Control Systems

G V V Sharma*

Contents

1 Signal Flow Graph 1
2 Bode Plot 1
3 Second order System 1
4 Routh Hurwitz Criterion 1
5 State-Space Model 1
6 Nyquist Plot 1
7 Compensators 1
8 Gain Margin 1
9 Phase Margin 1
10 Oscillator 1
11 Root Locus 1
12 Polar Plot 1
13 PID Controller 1
14 Feedback Circuits 1

***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```

*The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India e-mail: gadepall@iith.ac.in.

1 Signal Flow Graph

2 Bode Plot

3 Second order System

4 Routh Hurwitz Criterion

5 State-Space Model

6 Nyquist Plot

7 Compensators

8 Gain Margin

9 Phase Margin

10 Oscillator

11 Root Locus

12 Polar Plot

13 PID Controller

14 Feedback Circuits

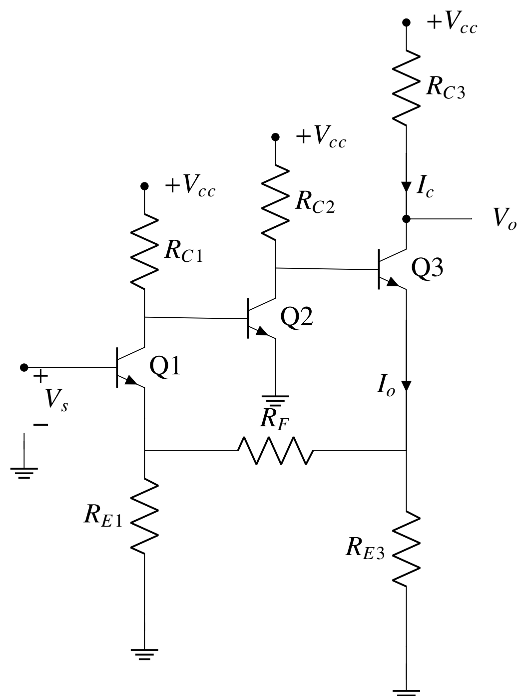

Fig. 14.0.0: circuit1

14.0.1. Part of the circuit of the MC1553 Amplifier is shown in circuit1 in fig.14.0.0 Assume the loop

Parameter	Value
R_{C1}	**9kΩ**
R_{E1}	**100Ω**
R_{C2}	**5kΩ**
R_F	**640Ω**
R_{E2}	**100Ω**
R_{C3}	**600Ω**
h_{fe}	**100**
r_o	∞Ω
I_{C1}	**0.6mA**
I_{C2}	**1mA**
I_{C3}	**4mA**
r_{e1}	41.7Ω
$r_{\pi 2}$	**2.5kΩ**
$\alpha 1$	**0.99**
g_{m2}	**40mA/V**
r_{e3}	6.25Ω
r_{o3}	$25k\Omega$
$r_{\pi 3}$	625Ω

TABLE 14.0.0: parameters

gain is large, find an approximate expression and value for the closed loop gain $A_f = \frac{I_0}{V_s}$ and for $\frac{I_c}{V_s}$,use values from Table 14.0.0

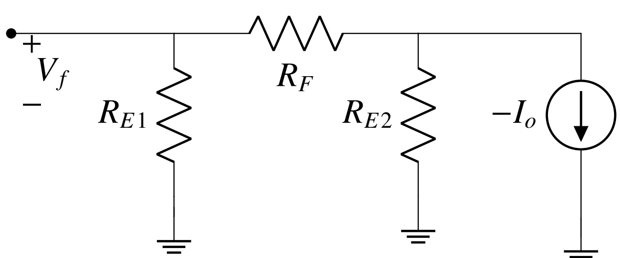

Fig. 14.0.1: circuit2

Solution: When GH >>1,

$$A_f = \frac{I_0}{V_s} \approx \frac{1}{H} \tag{14.0.1.1}$$

feedback factor H can be found from feedback network.The feedback network consists of resistors R_{E1}, R_F, R_{E2} using circuit2 in fig.14.0.1 we get

