Contents

***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```

1 Stability

1.1 Second order System

2 Routh Hurwitz Criterion

3 Compensators

4 Nyquist Plot

4.1. The open loop transfer function of a unity feedback system is given by

$$G(s) = \frac{\pi e^{-0.25s}}{s} \tag{4.1.1}$$

4.2. Find $\text{Re}\{G(\jmath\omega)\}$ and $\text{Im}\{G(\jmath\omega)\}$.
Solution: From (4.1.1),

$$G(j\omega) = -\frac{\pi}{\omega}(\sin 0.25\omega + j\cos 0.25\omega) \tag{4.2.1}$$

$$\text{Re}\{G(\jmath\omega)\} = -\frac{\pi}{\omega}(\sin 0.25\omega) \tag{4.2.2}$$

$$\text{Im}\{G(\jmath\omega)\} = -\frac{\pi}{\omega}(j\cos 0.25\omega) \tag{4.2.3}$$

4.3. Sketch the Nyquist plot.
Solution: The Nyquist plot is a graph of $\text{Re}\{G(\jmath\omega)\}$ vs $\text{Im}\{G(\jmath\omega)\}$. The following python code generates the Nyquist plot in Fig. 4.3

```
codes/ee18btech11007.py
```

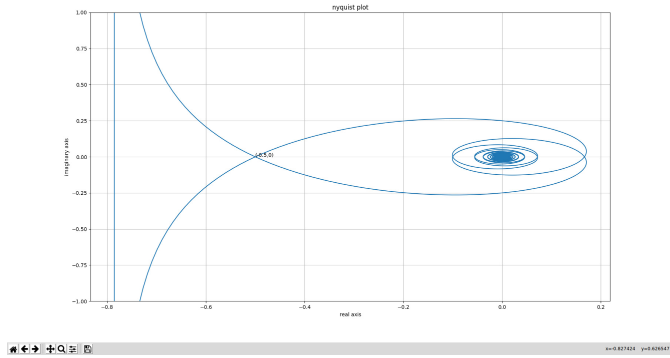

Fig. 4.3

4.4. Find the point at which the Nyquist plot of G(s) passes through the negative real axis

Solution: Nyquist plot cuts the negative real axis at ω for which

$$\angle G(\jmath\omega) = -\pi \tag{4.4.1}$$

From (4.1.1),

$$G(\jmath\omega) = \frac{\pi e^{-\frac{\jmath\omega}{4}}}{\jmath\omega} = \frac{\pi e^{-\jmath\left(\frac{\omega}{4}+\frac{\pi}{2}\right)}}{\omega} \tag{4.4.2}$$

$$\implies \angle G(\jmath\omega) = -\left(\frac{\omega}{4} + \frac{\pi}{2}\right) \tag{4.4.3}$$

From (4.4.3) and (4.4.1),

$$\frac{\omega}{4} + \frac{\pi}{2} = \pi \tag{4.4.4}$$

$$\implies \omega = 2\pi \tag{4.4.5}$$

Also, from (4.1.1),

$$\left|G(\jmath\omega)\right| = \frac{\pi}{|\omega|} \tag{4.4.6}$$

$$\implies \left|G(\jmath 2\pi)\right| = \frac{1}{2} \tag{4.4.7}$$

Variable	Value	Description
Z	0	Poles of $\frac{G(s)}{1+G(s)H(s)}$in right half of s plane
N	0	No of clockwise encirclements of nyquist plot of G(s)H(s) about the critical point -1+0j
P	0	No of poles of G(s)H(s) in the right half of s-plane

TABLE 4.4

NOTE-

- If a clockwise contour does not encircle zeros or poles, then the plot will not encircle the origin.
- If a clockwise contour encircles a zero, then the plot will encircle the origin clockwise once.
- If a clockwise contour encircles a pole, then the plot will encircle the origin counterclockwise once.
- clockwise encirclement of nyquist plot is taken as positive and counterclockwise encirclement of nyquist plot is taken as negative

4.5. Find the value of N defined in Table 4.4 from Fig. 4.3.

Solution: $N = 0$. as there are no encirclements of the nyquist plot of G(s) about -1+0j

4.6. Find the value of P defined in Table 4.4 from (4.1.1)

Solution: $\because H(s) = 1$, $G(s)H(s) = G(s)$. Also, $G(s)$ has a pole at $s = 0$, hence $P = 0$.

4.7. Use the Nyquist Stability criterion to determine if the system in 4.1.1 is stable.

Solution: According to the Nyquist criterion, the system is stable if

$$Z = P + N = 0, \tag{4.7.1}$$

where Z is defined in Table 4.4. $\because Z = 0$ from 4.7.1 , so the system is stable.