

# Control Systems

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### 1 Feedback Circuits

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**Abstract**—This manual is an introduction to control systems based on GATE problems. Links to sample Python codes are available in the text.

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/control/codes
```

#### 1 FEEDBACK CIRCUITS

1.0.1. Part of the circuit of the MC1553 Amplifier is shown in circuit1 in fig.1.0.1 ,Answer below questions using values from Table 1.0.1

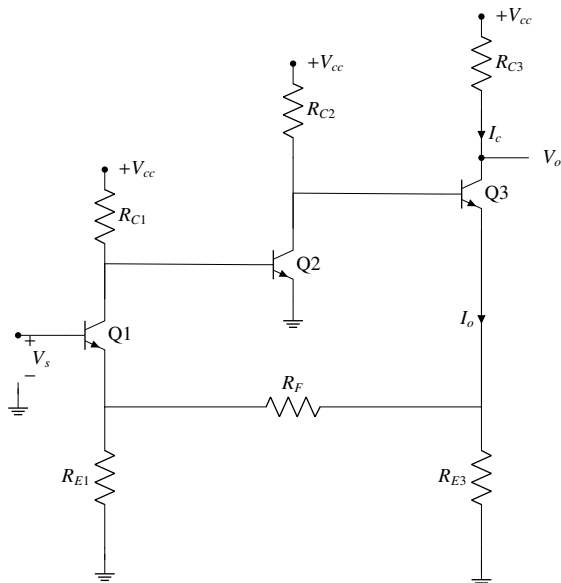


Fig. 1.0.1: circuit1

1.0.2. use feedback analysis to find open loop gain G

**Solution:** employing loading rules in fig.1.0.1, we obtain circuit3 given in fig.1.0.2 to find  $G = \frac{I_o}{V_i}$  we determine

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Parameter	Value
$R_{C1}$	$9k\Omega$
$R_{E1}$	$100\Omega$
$R_{C2}$	$5k\Omega$
$R_F$	$640\Omega$
$R_{E2}$	$100\Omega$
$R_{C3}$	$600\Omega$
$h_{fe}$	<b>100</b>
$r_o$	$\infty\Omega$
$I_{C1}$	<b>0.6mA</b>
$I_{C2}$	<b>1mA</b>
$I_{C3}$	<b>4mA</b>
$r_{e1}$	$41.7\Omega$
$r_{\pi2}$	$2.5k\Omega$
$\alpha_1$	<b>0.99</b>
$g_{m2}$	<b>40mA/V</b>
$r_{e3}$	$6.25\Omega$
$r_{o3}$	$25k\Omega$
$r_{\pi3}$	$625\Omega$

TABLE 1.0.1: parameters

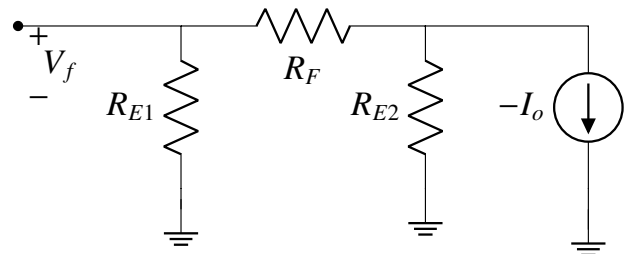


Fig. 1.0.1: circuit2

the gain of first stage, this is written by inspection as-

$$\frac{V_{c1}}{V_i} = \frac{-\alpha(R_{c1} || r_{\pi2})}{r_{e1} + (R_{E1} || (R_F + R_{E2}))} \quad (1.0.2.1)$$

using values from 1.0.1

$$\frac{V_{c1}}{V_i} = -14.92V/V \quad (1.0.2.2)$$

Next, we determine the gain of the second stage, which

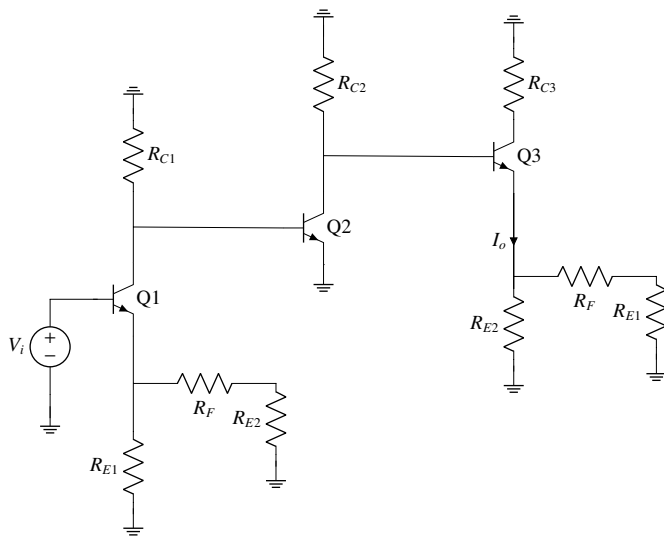


Fig. 1.0.2: circuit3

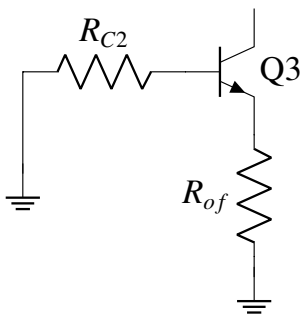


Fig. 1.0.2: circuit4

can be written by inspection (noting that  $V_{b2} = V_{c1}$ ) as

$$\frac{V_{c2}}{V_{c1}} = -g_{m2}R_{c2}[(h_{fe} + 1)(r_{e3} + (R_{E2} \parallel (R_F + R_{E1})))] \quad (1.0.2.3)$$

