

# Control Systems

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**Abstract**—This manual is an introduction to control systems based on GATE problems. Links to sample Python codes are available in the text.

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
control/codes
```

## 1 MASON'S GAIN FORMULA

1.1. The Block diagram of a system is illustrated in the figure shown, where  $X(s)$  is the input and  $Y(s)$  is the output. Draw the equivalent signal flow graph.

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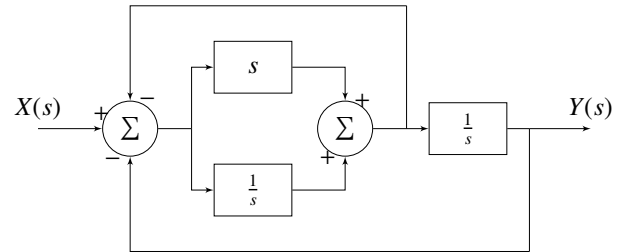


Fig. 1.1.1: Block Diagram

**Solution:** The signal flow graph of the block diagram in Fig. 1.1.1 is available in Fig. 1.1.2

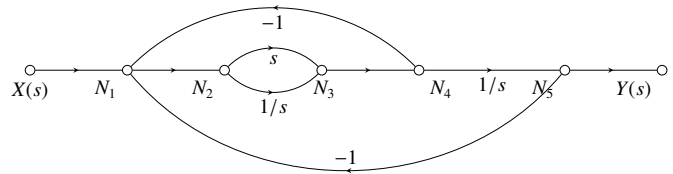


Fig. 1.1.2: Signal Flow Graph

1.2. Draw all the forward paths in Fig. 1.1.2 and compute the respective gains.

**Solution:** The forward paths are available in Figs. 1.2.3 and 1.2.4. The respective gains are

$$P_1 = s \left( \frac{1}{s} \right) = 1 \quad (1.2.1)$$

$$P_2 = (1/s)(1/s) = 1/s^2 \quad (1.2.2)$$

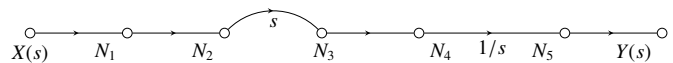


Fig. 1.2.3:  $P_1$

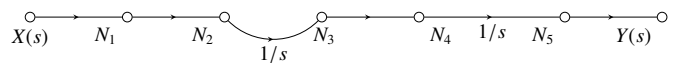


Fig. 1.2.4:  $P_2$

1.3. Draw all the loops in Fig. 1.1.2 and calculate the respective gains.

**Solution:** The loops are available in Figs. 1.3.5-1.3.8 and the corresponding gains are

$$L_1 = (-1)(s) = -s \quad (1.3.1)$$

$$L_2 = s\left(\frac{1}{s}\right)(-1) = -1 \quad (1.3.2)$$

$$L_3 = \left(\frac{1}{s}\right)(-1) = -\frac{1}{s} \quad (1.3.3)$$

$$L_4 = \left(\frac{1}{s}\right)\left(\frac{1}{s}\right)(-1) = -\frac{1}{s^2} \quad (1.3.4)$$

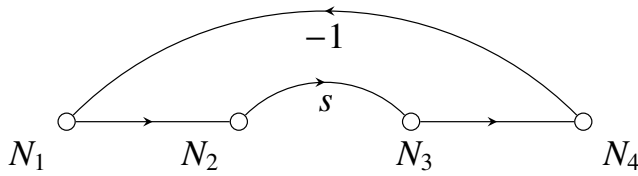


Fig. 1.3.5:  $L_1$

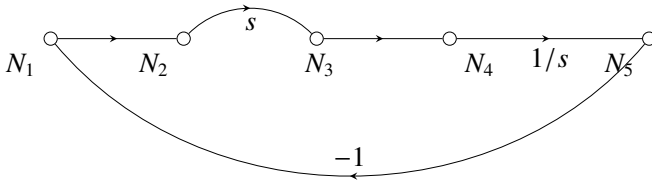


Fig. 1.3.6:  $L_2$

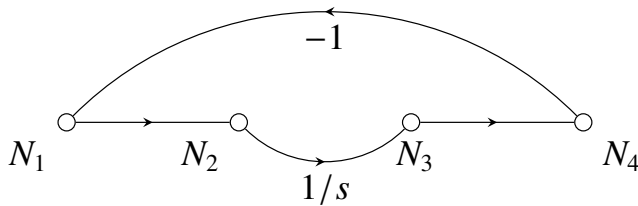


Fig. 1.3.7:  $L_3$

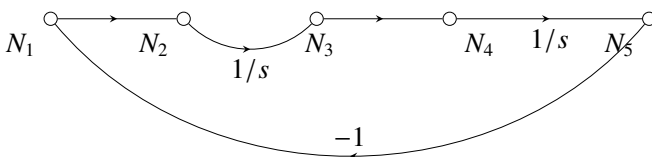


Fig. 1.3.8:  $L_4$

1.4. State Mason's Gain formula and explain the parameters through a table.

**Solution:** According to Mason's Gain Formula,

$$T = \frac{Y(s)}{X(s)} \quad (1.4.1)$$

$$= \frac{\sum_{i=1}^N P_i \Delta_i}{\Delta} \quad (1.4.2)$$

where the parameters are described in Table 1.4

Variable	Description
$P_i$	$i$ th forward path
$L_j$	$j$ th loop
$\Delta$	$1 - \sum L_i + \sum_{L_i \cap L_j = \phi} L_i L_j - \sum_{L_i \cap L_j \cap L_k = \phi} L_i L_j L_k + \dots$
$\Delta_i$	$1 - \sum_{L_k \cap P_i = \phi} L_k + \sum_{L_k \cap L_j \cap P_i = \phi} L_k L_j - \dots$

TABLE 1.4

1.5. List the parameters in Table 1.4 for Fig. 1.1.2.

