Control Systems

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***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```

1 Bode Plot

1.1 Introduction

1.1. For an LTI system, the Bode plot for its gain is as illustrated in the Fig.2.1 The number of system poles N_p and number of system zeros N_z in the frequency range 1 Hz $\leq$ f $\leq$ 10^7 Hz is

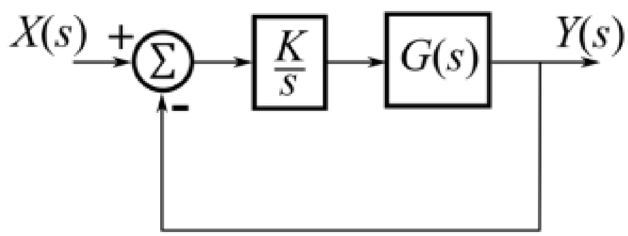

Fig. 1.1

Solution: Let us consider a generalized transfer gain

$$H(s) = k\frac{(s-z_1)(s-z_2)...(s-z_{m-1})(s-z_m)}{(s-p_1)(s-p_2)....(s-p_{n-1})(s-p_n)} \tag{1.1.1}$$

$$\begin{aligned} Gain = 20\log|H(s)| = 20\log|k| + 20\log|s-z_1| \\ +20\log|s-z_2| + \cdots + 20\log|s-z_m| - 20\log|s-p_1| \\ -20\log|s-p_2| - \cdots - 20\log|s-z_n| \end{aligned} \tag{1.1.2}$$

Let us consider a $20\log|s-z_1|$
Let $s = j\omega$

$$20\log|s-z_1| = 20\log\left|\sqrt{\omega^2+z_1^2}\right| \tag{1.1.3}$$

Based on log scale plot approximations,to the left of z_1 $\omega << z_1$ and towards right $\omega >> z_1$
For $\omega < z_1$

$$20\log|s-z_1| = 20\log\left|\sqrt{\omega^2+z_1^2}\right| = 20\log|z_1| \quad = constan \tag{1.1.4}$$

i.e. $Slope = 0$
For $\omega > z_1$

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$$20\log|s-z_1| = 20\log\left|\sqrt{\omega^2+z_1^2}\right| = 20\log|\omega| \tag{1.1.5}$$

i.e $Slope = 20$

When a zero is encountered the slope always increases by 20 dB/decade

Doing similar analysis for $-20\log|s-p_1|$ We conclude

When a pole is encountered the slope always decreases by 20 dB/decade

$$Slope = \frac{d(20\log H(f))}{df} \tag{1.1.6}$$

$$Slope = \begin{cases} 0 & 0 < f < 10^1 \\ -20 & 10 < f < 10^2 \\ -60 & 10^2 < f < 10^3 \\ -40 & 10^3 < f < 10^4 \\ 0 & 10^4 < f < 10^5 \\ -40 & 10^5 < f < 10^6 \\ -60 & 10^6 < f < 10^7 \end{cases} \tag{1.1.7}$$

Δ Slope = Change in slope at f

$$\Delta Slope = \begin{cases} -20 & f = 10^1 \\ -40 & f = 10^2 \\ +20 & f = 10^3 \\ +40 & f = 10^4 \\ -40 & f = 10^5 \\ -20 & f = 10^6 \end{cases} \tag{1.1.8}$$

Final Transfer function is

$$H(f) = \frac{K(f+10^3)(f+10^4)^2}{(f+10^1)(f+10^2)^2(f+10^5)^2(f+10^6)1} \tag{1.1.9}$$

