Reinforcement learning Episode 2

Value-based methods, Temporal Difference







Reinforcement learning Episode 2

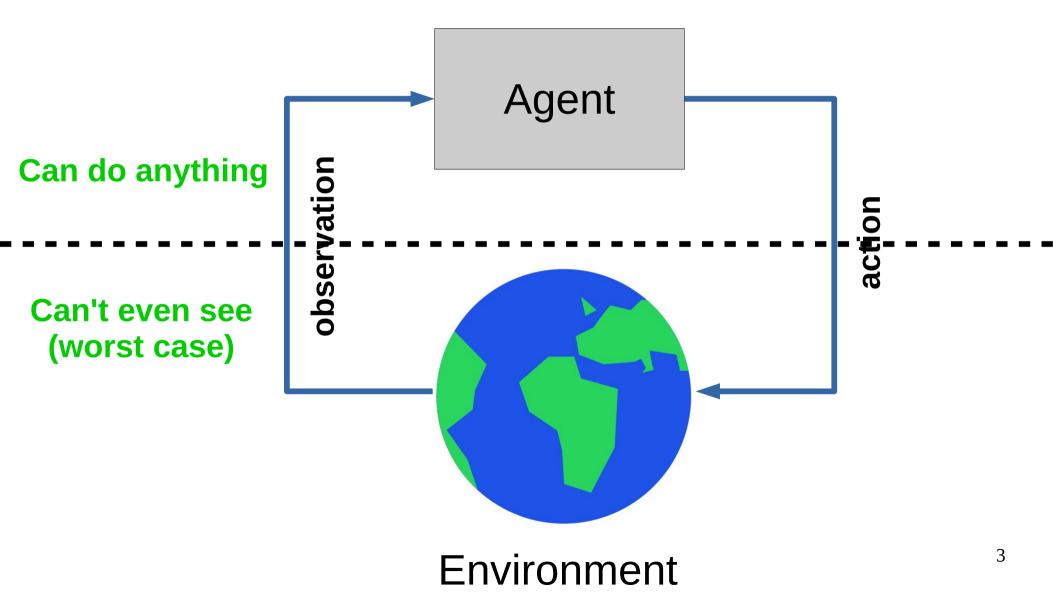
Oh gosh, we have 101 slides!



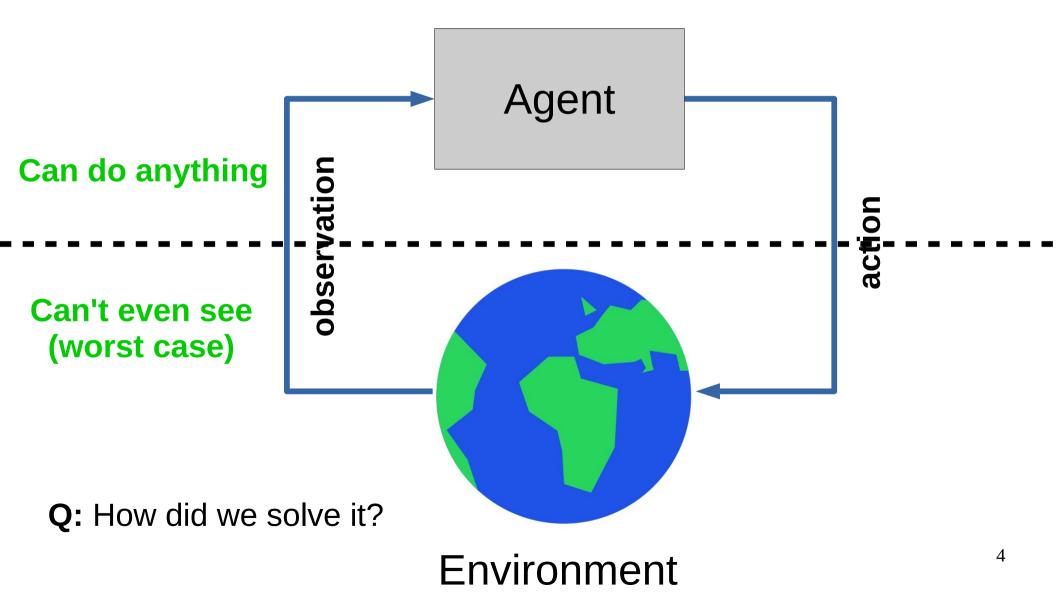




Recap: reinforcement learning



Recap: reinforcement learning



Black box methods

- Genetic algorithms
- Evolution strategies
- Crossentropy method

. . .

