

# Control Systems

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### 1 Feedback Circuits 1

**Abstract**—This manual is an introduction to control systems based on GATE problems. Links to sample Python codes are available in the text.

Download python codes using

svn co <https://github.com/gadepall/school/trunk/control/codes>

#### 1 FEEDBACK CIRCUITS

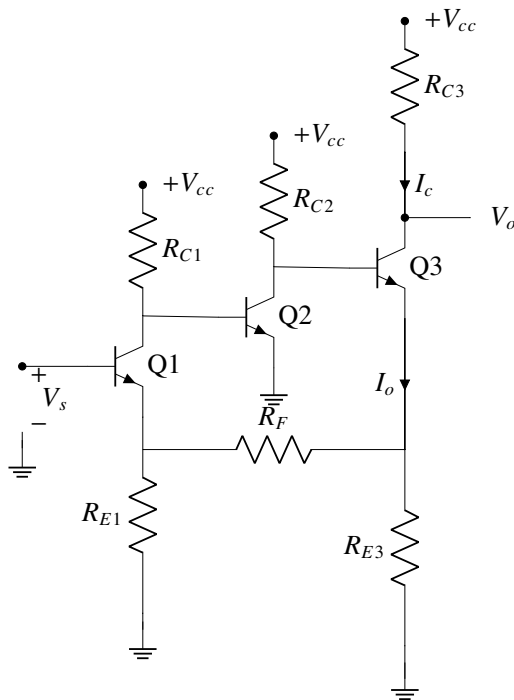


Fig. 1.0.0: circuit1

1.0.1. Part of the circuit of the MC1553 Amplifier is shown in circuit1 in fig.1.0.0 Assume the loop gain is large, find an approximate expression and value for the closed loop gain  $T = \frac{I_o}{V_s}$  use values from Table 1.0.1

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Parameter	Value
$R_{C1}$	9k $\Omega$
$R_{E1}$	100 $\Omega$
$R_{C2}$	5k $\Omega$
$R_F$	640 $\Omega$
$R_{E2}$	100 $\Omega$
$R_{C3}$	600 $\Omega$
$h_{fe}$	100
$r_o$	$\infty\Omega$
$I_{C1}$	0.6mA
$I_{C2}$	1mA
$I_{C3}$	4mA
$r_{e1}$	41.7 $\Omega$
$r_{\pi2}$	2.5k $\Omega$
$\alpha_1$	0.99
$g_{m2}$	40mA/V
$r_{e3}$	6.25 $\Omega$
$r_{o3}$	25k $\Omega$
$r_{\pi3}$	625 $\Omega$

TABLE 1.0.1: parameters

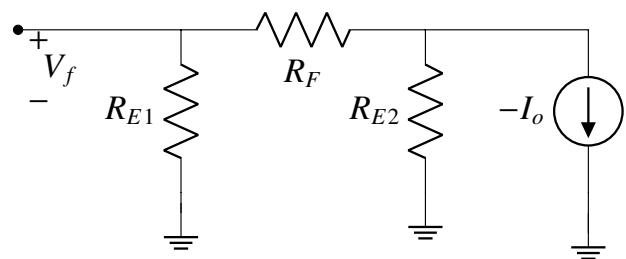


Fig. 1.0.1: circuit2

**Solution:** When  $GH \gg 1$ ,

$$T = \frac{I_o}{V_s} \approx \frac{1}{H} \quad (1.0.1.1)$$

feedback factor H can be found from feedback network. The feedback network consists of resistors