$$N_p = 6 \tag{1.1.10}$$

$$N_z = 3 \tag{1.1.11}$$

Python plot of the obtained transfer function is shown in fig 2.2

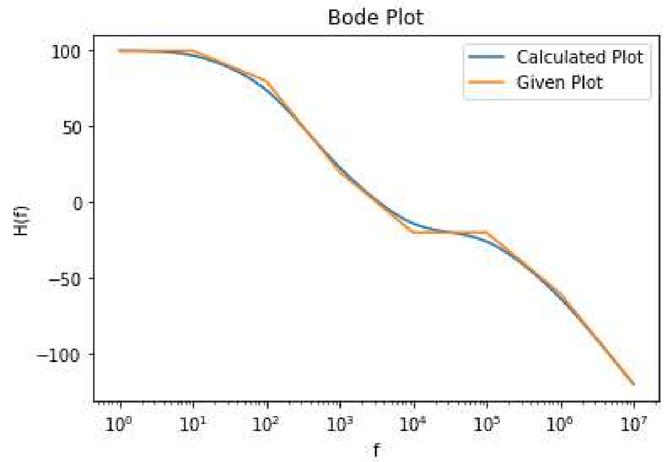

Fig. 1.1

1.2 Example

1.2.1. The asymptotic Bode magnitude plot of minimum phase transfer function G(s) is show below.

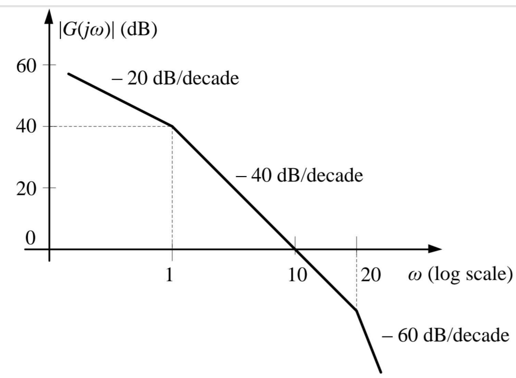

Fig. 1.2.1

1.2.2. Verify if the transfer function G(s) has 3 poles and one zero.

1.2.3. Verify if at very high frequency ($\omega \to \infty$), the phase angle $\angle G(j\omega) = -3\pi/2$ **Solution:** Since, each pole corresponds to -20 dB/decade and each zero corresponds to +20 dB/decade. Therefore, from the given Bode plot we can get the Transfer equation,

$$G(s) = \frac{k}{s(1+s)(20+s)} \tag{1.2.3.1}$$

Now, from the Transfer equation we can conclude that, there are three poles (0, -1 and -20) and no zeros.

$\therefore$ Statement 1 is false ....(1)

Calculating phase:

Since we know that,
phase ϕ is the sum of all the phases corresponding to each pole and zero.
phase corresponding to pole is =

$$-tan^{-1}(\frac{imaginary}{real}) \tag{1.2.3.2}$$

phase corresponding to zero is =

$$tan^{-1}(\frac{imaginary}{real}) \tag{1.2.3.3}$$

Now take,

$$s = j\omega \tag{1.2.3.4}$$

$$\Rightarrow G(j\omega) = \frac{k}{j\omega(1 + j\omega)(20 + j\omega)} \tag{1.2.3.5}$$

Therefore,

$$\phi = -tan^{-1}(\frac{\omega}{0}) - tan^{-1}(\omega) - tan^{-1}(\frac{\omega}{20}) \tag{1.2.3.6}$$

$$\phi = -90^\circ - tan^{-1}(\omega) - tan^{-1}(\frac{\omega}{20}) \tag{1.2.3.7}$$

$$\because \omega \to \infty \tag{1.2.3.8}$$

$$\phi = -90^\circ - 90^\circ - 90^\circ \tag{1.2.3.9}$$

$$\phi = -270^\circ \tag{1.2.3.10}$$

$$\phi = -3\pi/2 \tag{1.2.3.11}$$

$\therefore$ Statement 2 is true ....(2)
thus, from (1) and (2) option (B) is correct.

1.2.4.

2 Stability

2.1 Second order System

2.1. Consider the following second order system with the transfer function

$$G(s) = \frac{1}{1 + 2s + s^2} \tag{2.1.1}$$

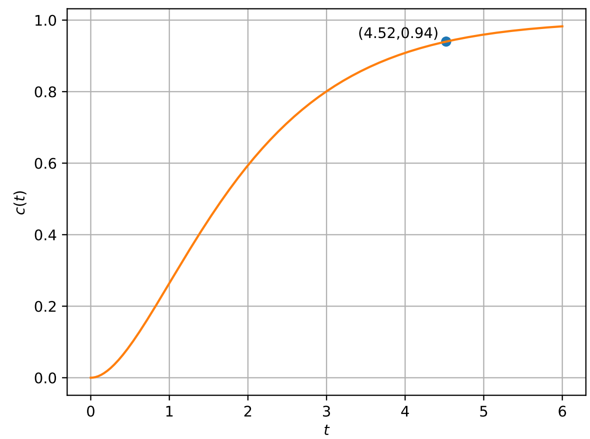

Fig. 2.2

Is the system stable?
Solution: The poles of

$$G(s) = \frac{1}{1 + 2s + s^2} \tag{2.1.2}$$

are at

$$s = -1 \tag{2.1.3}$$

i.e., the left half of s-plane. Hence the system is stable.

2.2. Find and sketch the step response $c(t)$ of the system.
Solution: For step-response, we take input as unit-step function u(t)

$$C(s) = U(s).G(s) = \left[\frac{1}{s}\right]\left[\frac{1}{1 + 2s + s^2}\right] \tag{2.2.1}$$

$$= \frac{1}{s(1 + s)^2} \tag{2.2.2}$$

$$= \frac{1}{s} - \frac{1}{(1 + s)} - \frac{1}{(1 + s)^2} \tag{2.2.3}$$

Taking the inverse Laplace transform,

$$c(t) = L^{-1}\left[\frac{1}{s}\right] - L^{-1}\left[\frac{1}{1 + s}\right] - L^{-1}\left[\frac{1}{(1 + s)^2}\right] \tag{2.2.4}$$

$$= \left(1 - e^{-t} - te^{-t}\right)u(t) \tag{2.2.5}$$

The following code plots $c(t)$ in Fig. 2.2

```
codes/ee18btech11002/plot.py
```

2.3. Find the steady state response of the system