Control Systems

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Contents

***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```

1 Feedback Circuits

1.0.1. Part of the circuit of the MC1553 Amplifier is shown in circuit1 in fig.1.0.1 Assume the loop gain is large, find an approximate expression and value for the closed loop gain $T = \frac{I_0}{V_s}$ use values from Table 1.0.1

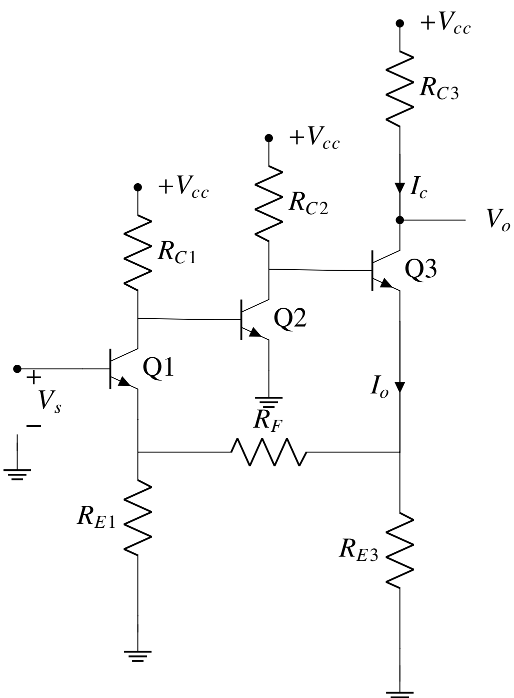

Fig. 1.0.1: circuit1

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Parameter	**Value**
R_{C1}	**9kΩ**
R_{E1}	**100Ω**
R_{C2}	**5kΩ**
R_F	**640Ω**
R_{E2}	**100Ω**
R_{C3}	**600Ω**
h_{fe}	**100**
r_o	∞Ω
I_{C1}	**0.6mA**
I_{C2}	**1mA**
I_{C3}	**4mA**
r_{e1}	41.7Ω
$r_{\pi 2}$	**2.5kΩ**
$\alpha 1$	**0.99**
g_{m2}	**40mA/V**
r_{e3}	6.25Ω
r_{o3}	$25k\Omega$
$r_{\pi 3}$	625Ω

TABLE 1.0.1: parameters

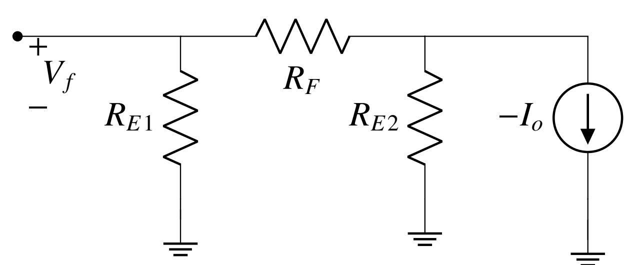

Fig. 1.0.1: circuit2

Solution: When GH >>1,

$$T = \frac{I_0}{V_s} \approx \frac{1}{H} \tag{1.0.1.1}$$

feedback factor H can be found from feedback network.The feedback network consists of resistors

R_{E1}, R_F, R_{E2} using circuit2 in fig.1.0.1 we get

$$H = \frac{V_f}{I_0} = \frac{R_{E2}}{R_{E2} + R_F + R_{E1}} \times R_{E1} \quad (1.0.1.2)$$

$$= \frac{100}{100 + 640 + 100} \times 100 = 11.9\Omega \quad (1.0.1.3)$$

thus,

$$T \approx \frac{1}{H} \quad (1.0.1.4)$$

$$= \frac{1}{R_{E2}}(1 + \frac{R_{E2} + R_F}{R_{E1}}) \quad (1.0.1.5)$$

$$= \frac{1}{11.9} = 84mA/V \quad (1.0.1.6)$$

$$\frac{I_c}{V_s} \approx \frac{I_0}{V_s} = 84mA/V \quad (1.0.1.7)$$

1.0.2. Find Voltage gain $\frac{V_0}{V_s}$ for above approximation
Solution:

$$\frac{V_0}{V_s} = \frac{-I_c R_{C3}}{V_s} = -84 \times 0.6 = -50.4V/V \quad (1.0.2.1)$$

1.0.3. use feedback analysis to find open loop gain G
Solution: employing loading rules in fig.1.0.1,we obtain circuit3 given in fig.1.0.3 to find $G = \frac{I_0}{V_i}$ we determine

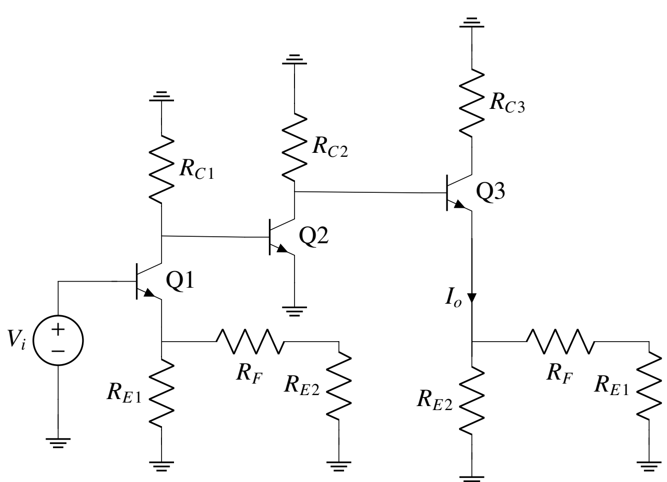

Fig. 1.0.3: circuit3

the gain of first stage,this is written by inspection as-

$$\frac{V_{c1}}{V_i} = \frac{-\alpha(R_{c1}||r_{\pi 2})}{r_{e1} + (R_{E1}||(R_F + R_{E2}))} \quad (1.0.3.1)$$

