Control Systems

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Contents

***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```

1 Mason's Gain Formula

1.1. The Block diagram of a system is illustrated in the figure shown, where $X(s)$ is the input and $Y(s)$ is the output. Draw the equivalent signal flow graph.

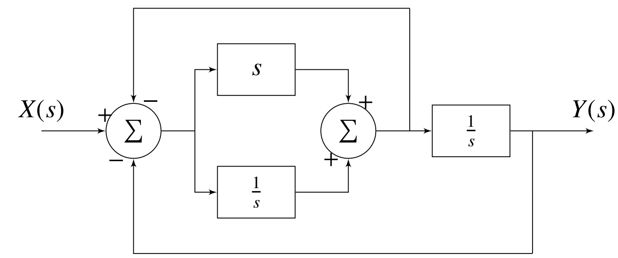

Fig. 1.1.1: Block Diagram

Solution: The signal flow graph of the block diagram in Fig. 1.1.1 is available in Fig. 1.1.2

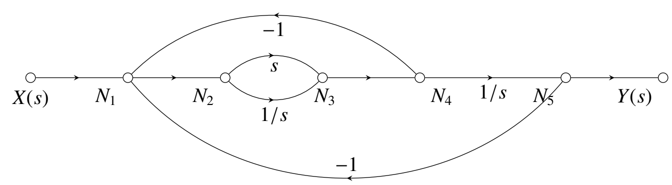

Fig. 1.1.2: Signal Flow Graph

1.2. Draw all the forward paths in Fig. 1.1.2 and compute the respective gains.
Solution: The forward paths are available in Figs. 1.2.3 and 1.2.4. The respective gains are

$$P_1 = s\left(\frac{1}{s}\right) = 1 \tag{1.2.1}$$

$$P_2 = (1/s)(1/s) = 1/s^2 \tag{1.2.2}$$

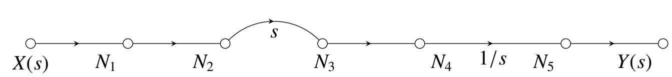

Fig. 1.2.3: P_1

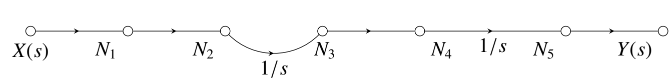

Fig. 1.2.4: P_2

1.3. Draw all the loops in Fig. 1.1.2 and calculate the respective gains.

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Solution: The loops are available in Figs. 1.3.5-1.3.8 and the corresponding gains are

$$L_1 = (-1)(s) = -s \tag{1.3.1}$$

$$L_2 = s\left(\frac{1}{s}\right)(-1) = -1 \tag{1.3.2}$$

$$L_3 = \left(\frac{1}{s}\right)(-1) = -\frac{1}{s} \tag{1.3.3}$$

$$L_4 = \left(\frac{1}{s}\right)\left(\frac{1}{s}\right)(-1) = -\frac{1}{s^2} \tag{1.3.4}$$

