

## Assignment 1

1. Sketch the root locus for the following systems.

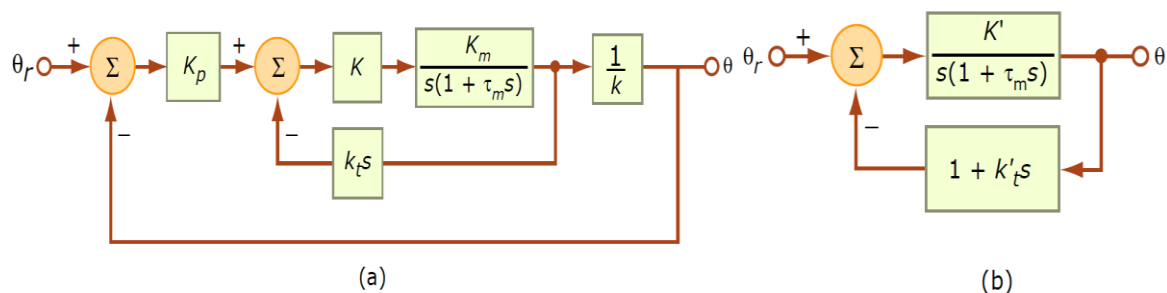
$$(a) G_c G_p(s) = \frac{K}{(s^2 + 6s + 8)(s^2 + 2s + 5)}$$

$$(b) G_c G_p(s) = \frac{K(s^2 - 2s + 2)}{s(s + 2)}$$

$$(c) G_c G_p(s) = \frac{K(s + 3)}{(s^4 - 4s^2)}$$

(d) After completing these sketches, verify the results using Matlab. How closely do your sketches resemble the actual plots?

2. Consider the DC motor control system with rate feedback shown in Figure 1(a).



(a) Find values for  $K^*$  and  $k_t^*$  so that the system of Figure 1(b) has the same transfer function as the system of Figure 1(a). Your answers should be in terms of  $K_p$ ,  $K$ ,  $k_t$ ,  $K_m$ , and  $k$ .

(b) Suppose  $\tau_m = 0.25$  sec. For the case with no rate feedback (i.e.,  $k_t = 0$ ), use root-locus techniques to select the proportional gain  $K^*$  to achieve a closed-loop damping ratio of  $\zeta = 0.2$ .

(c) Using the value of  $K^*$  found in part (b), sketch the locus of closed-loop pole locations for  $k_t^* > 0$ .

(d) Determine the system type number with respect to tracking  $\theta_r$ , and compute the corresponding error constant in terms of parameters  $K^*$  and  $k_t^*$ . What happens to the steady state error if  $K^*$  is increased? If  $k_t^*$  is increased?

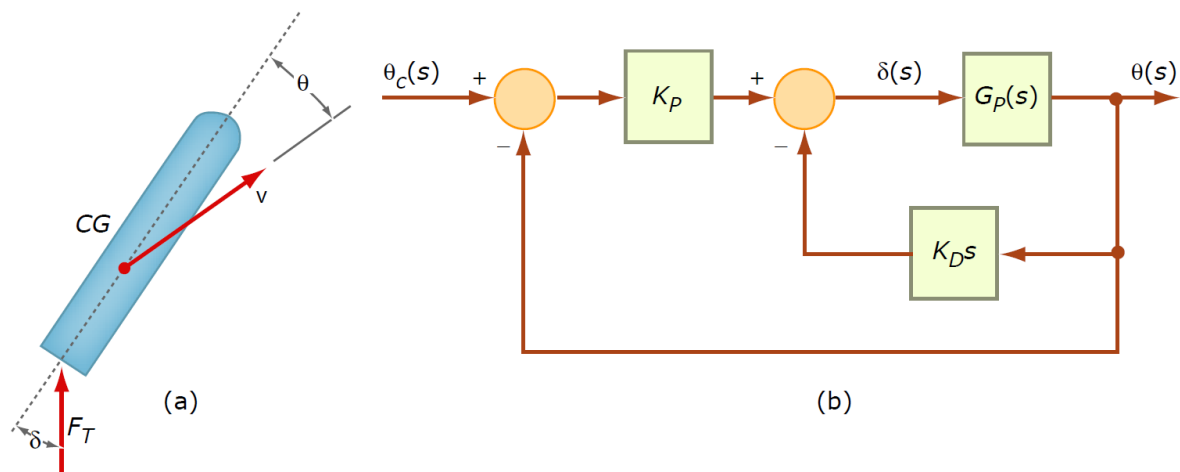
Hint: Your tracking error should take the form  $e(t) = \theta_r(t) - \theta(t)$ .

3. The attitude-control system of a space booster is shown in Figure 2. The attitude angle  $\theta$  is controlled by commanding the engine angle  $\delta$ , which is then the angle of the applied thrust,  $F_T$ . The vehicle velocity is denoted by  $v$ . A typical rigid-body transfer function for such a booster might take the form

$$G_c(s) = \frac{0.9}{s^2 - 0.03}$$

The rigid-body vehicle can be stabilized by the addition of rate feedback (Figure 2(b)).