$$H = \frac{V_f}{I_0} = \frac{R_{E2}}{R_{E2} + R_F + R_{E1}} \times R_{E1} \tag{14.0.1.2}$$

$$= \frac{100}{100 + 640 + 100} \times 100 = 11.9\Omega \tag{14.0.1.3}$$

thus,

$$A_f \approx \frac{1}{H} \tag{14.0.1.4}$$

$$= \frac{1}{R_{E2}}(1 + \frac{R_{E2} + R_F}{R_{E1}}) \tag{14.0.1.5}$$

$$= \frac{1}{11.9} = 84mA/V \tag{14.0.1.6}$$

$$\frac{I_c}{V_s} \approx \frac{I_0}{V_s} = 84mA/V \tag{14.0.1.7}$$

14.0.2. Find $\frac{V_0}{V_s}$
Solution:

$$\frac{V_0}{V_s} = \frac{-I_c R_{C3}}{V_s} = -84 \times 0.6 = -50.4V/V \tag{14.0.2.1}$$

14.0.3. use feedback analysis to find G , H , A_f , $\frac{V_0}{V_s}$, R_{in} and R_{out}.for calculating R_{out} assume r_0 of Q_3 is 25kΩ
Solution: employing loading rules in fig.14.0.0,we obtain circuit3 given in fig.14.0.3 to find $G = \frac{I_0}{V_i}$ we determine the gain of first

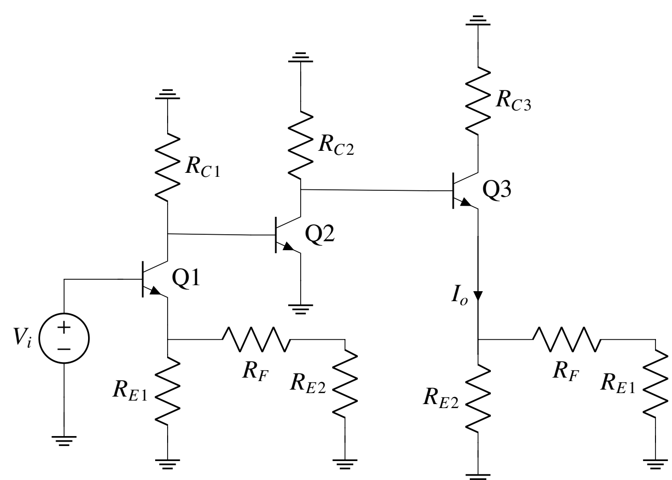

Fig. 14.0.3: circuit3

stage,this is written by inspection as-

$$\frac{V_{c1}}{V_i} = \frac{-\alpha(R_{c1} \| r_{\pi 2})}{r_{e1} + (R_{E1} \| (R_F + R_{E2}))} \tag{14.0.3.1}$$

using values from 14.0.0

$$\frac{V_{c1}}{V_i} = -14.92V/V \tag{14.0.3.2}$$

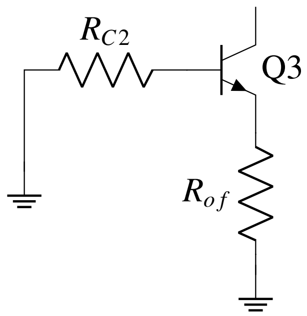

Fig. 14.0.3: circuit4

Next, we determine the gain of the second stage,which can be written by inspection(noting that $V_{b2} = V_{c1}$)as

$$\frac{V_c2}{V_{c1}} = -g_{m2}R_{c2}\|(h_{fe} + 1)[r_{e3} + (R_{E2}\|(R_F + R_{E1}))] \quad (14.0.3.3)$$

substituting ,results in

$$\frac{V_{c2}}{V_{c1}} = -131.2V/V \quad (14.0.3.4)$$

Finally,for the third stage we can write by inspection

$$\frac{I_0}{V_{c2}} = \frac{I_{e3}}{V_{b3}} = \frac{1}{r_{e3} + (R_{E2}\|(R_F + R_{E1}))} \quad (14.0.3.5)$$

