# EE 2227 - Control Systems

Assignment 4

Note that this assignment requires you to read some linear algebra and develop some proof writing skills. It is highly recommended that you attempt on your own after revising your linear algebra.

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1. The conceptual block diagram of a gas-fired heater is shown in Figure below. The commanded fuel pressure is proportional to the desired temperature. The difference between the commanded fuel pressure and a measured pressure related to the output temperature is used to actuate a valve and release fuel to the heater. The rate of fuel flow determines the temperature. When the output temperature equals the equivalent commanded temperature as determined by the commanded fuel pressure, the fuel flow is stopped and the heater shuts off.

   Assume that the transfer function of the heater, $G_H(s)$, is

   $$G_H(s) = \frac{1}{(s+0.4)(s+0.8)} \quad \frac{\text{degrees F}}{\text{ft}^3/\text{min}} \tag{1}$$

   and the transfer function of the fuel valve, $G_v(s)$, is

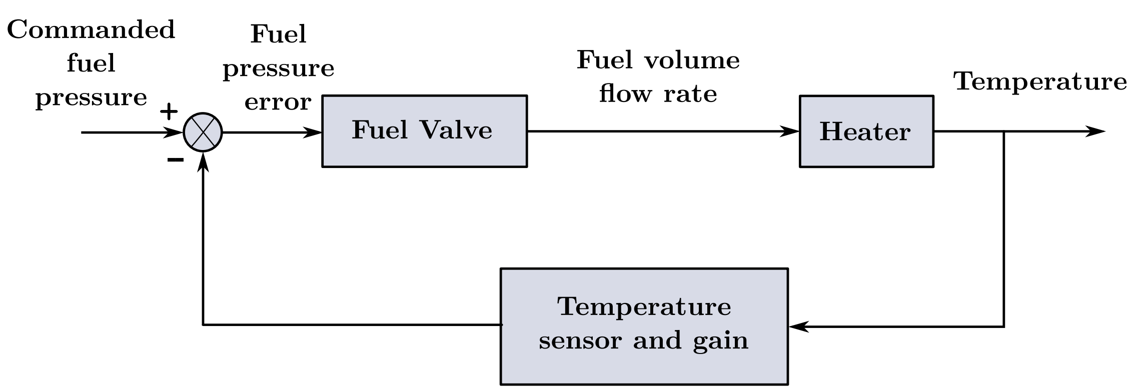
   $$G_v(s) = \frac{5}{s+5} \quad \frac{\text{ft}^3/\text{min}}{\text{psi}}. \tag{2}$$

   In the Figure you are shown a temperature feedback path. This feedback is replaced with a controller that yields a 5% overshoot and a settling time of 10 minutes.

   (a) Design an observer that will respond 10 times faster than the controller but with the same percentage overshoot.

   (b) Draw a signal flow graph depicting the observer. You are not required to draw the controller part of the signal flow graph.

   (c) Plot the system response to a suitable input and check for the difference between the closed-loop and the open-loop observer.




**Block diagram of a gas-fired heater**

2. A dc-dc converter is a device that takes as an input an unregulated dc voltage and provides a regulated dc voltage as its output. The output voltage may be lower (buck converter), higher (boost converter), or the same as the input voltage. Switching dc-dc converters have a semiconductor active switch (BJT or FET) that is closed periodically with a duty cycle $d$ in a pulse width modulated (PWM) manner. For a boost converter, averaging techniques lead to the Fig. shown below, which can be used to arrive at the following equations:

$$L\frac{di_L}{dt} = -(1-d)u_c + E_s,$$
$$C\frac{du_C}{dt} = (1-d)i_L - \frac{u_C}{R}$$

where $L$ and $C$ are respectively the values of internal inductance and capacitance; $i_L$ is the current through the internal inductor; $R$ is the resistive load connected to the converter; $E_s$ is the dc input voltage; and the capacitor voltage, $u_C$, is the converters output.

(1) Write the state equations assuming $L = 6$ mH, $C = 1$ mF, $R = 100\Omega$, a 50% PWM duty cycle.

(2) Is the system is observable or not?

(3) The controller is designed to obtain 20% overshoot and a settling time of 0.5. Design an observer that uses feedback of the difference of the output $y$ of the plant along with its own output $\hat{y}$ to provide 10 times faster response than the controller

(4) Plot $y$ and $\hat{y}$ with time for a ramp. Compare the performance of the closed-loop observer with the open-loop. Comment on the settling time in both scenarios.

3. Consider the standard state space equations

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu} \tag{3}$$
$$\mathbf{y} = \mathbf{Cx} + \mathbf{Du} \tag{4}$$

The solution of the state equation is given by

$$\mathbf{x}(t) = e^{\mathbf{A}t}\mathbf{x}(0) + \int_0^t e^{\mathbf{A}(t-\tau)}\mathbf{Bu}(\tau)d\tau. \tag{5}$$

Use the Cayley-Hamilton Theorem to argue that the observability matrix has to be rank $n$ for complete observability, where $n$ is order of the system.

4. For any system described in the state space representation, assume that the conversion to the classical representation results in a pole-zero cancellation. Comment whether the system is controllable and/or observable.

5. **Separation into observable and unobservable subspaces:** Prove that an unobservable system $(\mathbf{C}, \mathbf{A})$ can be taken using a similarity transform to

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_1 & \mathbf{A}_{12} \\ 0 & \mathbf{A}_2 \end{bmatrix} \tag{6}$$
$$\mathbf{C} = [0 \quad \mathbf{C}_2], \tag{7}$$

where $(\mathbf{C_2}, \mathbf{A_2})$ is observable. The observable subspace of the system $(\mathbf{C}, \mathbf{A})$ is given by the states of the form $\begin{bmatrix} * \\ 0 \end{bmatrix}$. The eigenvalues of $\mathbf{A}_1$ are unobservable eigenvalues. while those of $\mathbf{A}_2$ are observable.

6. **PBH test for observability:** Prove that the the system $(\mathbf{C}, \mathbf{A})$ is observable if and only if

$$\operatorname{rank} \begin{bmatrix} \mathbf{A} - \lambda \mathbf{I} \\ \mathbf{C} \end{bmatrix} = n \tag{8}$$

for all $\lambda \in \mathbb{C}$.

7. Consider a system $(\mathbf{C}, \mathbf{A})$ and define the observability Gramian as

$$\mathcal{G}_t = \int_0^t (e^{\mathbf{A}\tau})^T \mathbf{C}^T \mathbf{C} e^{\mathbf{A}\tau} d\tau. \tag{9}$$

For a stable $\mathbf{A}$ one may define the Gramian as

$$\mathcal{G} = \int_0^\infty (e^{\mathbf{A}\tau})^T \mathbf{C}^T \mathbf{C} e^{\mathbf{A}\tau} d\tau. \tag{10}$$

Another test, for observability is that the rank of $\mathcal{G}_t$ is $n$, while for stable $\mathbf{A}$ rank of $\mathcal{G}$ is $n$. Show that $\mathcal{G}$ satisfies the following Lyapunov equation:

$$\mathbf{A}^T \mathcal{G} + \mathcal{G} \mathbf{A} + \mathbf{C}^T \mathbf{C} = 0 \tag{11}$$