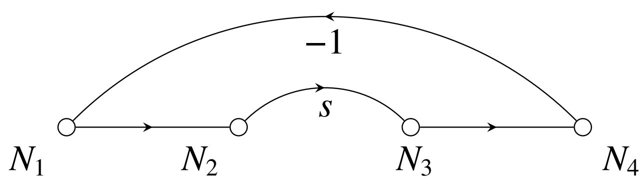

Fig. 1.3.5: L_1

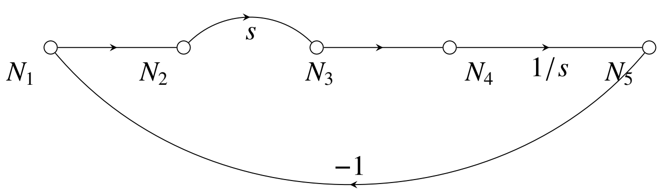

Fig. 1.3.6: L_2

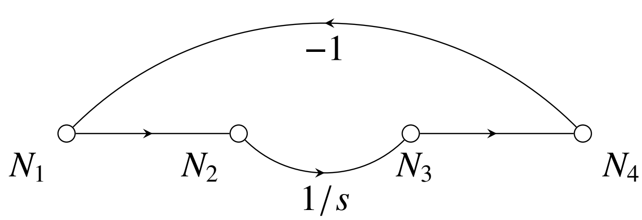

Fig. 1.3.7: L_3

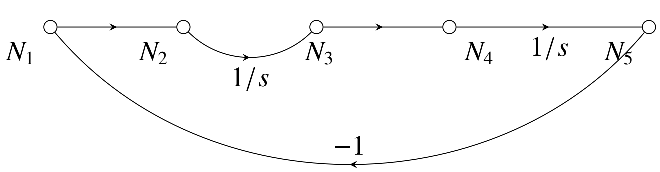

Fig. 1.3.8: L_4

1.4. State Mason's Gain formula and explain the parameters through a table.
Solution: According to Mason's Gain Formula,

$$T = \frac{Y(s)}{X(s)} \tag{1.4.1}$$

$$= \frac{\sum_{i=1}^{N} P_i \Delta_i}{\Delta} \tag{1.4.2}$$

where the parameters are described in Table 1.4

Variable	Description
P_i	ith forward path
L_j	jth loop
Δ	$1 - \sum L_i + \sum_{L_i \cap L_j = \phi} L_i L_j - \sum_{L_i \cap L_j \cap L_k = \phi} L_i L_j L_k + \dots$
Δ_i	$1 - \sum_{L_k \cap P_i = \phi} L_k + \sum_{L_k \cap L_j \cap P_i = \phi} L_k L_j - \dots$

TABLE 1.4

1.5. List the parameters in Table 1.4 for Fig. 1.1.2.
Solution: The parameters are available in Table 1.5

<table>
<tr><th>Path</th><th>Value</th><th>Parameter</th><th>Value</th><th>Remarks</th></tr>
<tr><td>P_1</td><td>1</td><td>Δ_1</td><td>1</td><td>All loops intersect with P_1</td></tr>
<tr><td>P_2</td><td>$\frac{1}{s^2}$</td><td>Δ_2</td><td>1</td><td>All loops intersect with P_2</td></tr>
<tr><td>L_1</td><td>$-s$</td><td rowspan="4">Δ</td><td rowspan="4">$1 - \sum_i L_i$</td><td rowspan="4">All loops intersect</td></tr>
<tr><td>L_2</td><td>-1</td></tr>
<tr><td>L_3</td><td>$-\frac{1}{s}$</td></tr>
<tr><td>L_4</td><td>$-\frac{1}{s^2}$</td></tr>
</table>

TABLE 1.5

1.6. Find the transfer function using Mason's Gain Formula.
Solution: From (1.4.2) and 1.5,

$$T(s) = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} \tag{1.6.1}$$

$$= \frac{1 + \frac{1}{s^2}}{1 - (-s - 1 - \frac{1}{s} - \frac{1}{s^2})} \tag{1.6.2}$$

$$= \frac{s^2 + 1}{s^3 + 2s^2 + s + 1} \tag{1.6.3}$$

after simplification.

1.7. State the Equivalent Matrix Form of Masons Gain formula

Solution: Mason's rule can be stated in a simple matrix form. Assume T is the transient matrix of the graph where

$$t_{nm} = [T_{nm}]$$

is sum transmittance of branches from node m toward node n. Then, the gain from node m to node n of the graph is equal to

$$u_{nm} = [U_{nm}]$$

where,

$$U = (I - T)^{-}1 \tag{1.7.1}$$

and I is the identity matrix.