using values from 1.0.1

$$\frac{V_{c1}}{V_i} = -14.92V/V \quad (1.0.3.2)$$

Next, we determine the gain of the second stage,which

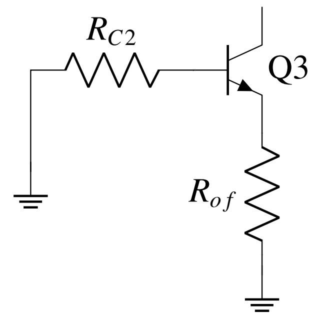

Fig. 1.0.3: circuit4

can be written by inspection(noting that $V_{b2} = V_{c1}$)as

$$\frac{V_c2}{V_{c1}} = -g_{m2}R_{c2}||(h_{fe} + 1)[r_{e3} + (R_{E2}||(R_F + R_{E1}))] \quad (1.0.3.3)$$

substituting ,results in

$$\frac{V_{c2}}{V_{c1}} = -131.2V/V \quad (1.0.3.4)$$

Finally,for the third stage we can write by inspection

$$\frac{I_0}{V_{c2}} = \frac{I_{e3}}{V_{b3}} = \frac{1}{r_{e3} + (R_{E2}||(R_F + R_{E1}))} \quad (1.0.3.5)$$

substituing values from 1.0.1 gives

$$\frac{I_0}{V_{c2}} = 10.6mA/V \quad (1.0.3.6)$$

combining the gains of the three stags results in

$$G = \frac{I_0}{V_i} = -14.92 \times -131.2 \times 10.6 \times 10^-3 = 20.7A/V \quad (1.0.3.7)$$

1.0.4. Find closed loop gain T and Voltage Gain V_0/V_s
Solution:

$$T = \frac{I_0}{V_s} = \frac{G}{1 + GH} = \frac{20.7}{1 + 20.7 \times 11.9} = 83.7mA/V \quad (1.0.4.1)$$

which we note is very close to the approximate value found in (1.0.1.7) and the voltage gain is found from

$$\frac{V_0}{V_s} = \frac{-I_c R_{c3}}{V_s} \approx \frac{-I_0 R_{C3}}{V_s} = -TR_{C3} \quad (1.0.4.2)$$

$$= -83.7 \times 10^{-3} \times 600 = -50.2V/V \quad (1.0.4.3)$$

1.0.5. Find R_{in} and R_{out}
Solution:

$$R_{in} = R_i f = R_i(1 + GH) \quad (1.0.5.1)$$

where R_i is the input resistance of the G circuit.The value of R_i can be found from the circuit in fig.1.0.3 as follows:

$$R_i = (h_{fe}+1)(r_{e1} + (R_{E1}||(R_F + R_{E2}))) = 13.65K\Omega \quad (1.0.5.2)$$

$$R_{if} = 13.65(1 + 20.7 \times 11.9) = 3.38M\Omega \quad (1.0.5.3)$$

$$R_{of} = R_o(1 + GH) \quad (1.0.5.4)$$

where R_o can be determined to be

$$R_o = (R_{E2}||(R_F + R_{E1})) + r_{e3} + \frac{R_{C2}}{h_{fe}+1} \quad (1.0.5.5)$$

from values in Table 1.0.1, yields $R_o = 143.9\Omega$. The output resistance R_{of} of the feedback amplifier can now be found as

$$R_{of} = R_o(1 + GH) = 143.9(1 + 20.7 \times 11.9) = 35.6K\Omega \quad (1.0.5.6)$$

R_{out} is found by using circuit4 in fig.1.0.3

$$R_{out} = r_o3 + [R_{of}||(r_{\pi 3} + R_{C2})](1 + g_{m3}r_{o3}\frac{r_{\pi 3}}{r_{\pi 3} + R_{C2}}) \quad (1.0.5.7)$$

$$= 25 + [35.6||(5.625)][1 + 160 \times 25\frac{0.625}{5.625}] = 2.19M\Omega \quad (1.0.5.8)$$

thus R_{out} is increased (from r_{o3}) but not by (1+GH)

1.0.6. put the obtained parameters in a table
Solution:

Parameter	**Value**
G	**20.7A/V**
H	11.9Ω
T	**83.7mA/V**
V_o/V_s	**-50.2V/V**
R_{in}	$3.38M\Omega$
R_{out}	$2.19M\Omega$
R_{of}	$35.6k\Omega$

TABLE 1.0.6: parameters

1.0.7. Represent this amplifier in a control system Block Diagram
Solution: figure in fig.1.0.7 represents our control system

1.0.8. write a code for doing calculations and verify the values obtained in 1.0.6
Solution: following code does all the calculations of above equations to give parameters in 1.0.6

```
codes/ee18btech11007/circuit_calc.py
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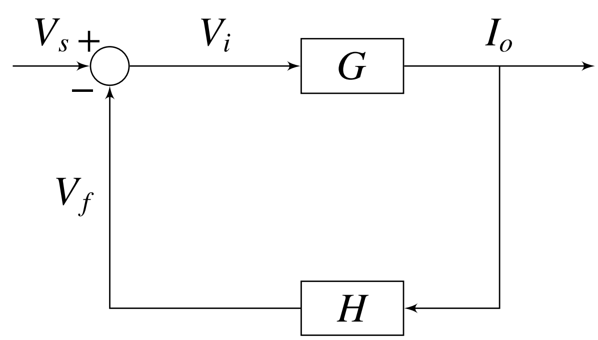

Fig. 1.0.7: block diagram