Problem Statement

Find the equation of the tangent to the circle, at the point

$$\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

whose centre is the point of intersection of the straight lines
$(2\ 1)\mathbf{x} = 3$
$(1\ -1)\mathbf{x} = 1$

Answer

First find centre of circle by finding point of intersection of lines The given lines are

$$(2\ 1)x = 3$$

$$(1\ -1)x = 1$$

(1)

$$\begin{pmatrix} 2 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} x \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

The point of intersection is found by multiplying both sides of the equation by the inverse matrix.

$$\begin{pmatrix} \end{pmatrix} \begin{pmatrix} 1/3 & 1/3 \\ \end{pmatrix} \begin{pmatrix} 3 \end{pmatrix}$$

So centre of circle is

$$\left(\ x \ \right) = \left(\begin{array}{c} 4/3 \\ 1/3 \end{array} \right)$$

The normal vector of the tangent will have the same direction of the line joining the centre of circle C(4/3 , 1/3) and Point of contact

$$\left(\begin{array}{c} 4/3 \\ 1/3 \end{array} \right) - \left(\begin{array}{c} 1 \\ -1 \end{array} \right) = \left(\begin{array}{c} 1/3 \\ 4/3 \end{array} \right)$$

The equation of the line is of the form:

$$\left(\begin{array}{c} 1/3 \\ 4/3 \end{array} \right) \left(\ X - P \ \right) = \left(\ 0 \ \right)$$

Where P is

$$\left(\begin{array}{c} 1 \\ -1 \end{array} \right)$$