Black box drawbacks

- Both need a full session to start learning
- Requires a lot of interaction
 - A lot of crashed robots / simulations

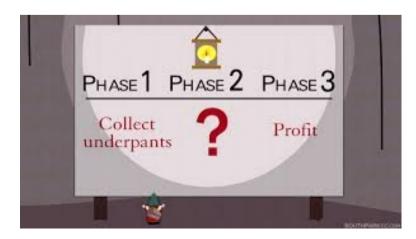


Today: value-based methods

- Core idea:
 - 1. You are at state **s**
 - 2. Compute expected reward for session if you take each possible action (a1,a2,a3,...)

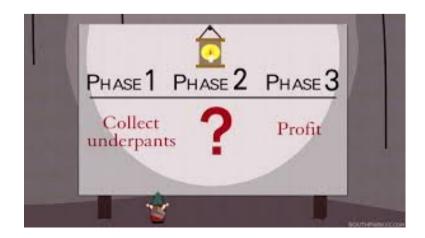
Then what? (e.g. chess)

You can compute expected win rate for each move. What do you do?

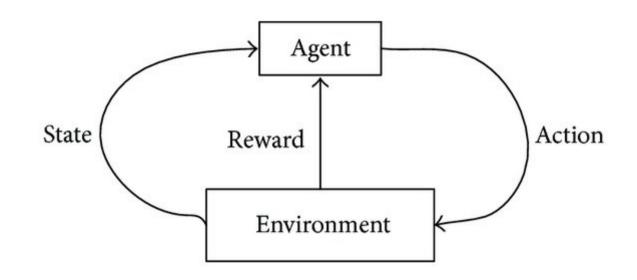


Today: value-based methods

- Core idea:
 - 1. You are at state **s**
 - 2. Compute expected reward for session if you take each possible action (a1,a2,a3,...)
 - 3. Take action with highest expected reward!



MDP formalism: reward on each tick



Classic MDP(Markov Decision Process) Agent interacts with environment

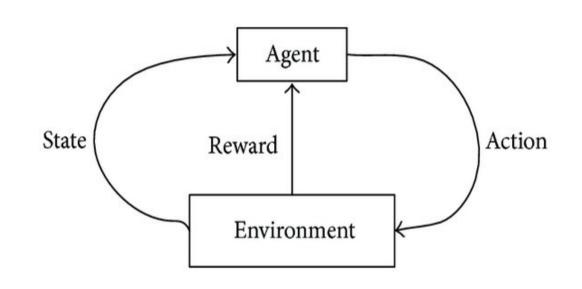
- Environment states: $s \in S$
- Agent actions: $a \in A$
- State transition: $P(s_{t+1}|s_t, a_t)$
- Reward: $r_t = r(s_t, a_t)$

Model-based setup

What we know

State transitions

$$P(s_{next}|s,a)$$
 or $s_{next}=T(s,a)$



• Rewards r(s, a)

