

Control Systems



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Abstract—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.

Download python codes using

svn co https://github.com/gadepall/school/trunk/control/codes

1 STABILITY

1.1. Consider the following second order system with the transfer function:

$$G(s) = \frac{1}{1 + 2s + s^2} \quad (1.1.1)$$

with input unit step

$$R(s) \qquad \qquad = \qquad \qquad \frac{1}{s} \quad (1.1.2)$$

Let C(s) be the corresponding output. The time taken by the system output c(t) to reach 94% of its steady state value, rounded off to two decimal places is

Solution:

$$G(s) = \left(\frac{1}{1 + 2s + s^2}\right) \quad (1.1.3)$$

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From given expression of G(s), both poles of G(s) are at (-1,0) which is on the left half of s-plane, therefore we can conclude that the system is stable.

$$C(s) = R(s).G(s) = (\frac{1}{s})(\frac{1}{1+2s+s^2})$$
 (1.1.4)

$$C(s) = \frac{1}{s(1+s)^2} \quad (1.1.5)$$

We found C(s) as:

$$C(s) = \frac{1}{s(1+s)^2} \quad (1.1.6)$$

Therefore,

$$C(s) = \frac{1}{s} - \frac{1}{(1+s)} - \frac{1}{(1+s)^2}$$
 (1.1.7)

Therefore:

$$c(t) = L^{-1}(\frac{1}{s}) - L^{-1}(\frac{1}{(1+s)}) - L^{-1}(\frac{1}{(1+s)^2})$$
(1.1.8)

Using the Known inverse transforms:

$$c(t) = (1 - e^{-t} - te^{-t}).u(t)$$
 (1.1.9)

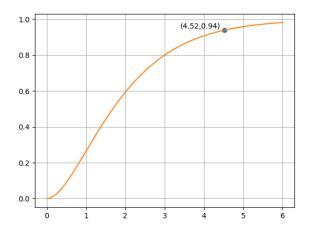
To know the steady state value of c(t), we calculate

$$\lim_{t \to \infty} c(t) = (1 + 0 + 0).(1) = 1 \quad (1.1.10)$$

Now, 94% of 1 is 0.94, so we should now solve for a positive t such that

$$(1 - e^{-t} - te^{-t}) = 0.94 (1.1.11)$$

$$t = 4.5228 \quad (1.1.12)$$



2 Routh Hurwitz Criterion

- 3 Compensators
- 4 NYQUIST PLOT