Contents

***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```

1 Stability

1.1 Second order System

2 Routh Hurwitz Criterion

3 Compensators

4 Nyquist Plot

4.1. Question-The open loop transfer function of a unity feedback system is given by

$$G(s) = \frac{\pi e^{-0.25s}}{s}$$

4.2. Find $\text{Re}\{G(\jmath\omega)\}$ and $\text{Im}\{G(\jmath\omega)\}$
Solution: substitute

$$S = j\omega \tag{4.2.1}$$

we get

$$G(j\omega) = \frac{\pi}{\omega}(-\sin 0.25\omega - j\cos 0.25\omega) \tag{4.2.2}$$

$$\text{Re}\{G(\jmath\omega)\} = \frac{\pi}{\omega}(-\sin 0.25\omega) \tag{4.2.3}$$

$$\text{Im}\{G(\jmath\omega)\} = \frac{\pi}{\omega}(-j\cos 0.25\omega) \tag{4.2.4}$$

4.3. Sketch the Nyquist plot.
Solution: The Nyquist plot is a graph of $\text{Re}\{G(\jmath\omega)\}$ vs $\text{Im}\{G(\jmath\omega)\}$. The following python code generates the Nyquist plot in Fig. 4.3

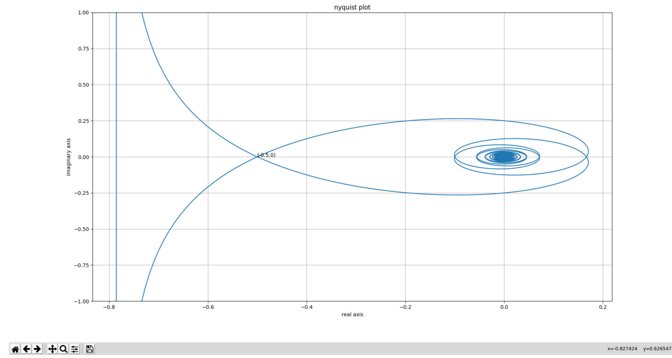

Fig. 4.3

```
codes/ee18btech11007/nyquist.py
```

4.4. Find the point at which the Nyquist plot of G(s) passes through negative real axis
Solution:

$$G(s) = \frac{\pi e^{-0.25s}}{s} \tag{4.4.1}$$

Nyquist plot cuts the negative real Axis at $\omega =$ phase cross over frequency,at phase cross over frequency the phase of nyquist plot becomes -π radians.
substitute

$$s = j\omega. \tag{4.4.2}$$

$$G(j\omega) = \frac{\pi}{\omega}(-\sin 0.25\omega - j\cos 0.25\omega) \tag{4.4.3}$$

$$\angle G(j\omega) = -\pi/2 - 0.25\omega. \tag{4.4.4}$$

$$\angle G(j\omega)|_{\omega=\omega_{pc}} = -\pi \tag{4.4.5}$$

by solving for ω we get $\omega_{pc} = 2\pi$.
magnitude at any point is

$$X = |G(j\omega)| = \frac{\pi}{\omega}. \tag{4.4.6}$$

substituting $\omega = 2\pi$ in magnitude equation we get X=0.5.

so it intersects at (-0.5,0j)

we can verify with the following plot that it intersects at (-0.5,0j)

4.5. Use Nyquist stability criterion to determine if the system is stable?

Solution: Nyquist Stability Criterion - for the stability of a closed loop transfer function G(s)/(1+G(s)*H(s)) ,the number of poles of G(s)*H(s) on right half of s-plane must equal the number of encirclement of nyquist contour of G(s)*H(s) about the critical point -1+0j

we must find

$$Z = P - N \tag{4.5.1}$$

Z=number of poles of closed loop transfer function in right half of s-plane.

P=number of poles of G(s)*H(s) in right half of s-plane

N=number of encirclement of nyquist contour of G(s)*H(s) about the critical point -1+0j, here H(S)=1.

from plot we get N=0,and we already know P=0 since our G(s) doesnt have any poles on right half of s-plane

$$Z = 0 - 0 = 0 \tag{4.5.2}$$

Z=0 implies the system is stable because we dont have any poles on right half of the s-plane therefore the system is stable.