Model-based setup

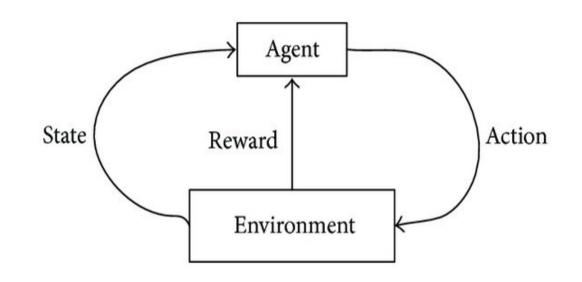
What we know

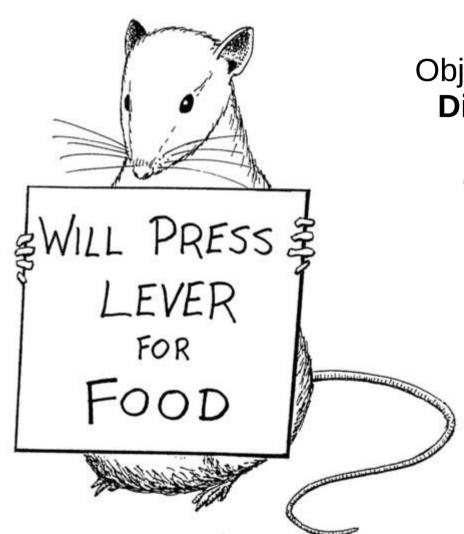
State transitions

$$P(s_{next}|s,a)$$
 or $s_{next}=T(s,a)$

Weaker version: we can only sample from P(s'|s,a)

• Rewards r(s, a)





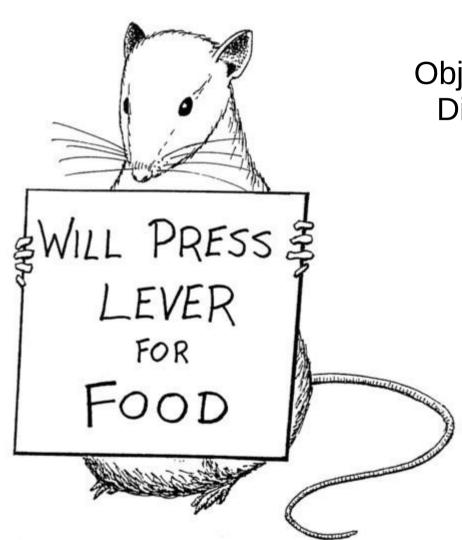
Objective:

Discounted return G

$$G_t = r_t + \gamma \cdot r_{t+1} + \gamma^2 \cdot r_{t+2} + \dots + \gamma^n \cdot r_{t+n}$$

$$G_t = \sum_{i} \gamma^{i} \cdot r_{t+i} \quad \gamma \in (0,1) const$$

y ~ patience Cake tomorrow is γ as good as now



Objective:

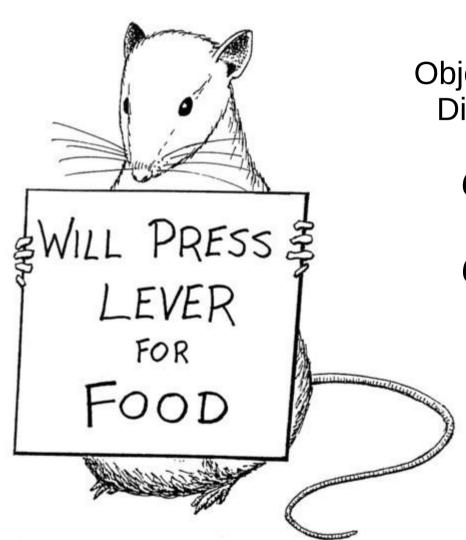
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$$G_t = \sum_i \gamma^i \cdot r_{t+i} \quad \gamma \in (0,1) const$$

γ ~ patience Cake tomorrow is γ as good as now

Q1: which γ corresponds to "only current reward matters"? **Q2:** with which γ G is just a sum of rewards?



