 - \sum L_i$	All loops intersect
L_2	-1			
L_3	$-\frac{1}{s}$			
L_4	$-\frac{1}{s^2}$			

TABLE 1.5

1.6. Find the transfer function using Mason's Gain Formula.

Solution: From (1.4.2) and 1.5,

$$T(s) = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta} \quad (1.6.1)$$

$$= \frac{1 + \frac{1}{s^2}}{1 - (-s - 1 - \frac{1}{s} - \frac{1}{s^2})} \quad (1.6.2)$$

$$= \frac{s^2 + 1}{s^3 + 2s^2 + s + 1} \quad (1.6.3)$$

after simplification.

1.7. Write a program to compute Mason's gain formula, given the branch nodes and gains for

each path.

Solution: The following code computes Masons Gain

codes/MasonsGain.py

2 BODE PLOT

2.1 Introduction

2.2 Example

3 SECOND ORDER SYSTEM

3.1 Damping

3.2 Example

4 ROUTH HURWITZ CRITERION

4.1 Routh Array

4.2 Marginal Stability

4.3 Stability

5 STATE-SPACE MODEL

5.1 Controllability and Observability

5.2 Second Order System

6 NYQUIST PLOT

7 COMPENSATORS

8 PHASE MARGIN