substituting, results in

$$\frac{V_{c2}}{V_{c1}} = -131.2V/V \quad (1.0.2.4)$$

Finally, for the third stage we can write by inspection

$$\frac{I_0}{V_{c2}} = \frac{I_{e3}}{V_{b3}} = \frac{1}{r_{e3} + (R_{E2} \parallel (R_F + R_{E1}))} \quad (1.0.2.5)$$

substituting values from 1.0.1 gives

$$\frac{I_0}{V_{c2}} = 10.6mA/V \quad (1.0.2.6)$$

combining the gains of the three stages results in

$$G = \frac{I_0}{V_i} = -14.92 \times -131.2 \times 10.6 \times 10^{-3} = 20.7A/V \quad (1.0.2.7)$$

### 1.0.3. Find Feedback Factor H

**Solution:** feedback factor H can be found from feed-

back network, The feedback network consists of resistors  $R_{E1}, R_F, R_{E2}$  using circuit2 in fig.1.0.1 we get

$$H = \frac{V_f}{I_0} = \frac{R_{E2}}{R_{E2} + R_F + R_{E1}} \times R_{E1} \quad (1.0.3.1)$$

$$H = \frac{100}{100 + 640 + 100} \times 100 = 11.9\Omega \quad (1.0.3.2)$$

### 1.0.4. Find closed loop gain T and Voltage Gain $V_0/V_s$

**Solution:**

$$T = \frac{I_0}{V_s} = \frac{G}{1 + GH} = \frac{20.7}{1 + 20.7 \times 11.9} = 83.7mA/V \quad (1.0.4.1)$$

the voltage gain is found from

$$\frac{V_0}{V_s} = \frac{-I_c R_{c3}}{V_s} \approx \frac{-I_0 R_{c3}}{V_s} = -T R_{c3} \quad (1.0.4.2)$$

$$= -83.7 \times 10^{-3} \times 600 = -50.2V/V \quad (1.0.4.3)$$

### 1.0.5. Now assume Loop gain is large and find approximate expression for closed loop gain $T = \frac{I_0}{V_s}$

**Solution:** When  $GH \gg 1$ ,

$$T = \frac{I_0}{V_s} \approx \frac{1}{H} \quad (1.0.5.1)$$

as

$$H = \frac{V_f}{I_0} = \frac{R_{E2}}{R_{E2} + R_F + R_{E1}} \times R_{E1} \quad (1.0.5.2)$$

$$= \frac{100}{100 + 640 + 100} \times 100 = 11.9\Omega \quad (1.0.5.3)$$

thus,

$$T = \frac{1}{11.9} = 84mA/V \quad (1.0.5.4)$$

$$\frac{I_c}{V_s} \approx \frac{I_0}{V_s} = 84mA/V \quad (1.0.5.5)$$

which we note is very close to the approximate value found in (1.0.4.1)

### 1.0.6. Find Voltage gain $\frac{V_0}{V_s}$ for above approximation

**Solution:**

$$\frac{V_0}{V_s} = \frac{-I_c R_{c3}}{V_s} = -84 \times 0.6 = -50.4V/V \quad (1.0.6.1)$$