Objective:

Discounted return G

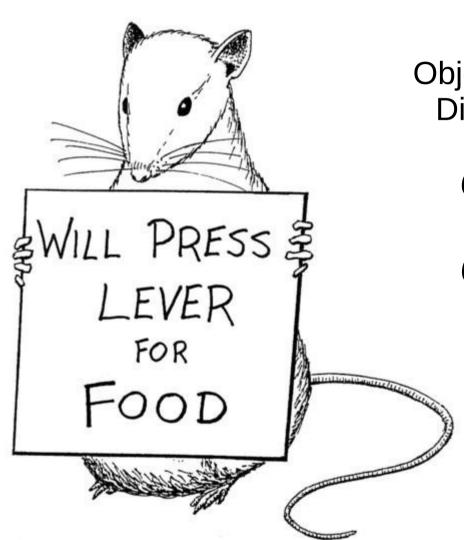
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$$G_t = \sum_{i} \gamma^{i} \cdot r_{t+i} \quad \gamma \in (0,1) const$$

Reinforcement learning:

 Find policy that maximizes the expected reward

$$\pi = P(a|s) : E[G] \rightarrow max$$



Objective:

Discounted return G

$$G_t = r_t + \gamma \cdot r_{t+1} + \gamma^2 \cdot r_{t+2} + \dots + \gamma^n \cdot r_{t+n}$$

$$G_t = \sum_{i} \gamma^{i} \cdot r_{t+i} \quad \gamma \in (0,1) const$$

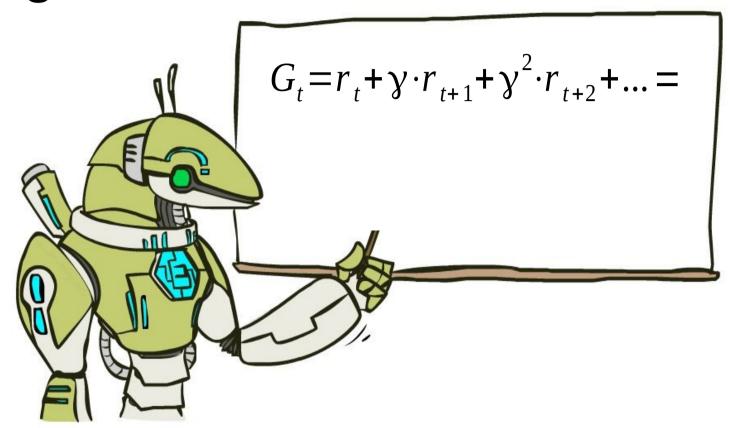
Reinforcement learning:

 Find policy that maximizes the expected reward

$$\pi = P(a|s) : E[G] \rightarrow max$$

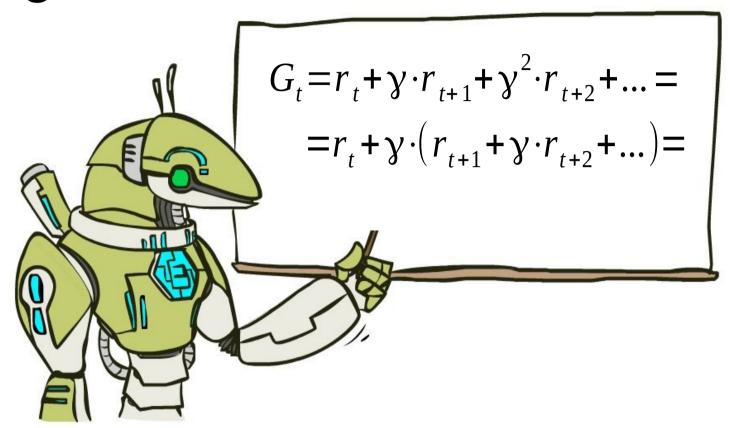
Does not maximize sum of rewards Unless y = 1

Dealing with G



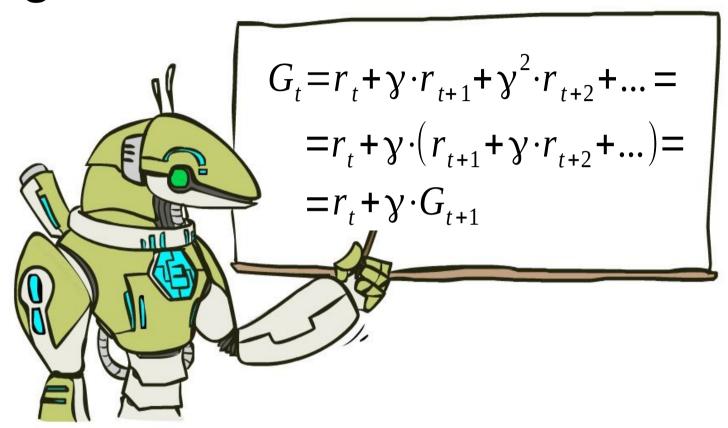
We rewrite G with sheer power of math!

