EE2227 Assignment 3 (State Space representation and Controllability)

Due – 02-04-2018 (Before Class)

1. Consider the following open-loop system

$$G_s = \frac{20}{(s+2)(s+4)(s+8)}$$

Design a controller to yield a 15% overshoot and a settling time of 0.75 seconds. Appropriately place the third pole for second order approximation to be valid. Make two choices for this. Plot the transient response using Matlab for both the cases and check how different is the transient response for the desired transient response.

2. We have seen in class the ease of implementing the controller using the phase variable choice of state variables. As such we performed transformation of the state coordinates that were not in phase variable form, computed the required state feedback vector for the phase variable choice of state and then we found the state feedback vector for the original variable choice of state vector. An alternate and more direct approach to compute the weight is to use the Ackermann's Formula which is given as follows

$$K_z = \underbrace{[0\ 0\ \cdots 1]}_{n-length} C_{M_z}^{-1} \varphi_{\mathrm{d}}(\mathbf{A}_{\mathrm{z}})$$

where C_{M_z} is the controllability matrix of the given system $\varphi_{\mathrm{d}}(\mathbf{A}_{\mathrm{z}})$ is the desired characteristic equation evaluated for the Matrix $\mathbf{A}_z$. Prove this formula. Caution: Please give it a sincere try before looking up online. Hint: Prove it for a small 3x3 matrix and get an idea of how to generalize.

3. In class we $K_z = K_x C_{M_x} C_{M_z}^{-1}$, where K_x is state feedback vector derived for the phase variable choice and C_{M_x} is the corresponding controllability matrix. Take a 3rd order system of your choice that is described in non-phase variable form. Use the transformation method to obtain state feedback vector. Use the Ackermann's formula to verify your answer.

4. In the past, Type-1 diabetes patients had to inject themselves with insulin three to four times a day. New delayed-action insulin analogues such as insulin Glargine require a single daily dose. For a specific patient, state-space model matrices are given by (Tarin, 2007)

$$\mathbf{A} = \begin{bmatrix} -0.435 & 0.209 & 0.02 \\ 0.268 & -0.394 & 0 \\ 0.227 & 0 & -0.02 \end{bmatrix}; \mathbf{B} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix};$$

$$\mathbf{C} = [0.0003\ \ 0\ \ 0];\quad \mathbf{D} = 0$$

The state variables are

x_1 = insulin amount in plasma compartment
x_2 = insulin amount in liver compartment
x_3 = insulin amount in interstitial (in body tissue) compartment

The system's input is u = external insulin flow. The system's output is y = plasma insulin concentration. Find the systems transfer function and verify your result using Matlab.

5. Consider a completely state controllable system

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$$
$$y = \mathbf{C}\mathbf{x}$$

where $\mathbf{x}$ = state vector (n-vector)
u = control signal (scalar)
y = output signal (scalar)
$\mathbf{A}$ = $n \times n$ constant matrix
$\mathbf{B}$ = $n \times 1$ constant matrix
$\mathbf{C}$ = $1 \times n$ constant matrix

Suppose that the rank of the following $(n + 1) \times (n + 1)$ matrix

$$\begin{bmatrix} \mathbf{A} & \mathbf{B} \\ -\mathbf{C} & 0 \end{bmatrix}$$

is $n + 1$. Show that the system defined by

$$\dot{\mathbf{e}} = \hat{\mathbf{A}}\mathbf{e} + \hat{\mathbf{B}}u_e$$

where

$$\hat{\mathbf{A}} = \begin{bmatrix} \mathbf{A} & \mathbf{0} \\ -\mathbf{C} & 0 \end{bmatrix}, \qquad \hat{\mathbf{B}} = \begin{bmatrix} \mathbf{B} \\ \mathbf{0} \end{bmatrix}, \qquad u_e = u(t) - u(\infty)$$

is completely state controllable.

6.

7. Consider the system defined by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

Show that this system cannot be stabilized by the state-feedback control $u = -\mathbf{K}\mathbf{x}$, whatever matrix $\mathbf{K}$ is chosen.

8. Consider the inverted-pendulum system show in the Figure below. Assume that

$$M = 2\text{ kg}, \qquad m = 0.5\text{ kg}, \qquad l = 1\text{ m}$$

Define state variables as $x_1 = \theta, \quad x_2 = \dot{\theta}, \quad x_3 = x, \quad x_4 = \dot{x}$
and output variables as $y_1 = \theta = x_1, \quad y_2 = x = x_3$

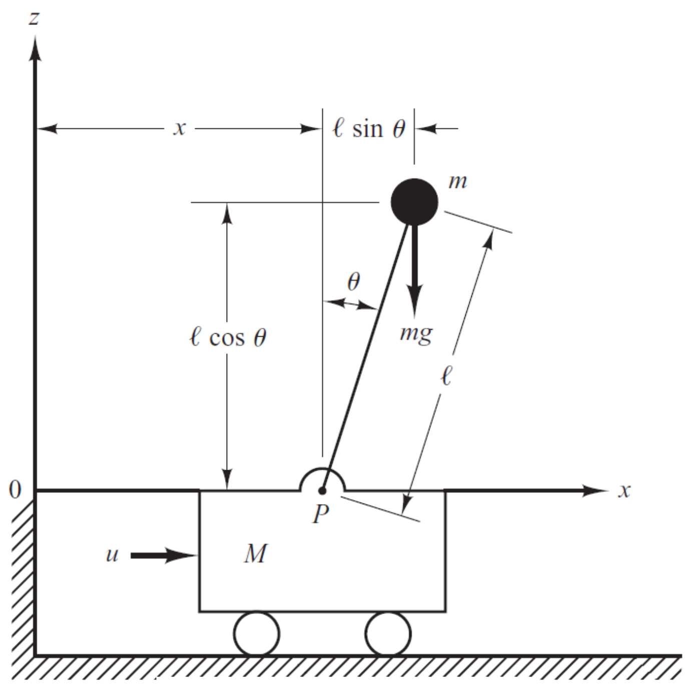
Derive the state-space equations for this system.

It is desired to have closed-loop poles at $s = -4 + j4,\ s = -4 - j4,\ s = -20,\ s = -20$

Determine the state-feedback gain matrix $\mathbf{K}$.

Using the state-feedback gain matrix $\mathbf{K}$ thus determined, examine the performance of the system by computer simulation. Write a MATLAB program to obtain the response of the system to an arbitrary initial condition. Obtain the response curves $x_1(t)$ versus t, $x_2(t)$ versus t, $x_3(t)$ versus t, and $x_4(t)$ versus t for the following set of initial condition:

$$x_1(0) = 0, \quad x_2(0) = 0, \quad x_3(0) = 0, \quad x_4(0) = 1\text{ m/s}$$



Inverted-pendulum system