**Solution:** The parameters are available in Table 1.5

Path	Value	Parameter	Value	Remarks
$P_1$	1	$\Delta_1$	1	All loops intersect with $P_1$
$P_2$	$\frac{1}{s^2}$	$\Delta_2$	1	All loops intersect with $P_2$
$L_1$	$-s$	$\Delta$	$1 - \sum L_i$	All loops intersect
$L_2$	$-1$			
$L_3$	$-\frac{1}{s}$			
$L_4$	$-\frac{1}{s^2}$			

TABLE 1.5

1.6. Find the transfer function using Mason's Gain Formula.

**Solution:** From (1.4.2) and 1.5,

$$T(s) = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta} \quad (1.6.1)$$

$$= \frac{1 + \frac{1}{s^2}}{1 - (-s - 1 - \frac{1}{s} - \frac{1}{s^2})} \quad (1.6.2)$$

$$= \frac{s^2 + 1}{s^3 + 2s^2 + s + 1} \quad (1.6.3)$$

after simplification.

1.7. State the Equivalent Matrix Form of Mason's Gain formula

**Solution:** Mason's rule can be stated in a simple matrix form. Assume  $T$  is the transient matrix of the graph where

$$t_{nm} = [T_{nm}]$$

is sum transmittance of branches from node  $m$  toward node  $n$ . Then, the gain from node  $m$  to node  $n$  of the graph is equal to

$$u_{nm} = [U_{nm}]$$

where,

$$U = (I - T)^{-1} \quad (1.7.1)$$

and  $I$  is the identity matrix.

- 1.8. Find the Transfer Function using the matrix equivalent form of Masons Gain Formula

**Solution:** The transient matrix for 1.1.2 is

$$\mathbf{T} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & s + 1/s & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & 1/s \\ -1 & 0 & 0 & 0 & 0 \end{pmatrix} \quad (1.8.1)$$

$$\mathbf{I} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad (1.8.2)$$

$$\mathbf{I} - \mathbf{T} = \begin{pmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -s - 1/s & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 1 & 0 & 0 & 1 & -1/s \\ 1 & 0 & 0 & 0 & 1 \end{pmatrix} \quad (1.8.3)$$

$$U = (\mathbf{I} - \mathbf{T})^{-1} \quad (1.8.4)$$

finding

$$U_{01} = \frac{s^2 + 1}{s^3 + 2s^2 + s + 1} \quad (1.8.5)$$

gives us the transfer function

- 1.9. Write a program to compute Mason's gain formula, given the branch nodes and gains for each path.

**Solution:** The following code computes Masons Gain

```
codes/MasonsGain.py
```

## 2 BODE PLOT

### 2.1 Introduction

### 2.2 Example

## 3 SECOND ORDER SYSTEM

### 3.1 Damping

### 3.2 Example

## 4 ROUTH HURWITZ CRITERION

### 4.1 Routh Array

### 4.2 Marginal Stability

### 4.3 Stability

## 5 STATE-SPACE MODEL

### 5.1 Controllability and Observability

### 5.2 Second Order System

## 6 NYQUIST PLOT

## 7 COMPENSATORS

## 8 PHASE MARGIN