JEE problem

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Question

A circle C of radius 1 is inscribed in an equilateral triangle PQR. The points of contact of C with the sides PQ, QR, RP are D, E, F, respectively. The line PQ is given by the equation $\sqrt{3}x + y - 6 = 0$
the point D is $(3\sqrt{3}/2, 3/2)$
Further, it is given that the origin and the centre of C are on the same side of the line PQ.
Q.18. The equation of circle C is?

Matrix form of the Question

A circle C of radius 1 is inscribed in an equilateral triangle PQR. The points of contact of C with the sides PQ, QR, RP are D, E, F, respectively. The line PQ is given by the equation

$$\begin{bmatrix}\sqrt{3} & 1\end{bmatrix} \times \begin{bmatrix}x \\ y\end{bmatrix} = 6$$

the point D $= \begin{bmatrix}3\sqrt{3}/2 \\ 3/2\end{bmatrix}$

Further, it is given that the origin and the centre of C are on the same side of the line PQ.

Q.18. The equation of circle C is?

Solution

PQ= $\begin{bmatrix} 1 \\ -\sqrt{3} \end{bmatrix}$ is direction vector of PQ

omat= $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$.

direction of normal to PQ is

N1 = $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \times \begin{bmatrix} -1 \\ \sqrt{3} \end{bmatrix} = \begin{bmatrix} \sqrt{3} \\ 1 \end{bmatrix}$

centre C lies on C= D+K$\times N1$

here norm(C-D)=1

K is obtained as -1/norm(N1) i.e K= -1/2

so C is $\begin{bmatrix} \sqrt{3} \\ 1 \end{bmatrix}$

equation of circle is obtained as $(\text{norm(X-C)})^2 = 1$

$(\text{X-C})(\text{X-C})^T = 1$

Where X= $\begin{bmatrix} x \\ y \end{bmatrix}$

Q19-Find points E and F -

Solution-

P =D +K$\times PQ$

is the equation of line PQ

norm(P-D)=$\sqrt{3}$

K=$\sqrt{3}/norm(PQ) = \sqrt{3}/2$

P=$\begin{bmatrix} 2\sqrt{3} \\ 0 \end{bmatrix}$

since D is mid point of PQ

Q=2D-P

M= $\begin{bmatrix} cos(i) & -sin(i) \\ sin(i) & cos(i) \end{bmatrix}$

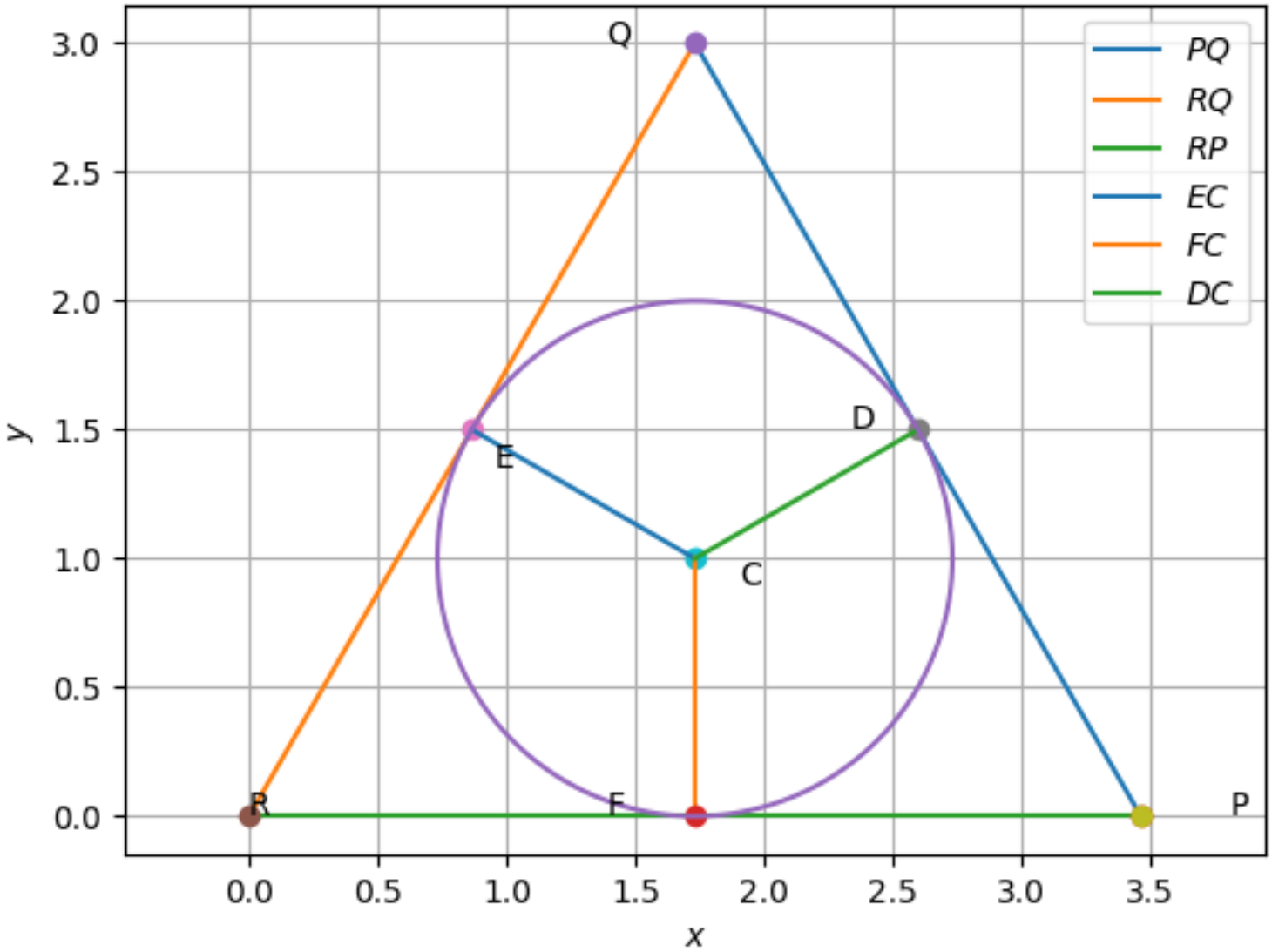
i=$\pi/3$

Multiply M with PQ to obtain Direction of RQ

$$x = \begin{bmatrix} cos(i) & -sin(i) \\ sin(i) & cos(i) \end{bmatrix}^T \begin{bmatrix} -1 \\ +\sqrt{3} \end{bmatrix} \tag{1}$$

$$\therefore RQ = \begin{bmatrix} 1 \\ \sqrt{3} \end{bmatrix} isdirectionofRQ$$

R-Q=K$\times RQ$
is the equation of line RQ
norm(R-Q)=$2\sqrt{3}$
K=-$2\sqrt{3}/norm(RQ) = -\sqrt{3}$
R=$\begin{bmatrix} 0 \\ 0 \end{bmatrix}$
E=(Q+R)/2,F=(R+P)/2
E=$\begin{bmatrix} \sqrt{3}/2 \\ 3/2 \end{bmatrix}$
F=$\begin{bmatrix} \sqrt{3} \\ 0 \end{bmatrix}$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