substituing values from 14.0.0 gives

$$\frac{I_0}{V_{c2}} = 10.6mA/V \quad (14.0.3.6)$$

combining the gains of the three stags results in

$$G = \frac{I_0}{V_i} = -14.92 \times -131.2 \times 10.6 \times 10^-3 = 20.7A/V \quad (14.0.3.7)$$

the closed loop gain A_f is found from

$$A_f = \frac{I_0}{V_s} = \frac{G}{1 + GH} = \frac{20.7}{1 + 20.7 \times 11.9} = 83.7mA/V \quad (14.0.3.8)$$

which we note is very close to the approximate value found in (14.0.1.7),above the voltage gain is found from

$$\frac{V_0}{V_s} = \frac{-I_c R_{c3}}{V_s} \approx \frac{-I_0 R_{C3}}{V_s} = -A_f R_{C3} \quad (14.0.3.9)$$

$$= -83.7 \times 10^{-3} \times 600 = -50.2V/V \quad (14.0.3.10)$$

which is also very close to the approximate value found in (14.0.1.7) above given by

$$R_{in} = R_i f = R_i(1 + GH) \quad (14.0.3.11)$$

where R_i is the input resistance of the G circuit.The value of R_i can be found from the circuit in fig.14.0.3 as follows:

$$R_i = (h_{fe} + 1)(r_{e1} + (R_{E1}\|(R_F + R_{E2}))) = 13.65K\Omega \quad (14.0.3.12)$$

$$R_{if} = 13.65(1 + 20.7 \times 11.9) = 3.38M\Omega \quad (14.0.3.13)$$

$$R_{of} = R_o(1 + GH) \quad (14.0.3.14)$$

where R_o can be determined to be

$$R_o = (R_{E2}\|(R_F + R_{E1})) + r_{e3} + \frac{R_{C2}}{h_{fe} + 1} \quad (14.0.3.15)$$

from values in Table 14.0.0, yields R_o = 143.9Ω. The output resistance R_{of} of the feedback amplifier can now be found as

$$R_{of} = R_o(1 + GH) = 143.9(1 + 20.7 \times 11.9) = 35.6K\Omega \quad (14.0.3.16)$$

R_{out} is found by using circuit4 in fig.14.0.3

$$R_{out} = r_o3 + [R_{of}\|(r_{\pi3} + R_{C2})](1 + g_{m3}r_{o3}\frac{r_{\pi3}}{r_{\pi3} + R_{C2}}) \quad (14.0.3.17)$$

$$= 25 + [35.6\|(5.625)][1 + 160 \times 25\frac{0.625}{5.625}] = 2.19M\Omega \quad (14.0.3.18)$$

thus R_{out} is increased (from r_{o3}) but not by (1+GH)

14.0.4. put the obtained parameters in a table
Solution:

Parameter	Value
G	**20.7A/V**
H	11.9Ω
A_f	**83.7mA/V**
V_o/V_s	**-50.2V/V**
R_{in}	$3.38M\Omega$
R_{out}	$2.19M\Omega$
R_{of}	$35.6k\Omega$

TABLE 14.0.4: parameters

14.0.5. Represent this amplifier in a control system Block Diagram

Solution: figure in fig.14.0.5 represents our control system

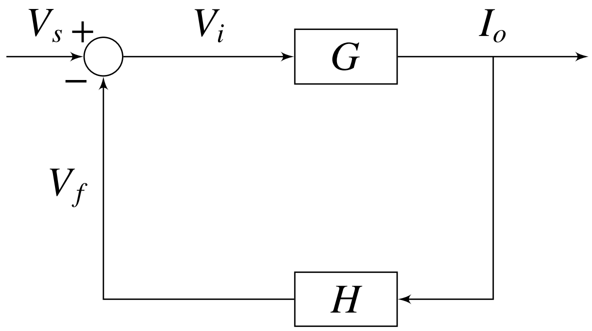

Fig. 14.0.5: block diagram

14.0.6. write a code for doing calculations and verify the values obtained in 14.0.4
Solution: following code does all the calculations of above equations to give parameters in 14.0.4

```
codes/ee18btech11007/circuit_calc.py
```