Control Systems

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Contents

***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```

1 Stability

1.1 Second order System

1.1. Consider the following second order system with the transfer function

$$G(s) = \frac{1}{1 + 2s + s^2} \tag{1.1.1}$$

Is the system stable?

Solution: The poles of

$$G(s) = \frac{1}{1 + 2s + s^2} \tag{1.1.2}$$

are at

$$s = -1 \tag{1.1.3}$$

i.e., the left half of s-plane. Hence the system is stable.

1.2. Find and sketch the step response $c(t)$ of the system.

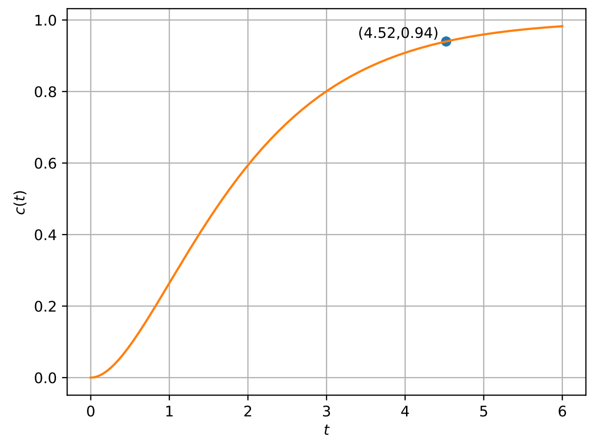

Fig. 1.2

Solution: For step-response, we take input as unit-step function u(t)

$$C(s) = U(s).G(s) = \left[\frac{1}{s}\right]\left[\frac{1}{1 + 2s + s^2}\right] \tag{1.2.1}$$

$$= \frac{1}{s(1+s)^2} \tag{1.2.2}$$

$$= \frac{1}{s} - \frac{1}{(1+s)} - \frac{1}{(1+s)^2} \tag{1.2.3}$$

Taking the inverse Laplace transform,

$$c(t) = L^{-1}\left[\frac{1}{s}\right] - L^{-1}\left[\frac{1}{1+s}\right] - L^{-1}\left[\frac{1}{(1+s)^2}\right] \tag{1.2.4}$$

$$= \left(1 - e^{-t} - te^{-t}\right) u(t) \tag{1.2.5}$$

The following code plots $c(t)$ in Fig. 1.2

```
codes/ee18btech11002/plot.py
```

1.3. Find the steady state response of the system using the final value theorem. Verify using 2.2.1

Solution: To know the steady response value

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of c(t), using final value theorem,

$$\lim_{t\to\infty} c(t) = \lim_{s\to 0} sC(s) \tag{1.3.1}$$

We get

$$\lim_{s\to 0} s\left(\frac{1}{s}\right)\left(\frac{1}{1+s+s^2}\right) = \frac{1}{1+0+0} = 1 \tag{1.3.2}$$

Using 2.2.1,

$$\lim_{t\to\infty} c(t) = \lim_{t\to\infty} \left(1 - e^{-t} - te^{-t}\right) u(t) \tag{1.3.3}$$

$$= (1 - 0 - 0) = 1 \tag{1.3.4}$$

1.4. Find the time taken for the system output c(t) to reach 94% of its steady state value.
Solution: Now, 94% of 1 is 0.94, so we should now solve for a positive t such that

$$1 - e^{-t} - te^{-t} = 0.94 \tag{1.4.1}$$

The following code

```
codes/ee18btech11002/solution.py
```

provides the necessary solution as

$$t = 4.5228 \tag{1.4.2}$$

2 Routh Hurwitz Criterion

2.1. Consider a unity feedback system as shown in Fig. 2.1, with an integral compensator $\frac{k}{s}$ and open-loop transfer function

$$G(s) = \frac{1}{s^2+3s+2} \tag{2.1.1}$$

where k greater than 0. Find its closed loop transfer function.

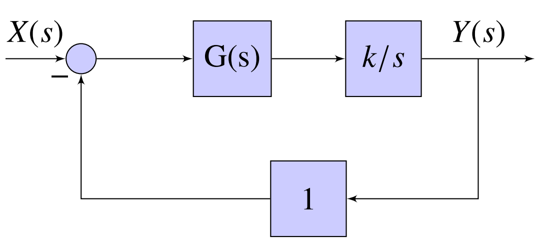

Fig. 2.1

Solution: The transfer function for negative feedback is given by

$$\frac{Y(s)}{X(s)} = \frac{G(s)}{1+G(s)H(s)} \tag{2.1.2}$$

where H(s) = 1 for unity feedback system and G(s) is net forward open loop gain

$$G(s) = \left[\frac{1}{s^2+3s+2}\right]\left[\frac{k}{s}\right] = \frac{k}{s^3+3s^2+2s} \tag{2.1.3}$$

2.2. Find the *characteristic* equation for $G(s)$.
Characteristic equation is..,

$$1 + G(s)H(s) = 0 \tag{2.2.1}$$

$$=> 1 + \left[\frac{k}{s^3+3s^2+2s}\right] = 0 \tag{2.2.2}$$

$$=> s^3 + 3s^2 + 2s + k = 0 \tag{2.2.3}$$

2.3. Using the tabular method for the Routh hurwitz criterion, find $k > 0$ for which there are two poles of unity feedback system on jω axis.
Solution: This criterion is based on arranging the coefficients of characteristic equation into an array called Routh array. For any characteristic equation q(s),

$$q(s) = a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \ldots.. + a_{n-1} s + a_n = 0 \tag{2.3.1}$$

Routh array can be constructed as follows..,

$$\begin{pmatrix} s^n \\ s^{n-1} \\ s^{n-2} \\ \vdots \end{pmatrix} \begin{pmatrix} a_0 & a_2 & a_4 & \cdots & \\ a_1 & a_3 & a_5 & \cdots & \\ b_1 & b_2 & b_3 & \cdots & \\ \vdots & \vdots & \vdots & \ddots & \vdots\ldots \end{pmatrix}$$

where

$$b_1 = \frac{a_1 a_2 - a_0 a_3}{a_1} \tag{2.3.2}$$

$$b_2 = \frac{a_1 a_4 - a_0 a_5}{a_1} \tag{2.3.3}$$

$$c_1 = \frac{b_1 a_3 - a_1 b_2}{b_1} \tag{2.3.4}$$

$$c_2 = \frac{b_1 a_5 - a_1 b_3}{b_1} \tag{2.3.5}$$

For poles to lie on imaginary axis any one entire row of hurwitz matrix should be zero. Constructing the routh array for the characteristic equation obtained in equation(2.2.1)

$$s^3 + 3s^2 + 2s + k = 0 \tag{2.3.6}$$

$$\begin{pmatrix} s^3 \\ s^2 \\ s^1 \\ s^0 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 3 & k \\ \frac{6-k}{3} & 0 \\ k & 0 \end{pmatrix} \tag{2.3.7}$$

For poles on jω axis any one of the row should be zero.

$$\frac{6-k}{3} = 0 \ or \ k = 0 \tag{2.3.8}$$

But given k greater than 0 ...

$$6 - k = 0 \tag{2.3.9}$$

$$k = 6 \tag{2.3.10}$$

2.4. Repeat the above using the determinant method.
2.5. Verify your answer using a python code for both the determinant method as well as the tabular method.

3 Compensators

4 Nyquist Plot