1.(a)(i) U = 150

F only = 25  $F \cap I$ = 15

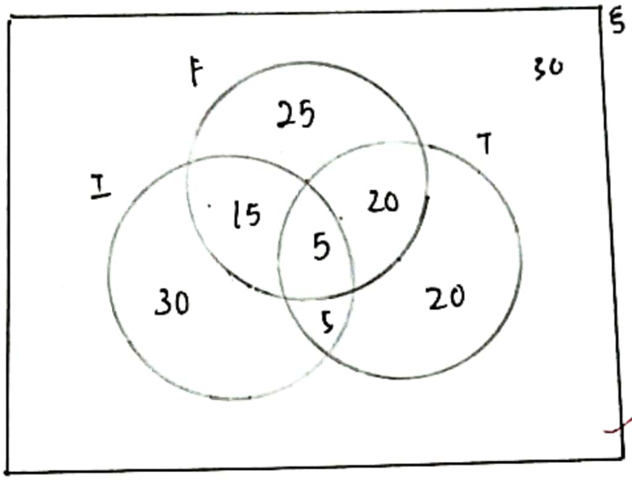
I only = 30  $F \cap I \cap T$ = 5

T only = 20

F = Facebook

I = Instagram

T = Twitter




2

(ii) Student does not have an account in any three sosial networks.

= 150 - 25 - 15 - 5 - 20 - 30 - 20 - 5

= 30 students

2

(iii) Student have exactly two social networks

= 15 + 20 + 5

= 40 students

2

(iv) student have social media account other than Facebook

= 30 + 5 + 20

= 55

2

(b)(i) $A = \{3, 5, 7, 9\}$ $\quad |A| = 4$

$B = \{2, 3, 5, 7\}$ $\quad |B| = 4$

$C = \{3, 6, 9\}$ $\quad |C| = 3$

3

(ii) $P(A) = 2^4 - 1$

$= 16 - 1$

$= 15$

3

(iii) $C \times B = \{(3,2), (3,3), (3,5), (3,7), (6,2), (6,3), (6,5),$
$(6,7), (6,9), (9,2), (9,3), (9,5), (9,7)\}$

2

2a. Using truth table:

| $p$ | $q$ | $\sim p$ | $(p \vee q)$ | $\sim(p \vee q)$ | $(\sim p \wedge q)$ | $\sim(p \vee q) \vee (\sim p \wedge q)$ |
|---|---|---|---|---|---|---|
| T | T | F | T | F | F | F |
| T | F | F | T | F | F | F |
| F | T | T | T | F | T | T |
| F | F | T | F | T | F | T |

$\therefore \sim(p \vee q) \vee (\sim p \wedge q) \equiv \sim p$ (Verified)

3

Using logic property law:

$\sim(p \vee q) \vee (\sim p \wedge q) = (\sim p \wedge \sim q) \vee (\sim p \wedge q)$ [De Morgan's laws]

$= \sim p \wedge (\sim q \vee q)$ [Distributive laws]

$= \sim p \wedge U$

$= \sim p$

$\therefore \sim(p \vee q) \vee (\sim p \wedge q) \equiv \sim p$ (Verified)

3

bi. $(r \wedge q) \rightarrow p$ 2

ii. $(\sim r \wedge \sim q) \rightarrow \sim p$ 4 2

iii. $\sim p \rightarrow (\sim r \wedge \sim q)$ 4 2

c. Negation of $\forall x (x^2 + 2x - 3 = 0)$: $\sim(\forall x (x^2 + 2x - 3 = 0)) = \exists x (\sim(x^2 + 2x - 3 = 0))$ 4

$\exists x (\sim(x^2 + 2x - 3 = 0))$ where the domain of discourse is Integer.

When $x = 2$, $x^2 + 2x - 3 = (2)^2 + 2(2) - 3$

$= 5 (\neq 0)$

$\therefore$ The proposition $\exists x (\sim(x^2 + 2x - 3 = 0))$ is TRUE.

5

2d. Let $P(x)$: x is student who can speak Russian.

$Q(x)$: x is student who know C++.

where the domain of discourse consist of all students at school.

i. $\exists x(P(x) \land \sim Q(x))$

ii. $\forall x(P(x) \lor Q(x))$

iii. $\forall x(\sim P(x) \land \sim Q(x))$

3a. Let $P(X)$: $a^2-3b$ is even

$Q(X)$: $a$ is even and $b$ is even

$\forall x(P(X) \to Q(X))$

$P(X) \to Q(X) \equiv \sim Q(X) \to \sim P(X)$

$\sim Q(X)$ is true : - Case 1 : $a$ is odd and $b$ is even

- Case 2 : $a$ is even and $b$ is odd
- Case 3 : $a$ is odd and $b$ is odd

2

Case 1 : if $a$ is odd and $b$ is even, let $a = 2m+1$, $b = 2n$

$$a^2-3b = (2m+1)^2 - 3(2n)$$
$$= 4m^2+4m+1-6n$$
$$= 2(2m^2+2m-3n)+1$$
$$t = 2m^2+2m-3n$$
$\therefore a^2-3b = 2t+1$ (odd)

$\therefore \sim Q(X)$ is true, $\sim P(X)$ is true, $\sim Q(X) \to \sim P(X)$ is true.

Case 2 : if $a$ is even and $b$ is odd, let $a = 2k$, $b = 2l+1$

$$a^2-3b = (2k)^2 - 3(2l+1)$$
$$= 4k^2-6l-3$$
$$= 4k^2-6l-4+1$$
$$= 2(2k^2-3l-2)+1$$
$$s = 2k^2-3l-2$$
$\therefore a^2-3b = 2s+1$ (odd)

$\therefore \sim Q(X)$ is true, $\sim P(X)$ is true, $\sim Q(X) \to \sim P(X)$ is true.

6

Case 3 : $a$ is odd and $b$ is odd, let $a = 2v+1$, $b = 2w+1$

$$a^2-3b = (2v+1)^2 - 3(2w+1)$$
$$= 4v^2+4v+1-6w-3$$
$$= 2(2v^2+2v-3w-1)$$
$$r = 2v^2+2v-3w-1$$
$\therefore a^2-3b = 2r$ (even)

$\therefore \sim Q(X)$ is true, $\sim P(X)$ is false, $\sim Q(X) \to \sim P(X)$ is false.

$\therefore$ The statement is proven false because $\sim Q(X) \to \sim P(X)$ is false in Case 3.