Control Systems

G V V Sharma*

CONTENTS

Abstract—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```

1 STABILITY

1.1 Second order System

1.1. Consider the following second order system with the transfer function

$$G(s) = \frac{1}{1 + 2s + s^2} \qquad (1.1.1)$$

Is the system stable?
Solution: The poles of

$$G(s) = \frac{1}{1 + 2s + s^2} \qquad (1.1.2)$$

are at

$$s = -1 \qquad (1.1.3)$$

i.e., the left half of s-plane. Hence the system is stable.

*The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India e-mail: gadepall@iith.ac.in.
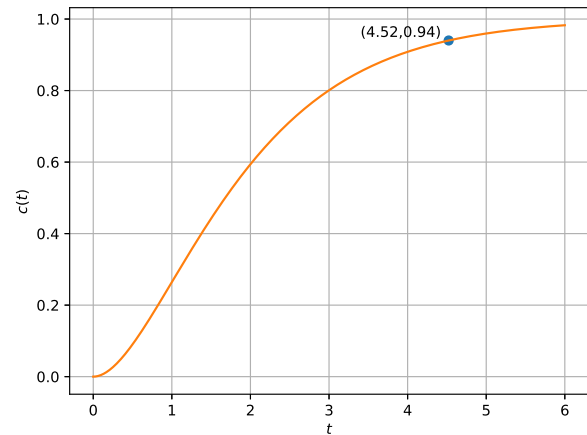
Fig. 1.2

1.2. Find and sketch the step response $c(t)$ of the system.
  Solution: For step-response, we take input as unit-step function u(t)

$$C(s) = U(s).G(s) = \left[\frac{1}{s}\right]\left[\frac{1}{1 + 2s + s^2}\right] \tag{1.2.1}$$

$$= \frac{1}{s(1 + s)^2} \tag{1.2.2}$$

$$= \frac{1}{s} - \frac{1}{(1 + s)} - \frac{1}{(1 + s)^2} \tag{1.2.3}$$

Taking the inverse Laplace transform,

$$c(t) = L^{-1}\left[\frac{1}{s}\right] - L^{-1}\left[\frac{1}{1 + s}\right] - L^{-1}\left[\frac{1}{(1 + s)^2}\right] \tag{1.2.4}$$

$$= (1 - e^{-t} - te^{-t})\,u(t) \tag{1.2.5}$$

The following code plots $c(t)$ in Fig. 1.2

```
codes/ee18btech11002/plot.py
```

1.3. Find the steady state response of the system

using the final value theorem. Verify using 1.2.5

Solution: To know the steady response value of c(t), we calculate

$$\lim_{t \to \infty} c(t) = (1 + 0 + 0).(1) = 1 \qquad (1.3.1)$$

1.4. Find the time taken for the system output c(t) to reach 94% of its steady state value.

Solution: Now, 94% of 1 is 0.94, so we should now solve for a positive t such that

$$1 - e^{-t} - te^{-t} = 0.94 \qquad (1.4.1)$$

The following code

codes/ee18btech11002/solution.py

provides the necessary solution as

$$t = 4.5228 \qquad (1.4.2)$$

2 ROUTH HURWITZ CRITERION

3 COMPENSATORS

4 NYQUIST PLOT