### Booster Control System



- With  $K_D = 0$ , plot the root locus and state the different types of responses possible (relate the response with the possible pole locations). Why is  $K_P$  alone not sufficient to stabilize the dynamics?
- Design the compensator shown (which is PD) to place the closed-loop poles at  $s = -0.2 \pm j0.3$  (the resulting time constant is 5 sec).
- Plot the root locus of the compensated system, with  $K_P$  variable and  $K_D$  set to the value found in (b). Compare with your answer in (a).
- For some value of  $K_P$ , use Matlab to compute the closed-loop response to an impulse for  $\theta_c$ .

4. Suppose that you are to design a unity gain feedback controller for a first order plant. The plant has a zero at 2 and a pole at the origin. The controller has a pole at  $-p$ , zero at  $-z$  and a gain  $K$ .

(a) Using root-locus methods, specify some  $p$  and  $z$  for which it is possible to make the closed-loop system strictly stable. Include a sketch of the closed-loop root locus, as well as the corresponding range of gains  $K$  for which the system is strictly stable.

(b) Suppose  $p$  and  $z$  are fixed to the values chosen in (a). Design  $K$  to meet the following specifications:

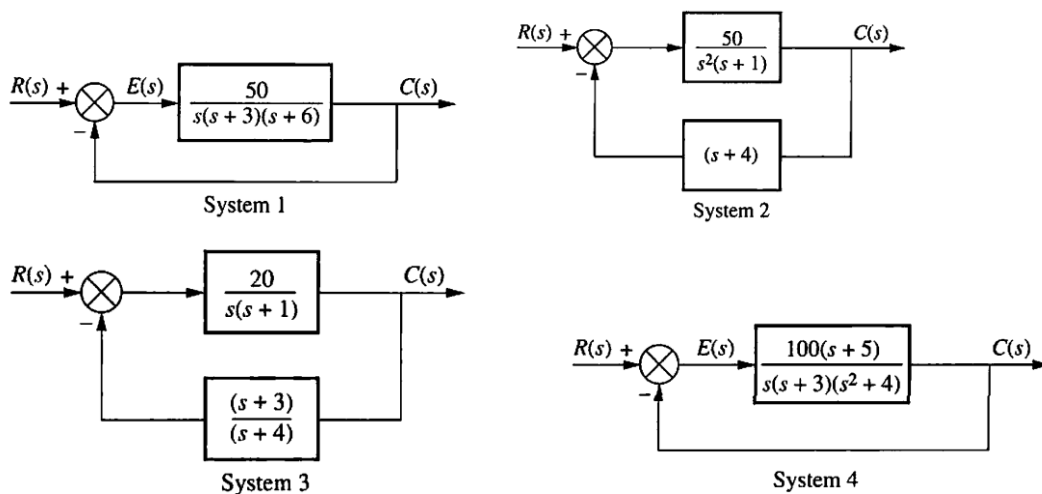
- The closed-loop system must be strictly stable.
- The damping ratio  $\zeta$  must be between 0.4 and 0.6.
- Given these constraints, minimize the natural frequency  $\omega_n$ .

### Optional Part – Bode/Nyquist plot and Nyquist criterion

1. Sketch the Bode asymptotic magnitude and asymptotic phase plots. Compare your results with the actual log-magnitude and phase plots

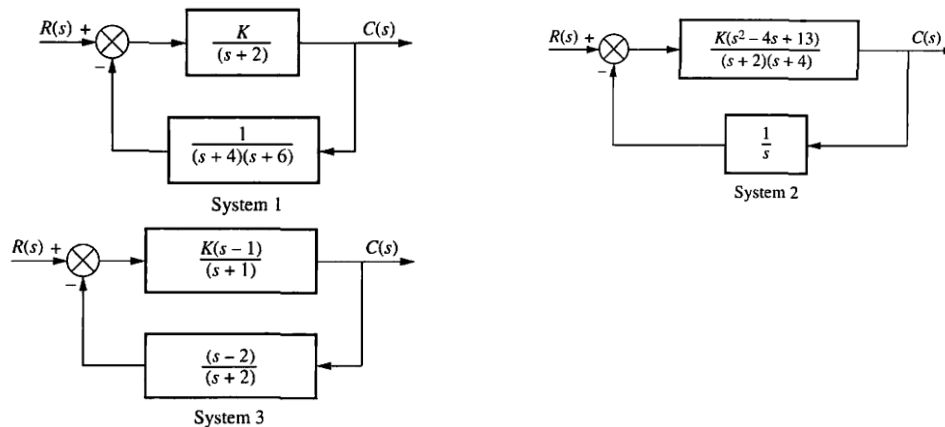
a.  $G(s) = \frac{1}{s(s+2)(s+4)}$       b.  $G(s) = \frac{(s+5)}{(s+2)(s+4)}$   
c.  $G(s) = \frac{(s+3)(s+5)}{s(s+2)(s+4)}$

2. Sketch the Nyquist diagram for each of the systems given below



3. Discuss the stability of systems in Problem 1 from the bode plots.
4. Using the Nyquist criterion, find out whether each system of Problem 2 is stable or not.

5. Using the Nyquist criterion, find the range of  $K$  for stability for each of the systems given below



6. For each system of Problem 5, find the gain margin and phase margin if the value of  $K$  in each part of Problem 5 is
- $K = 1000$
  - $K = 100$
  - $K = 0.1$
7. Given a unity feedback system with the forwardpath transfer function and a delay of 0.5 second, make a second-order approximation and estimate the percent overshoot if  $K = 40$ . Use Bode plots and frequency response techniques

$$G(s) = \frac{K}{s(s+3)(s+12)}$$