$R_{E1}, R_F, R_{E2}$  using circuit2 in fig.1.0.1 we get

$$H = \frac{V_f}{I_0} = \frac{R_{E2}}{R_{E2} + R_F + R_{E1}} \times R_{E1} \quad (1.0.1.2)$$

$$= \frac{100}{100 + 640 + 100} \times 100 = 11.9\Omega \quad (1.0.1.3)$$

thus,

$$T \approx \frac{1}{H} \quad (1.0.1.4)$$

$$= \frac{1}{R_{E2}} \left( 1 + \frac{R_{E2} + R_F}{R_{E1}} \right) \quad (1.0.1.5)$$

$$= \frac{1}{11.9} = 84mA/V \quad (1.0.1.6)$$

$$\frac{I_c}{V_s} \approx \frac{I_0}{V_s} = 84mA/V \quad (1.0.1.7)$$

1.0.2. Find Voltage gain  $\frac{V_0}{V_s}$  for above approximation

**Solution:**

$$\frac{V_0}{V_s} = \frac{-I_c R_{C3}}{V_s} = -84 \times 0.6 = -50.4V/V \quad (1.0.2.1)$$

1.0.3. use feedback analysis to find open loop gain G

**Solution:** employing loading rules in fig.1.0.0, we obtain circuit3 given in fig.1.0.3 to find  $G = \frac{I_0}{V_i}$  we determine

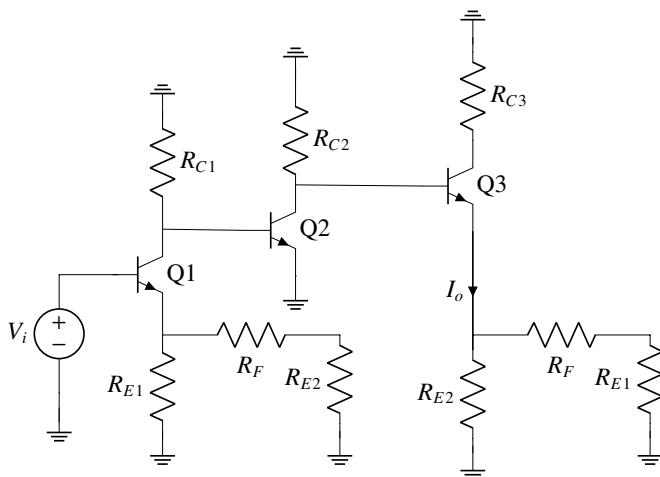


Fig. 1.0.3: circuit3

the gain of first stage, this is written by inspection as-

$$\frac{V_{c1}}{V_i} = \frac{-\alpha(R_{C1} || r_{\pi 2})}{r_{e1} + (R_{E1} || (R_F + R_{E2}))} \quad (1.0.3.1)$$

using values from 1.0.1

$$\frac{V_{c1}}{V_i} = -14.92V/V \quad (1.0.3.2)$$

Next, we determine the gain of the second stage, which

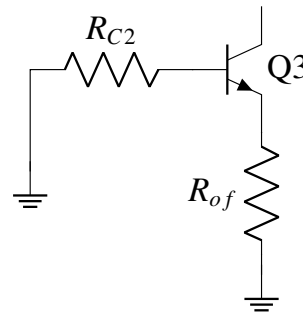


Fig. 1.0.3: circuit4

can be written by inspection (noting that  $V_{b2} = V_{c1}$ ) as

$$\frac{V_{c2}}{V_{c1}} = -g_{m2} R_{c2} || (h_{fe} + 1) [r_{e3} + (R_{E2} || (R_F + R_{E1}))] \quad (1.0.3.3)$$

substituting results in

$$\frac{V_{c2}}{V_{c1}} = -131.2V/V \quad (1.0.3.4)$$

Finally, for the third stage we can write by inspection

$$\frac{I_0}{V_{c2}} = \frac{I_{e3}}{V_{b3}} = \frac{1}{r_{e3} + (R_{E2} || (R_F + R_{E1}))} \quad (1.0.3.5)$$

substituting values from 1.0.1 gives

$$\frac{I_0}{V_{c2}} = 10.6mA/V \quad (1.0.3.6)$$

combining the gains of the three stages results in

$$G = \frac{I_0}{V_i} = -14.92 \times -131.2 \times 10.6 \times 10^{-3} = 20.7A/V \quad (1.0.3.7)$$