### 1.0.7. Find $R_{in}$ and $R_{out}$ for circuit in fig.1.0.1

**Solution:**

$$R_{in} = R_{if} = R_i(1 + GH) \quad (1.0.7.1)$$

where  $R_i$  is the input resistance of the G circuit. The value of  $R_i$  can be found from the circuit in fig.1.0.2 as follows:

$$R_i = (h_{fe} + 1)(r_{e1} + (R_{E1} \parallel (R_F + R_{E2}))) = 13.65K\Omega \quad (1.0.7.2)$$

$$R_{if} = 13.65(1 + 20.7 \times 11.9) = 3.38M\Omega \quad (1.0.7.3)$$

$$R_{of} = R_o(1 + GH) \quad (1.0.7.4)$$

where  $R_o$  can be determined to be

$$R_o = (R_{E2} || (R_F + R_{E1})) + r_{e3} + \frac{R_{C2}}{h_{fe} + 1} \quad (1.0.7.5)$$

from values in Table 1.0.1, yields  $R_o = 143.9\Omega$ . The output resistance  $R_{of}$  of the feedback amplifier can now be found as

$$R_{of} = R_o(1 + GH) = 143.9(1 + 20.7 \times 11.9) = 35.6K\Omega \quad (1.0.7.6)$$

$R_{out}$  is found by using circuit4 in fig.1.0.2

$$R_{out} = r_{o3} + [R_{of} || (r_{\pi3} + R_{C2})] (1 + g_{m3} r_{o3} \frac{r_{\pi3}}{r_{\pi3} + R_{C2}}) \quad (1.0.7.7)$$

$$= 25 + [35.6 || (5.625)] [1 + 160 \times 25 \frac{0.625}{5.625}] = 2.19M\Omega \quad (1.0.7.8)$$

thus  $R_{out}$  is increased (from  $r_{o3}$ ) but not by  $(1+GH)$

1.0.8. put the obtained parameters in a table

**Solution:**

Parameter	Value
<b>G</b>	<b>20.7A/V</b>
<b>H</b>	<b>11.9Ω</b>
<b>T</b>	<b>83.7mA/V</b>
$V_o/V_s$	<b>-50.2V/V</b>
$R_{in}$	<b>3.38MΩ</b>
$R_{out}$	<b>2.19MΩ</b>
$R_{of}$	<b>35.6kΩ</b>

TABLE 1.0.8: parameters

1.0.9. Represent this amplifier in a control system Block Diagram

**Solution:** figure in fig.1.0.9 represents our control system

1.0.10. write a code for doing calculations and verify the values obtained in 1.0.8

**Solution:** following code does all the calculations of above equations to give parameters in 1.0.8

```
codes/ee18btech11007/circuit_calc.py
```

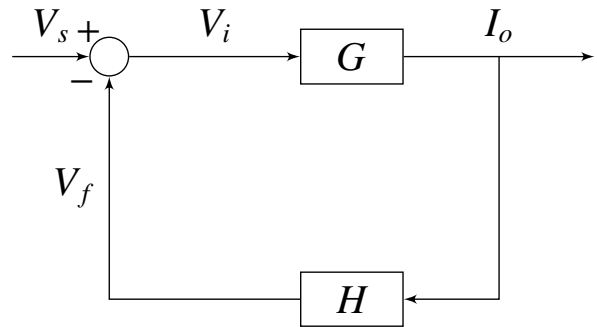


Fig. 1.0.9: block diagram