Dealing with G



We rewrite G with sheer power of math!

Dealing with G



We rewrite G with sheer power of math!

More new letters!

State values : $V_{\pi}(s)$

• **Definition:** $V_{\pi}(s)$ – expected total reward G that can be obtained starting from state s and following policy π .

VS



small potential rewards



More new letters!

State values :
$$V_{\pi}(s)$$

$$V_{\pi}(s) = E_{a \sim \pi(a|s)} \dots$$

State values : $V_{\pi}(s)$

$$V_{\pi}(s) = E E_{a \sim \pi(a|s) \ s', r \sim P(s'|s, a)} \dots$$

State values : $V_{\pi}(s)$

$$V_{\pi}(s) = E \qquad E \qquad E \qquad \dots$$
 $a \sim \pi(a|s) \quad s', r \sim P(s'|s,a) \quad a', r', s'', \dots$

State values : $V_{\pi}(s)$

$$V_{\pi}(s) = E E_{a \sim \pi(a|s) \ s', r \sim P(s'|s,a) \ a', r', s'', \dots} E_{t+1} + \gamma \cdot r_{t+1} + \gamma^2 \cdot r_{t+2} + \dots + \gamma^n \cdot r_{t+n}$$

State values : $V_{\pi}(s)$

• **Definition:** $V_{\pi}(s)$ – expected total reward G that can be obtained starting from state s and following policy π .

$$V_{\pi}(s) = E_{\tau \sim p(\tau|...)} G(\tau)$$

trajectory $\tau = (s,a,r,s',a',r',...)$

Bellman equation

State values : $V_{\pi}(s)$

$$V_{\pi}(s) = E E_{a \sim \pi(a|s) \quad s', r \sim P(s', r|s, a)} [r + \gamma \cdot V_{\pi}(s')]$$

Bellman equation

State values : $V_{\pi}(s)$

Definition: $V_{\pi}(s)$ – expected total reward R that can be obtained starting from state s and following policy π .

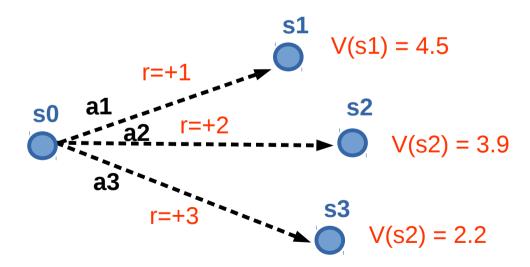
$$V_{\pi}(s) = E E_{a \sim \pi(a|s) \quad s', r \sim P(s', r|s, a)} [r + \gamma \cdot V_{\pi}(s')]$$

how I like $s = \text{what } r \mid \text{can get right now} + \gamma * \text{how I like } s'$

Optimal policy

Imagine we know exact value of any $V_{\pi}(s)$. (also P(s',r|s,a))

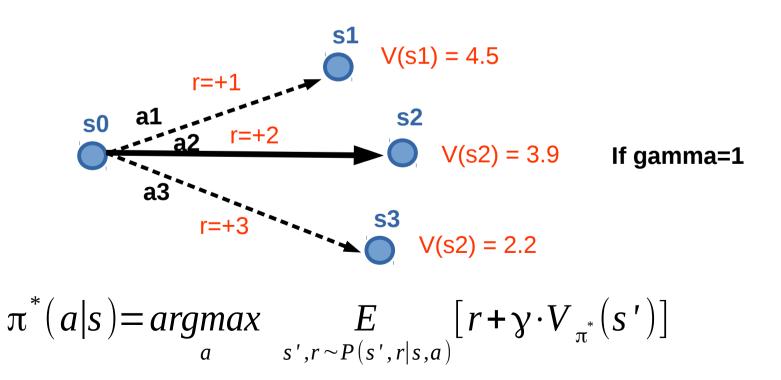
Q: how do we pick optimal action?