1.8. Find the Transfer Function using the matrix equivalent form of Masons Gain Formula
Solution: The the state equations of 1.1.2 are-

$$N_1 = X(s) - N_4 - N_5 \tag{1.8.1}$$

$$N_2 = N_1 \tag{1.8.2}$$

$$N_3 = N_2(s + 1/s) \tag{1.8.3}$$

$$N_4 = N_3 \tag{1.8.4}$$

$$N_5 = N_4/s \tag{1.8.5}$$

The transient matrix for 1.1.2 is obtained as

$$\mathbf{T} = \begin{pmatrix} 0 & 0 & 0 & -1 & -1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & s + 1/s & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1/s & 0 \end{pmatrix} \tag{1.8.6}$$

$$\mathbf{I} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{1.8.7}$$

$$\mathbf{I} - \mathbf{T} = \begin{pmatrix} 1 & 0 & 0 & 1 & 1 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & -s - 1/s & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & -1/s & 1 \end{pmatrix} \tag{1.8.8}$$

$$U = (\mathbf{I} - \mathbf{T})^{-}1 \tag{1.8.9}$$

our transfer function is U_{40}

$$U_{40} = C_{04}/|\mathbf{I} - \mathbf{T}| \tag{1.8.10}$$

$$C_{04} = \begin{vmatrix} -1 & 1 & 0 & 0 \\ 0 & -s - 1/s & 1 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & -1/s \end{vmatrix} \tag{1.8.11}$$

$$C_{04} = -1 \begin{vmatrix} -s - 1/s & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 1 & -1/s \end{vmatrix} \tag{1.8.12}$$

$$C_{04} = (s + 1/s)(1/s) = \frac{s^2 + 1}{s^2} \tag{1.8.13}$$

$$|\mathbf{I} - \mathbf{T}|^T = 1 \begin{vmatrix} 1 & -s - 1/s & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1/s \\ 0 & 0 & 0 & 1 \end{vmatrix} \tag{1.8.14}$$

$$+1 \begin{vmatrix} 0 & -s - 1/s & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 1 & 0 & 1 & -1/s \\ 1 & 0 & 0 & 1 \end{vmatrix} \tag{1.8.15}$$

$$|\mathbf{I} - \mathbf{T}|^T = 1 \begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1/s \\ 0 & 0 & 1 \end{vmatrix} + (s + 1/s) \begin{vmatrix} 0 & -1 & 0 \\ 0 & 1 & -1/s \\ 0 & 0 & 1 \end{vmatrix} \tag{1.8.16}$$

$$+(s + 1/s) \begin{vmatrix} 0 & -1 & 0 \\ 1 & 1 & -1/s \\ 1 & 0 & 1 \end{vmatrix} \tag{1.8.17}$$

$$|\mathbf{I} - \mathbf{T}|^T = 1 + 0 + (s + 1/s)(1 + 1/s) \tag{1.8.18}$$

$$|\mathbf{I} - \mathbf{T}|^T = s + 2 + 1/s + 1/s^2 \tag{1.8.19}$$

$$|\mathbf{I} - \mathbf{T}|^T = \frac{s^3 + 2s^2 + s + 1}{s^2} \tag{1.8.20}$$

$$|\mathbf{I} - \mathbf{T}|^T = |\mathbf{I} - \mathbf{T}| \tag{1.8.21}$$

from (1.8.10),(1.8.13),(1.8.20) we get

$$T(s) = \frac{s^2 + 1}{s^3 + 2s^2 + s + 1} \tag{1.8.22}$$

1.9. Write a program to compute Mason's gain formula, given the branch nodes and gains for each path.
Solution: below code finds Masons Gain

```
codes/MasonsGain.py
```

2 Bode Plot

2.1 Introduction

2.2 Example

3 Second order System

3.1 Damping

3.2 Example

4 Routh Hurwitz Criterion

4.1 Routh Array

4.2 Marginal Stability

4.3 Stability

5 State-Space Model

5.1 Controllability and Observabiity

5.2 Second Order System

6 Nyquist Plot

7 Compensators

8 Phase Margin