1.0.4. Find closed loop gain T and Voltage Gain  $V_0/V_s$

**Solution:**

$$T = \frac{I_0}{V_s} = \frac{G}{1 + GH} = \frac{20.7}{1 + 20.7 \times 11.9} = 83.7mA/V \quad (1.0.4.1)$$

which we note is very close to the approximate value found in (1.0.1.7) and the voltage gain is found from

$$\frac{V_0}{V_s} = \frac{-I_c R_{C3}}{V_s} \approx \frac{-I_0 R_{C3}}{V_s} = -T R_{C3} \quad (1.0.4.2)$$

$$= -83.7 \times 10^{-3} \times 600 = -50.2V/V \quad (1.0.4.3)$$

1.0.5. Find  $R_{in}$  and  $R_{out}$

**Solution:**

$$R_{in} = R_{if} = R_i (1 + GH) \quad (1.0.5.1)$$

where  $R_i$  is the input resistance of the G circuit. The value of  $R_i$  can be found from the circuit in fig.1.0.3 as

follows:

$$R_i = (h_{fe} + 1)(r_{e1} + (R_{E1} || (R_F + R_{E2}))) = 13.65 K\Omega \quad (1.0.5.2)$$

$$R_{if} = 13.65(1 + 20.7 \times 11.9) = 3.38 M\Omega \quad (1.0.5.3)$$

$$R_{of} = R_o(1 + GH) \quad (1.0.5.4)$$

where  $R_o$  can be determined to be

$$R_o = (R_{E2} || (R_F + R_{E1})) + r_{e3} + \frac{R_{C2}}{h_{fe} + 1} \quad (1.0.5.5)$$

from values in Table 1.0.1, yields  $R_o = 143.9\Omega$ . The output resistance  $R_{of}$  of the feedback amplifier can now be found as

$$R_{of} = R_o(1 + GH) = 143.9(1 + 20.7 \times 11.9) = 35.6 K\Omega \quad (1.0.5.6)$$

$R_{out}$  is found by using circuit4 in fig.1.0.3

$$R_{out} = r_{o3} + [R_{of} || (r_{\pi3} + R_{C2})] \left(1 + g_{m3} r_{o3} \frac{r_{\pi3}}{r_{\pi3} + R_{C2}}\right) \quad (1.0.5.7)$$

$$= 25 + [35.6 || (5.625)] \left[1 + 160 \times 25 \frac{0.625}{5.625}\right] = 2.19 M\Omega \quad (1.0.5.8)$$

thus  $R_{out}$  is increased (from  $r_{o3}$ ) but not by  $(1+GH)$

1.0.6. put the obtained parameters in a table

**Solution:**

Parameter	Value
<b>G</b>	<b>20.7A/V</b>
<b>H</b>	11.9 $\Omega$
<b>T</b>	<b>83.7mA/V</b>
$V_o/V_s$	<b>-50.2V/V</b>
$R_{in}$	3.38M $\Omega$
$R_{out}$	2.19M $\Omega$
$R_{of}$	35.6k $\Omega$

TABLE 1.0.6: parameters

1.0.7. Represent this amplifier in a control system Block Diagram

**Solution:** figure in fig.1.0.7 represents our control system

1.0.8. write a code for doing calculations and verify the values obtained in 1.0.6

**Solution:** following code does all the calculations of above equations to give parameters in 1.0.6

```
codes/ee18btech11007/circuit_calc.py
```

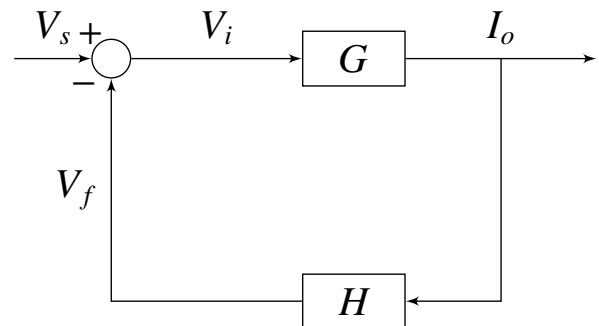


Fig. 1.0.7: block diagram