Optimal policy

Imagine we know exact value of any $V_{\pi}(s)$. (also P(s',r|s,a))

Q: how do we pick optimal action?



$\pi^*(a|s)$ and $V^*(s)$

Optimal policy : $\pi^*(a|s)$

$$\pi^*(a|s) = \underset{a}{\operatorname{argmax}} \quad \underset{s',r \sim P(s',r|s,a)}{E} [r + \gamma \cdot V_{\pi^*}(s')]$$

Optimal state value : $V^*(s)$

$$V^*(s)=V_{\pi^*}(s)=\max_{a} E_{s',r\sim P(s',r|s,a)}[r+\gamma\cdot V(s')]$$

Idea:

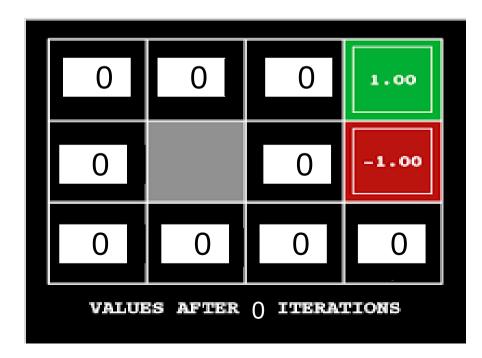
$$\forall s, V_0(s) := 0$$

$$V_{i+1}(s) := \max_{a} [r(s,a) + \gamma \cdot E_{s' \sim P(s'|s,a)} V_{i}(s')]$$

Idea:

$$\forall s, V_0(s) := 0$$

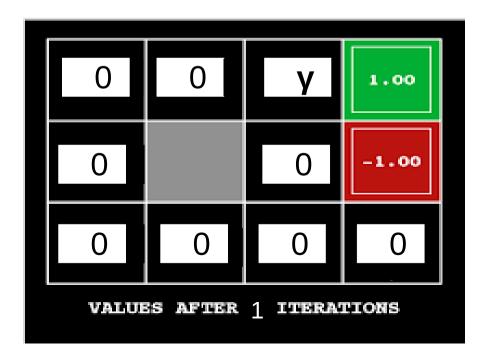
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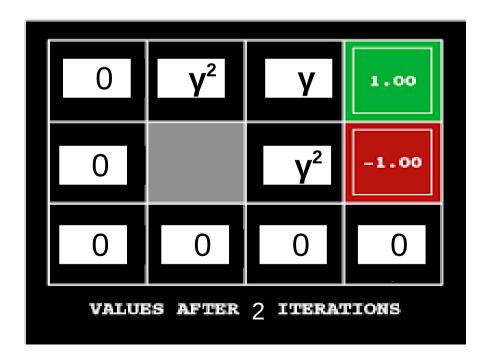
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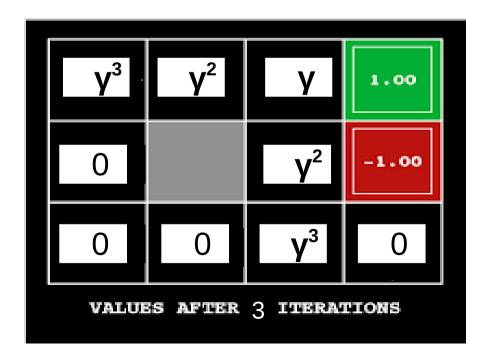
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Idea:

$$\forall s, V_0(s) := 0$$

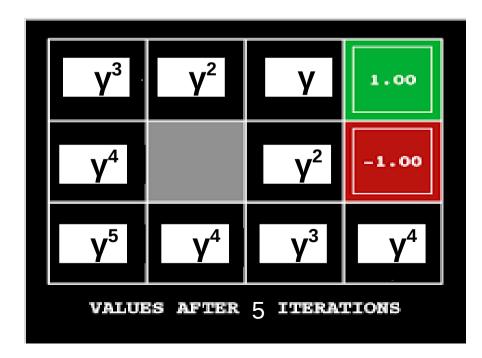
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Idea:

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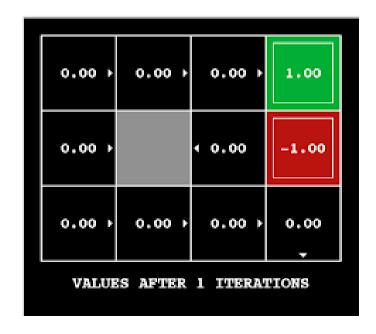
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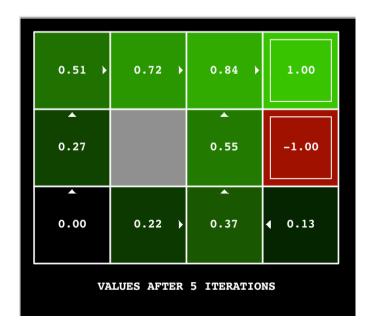


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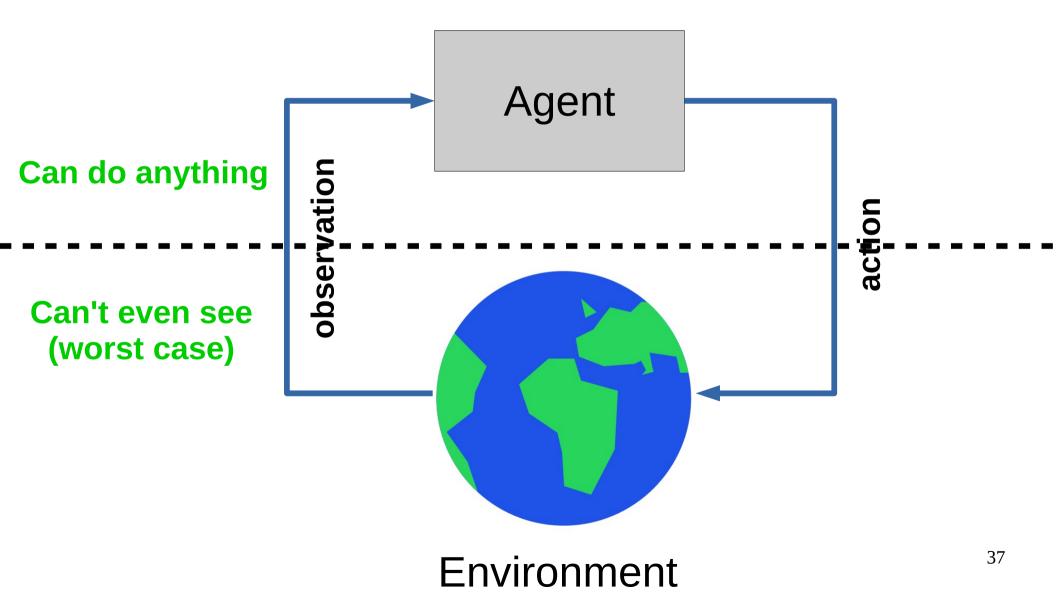
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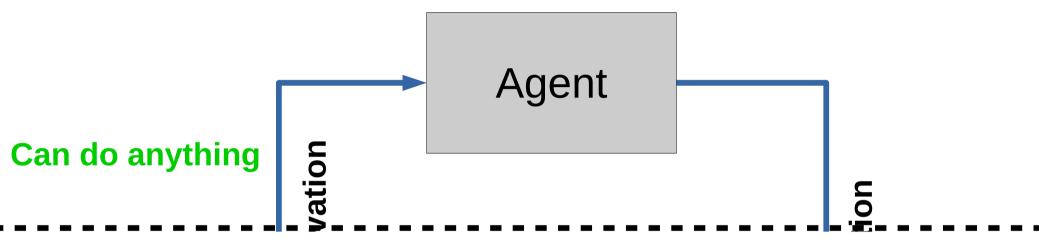


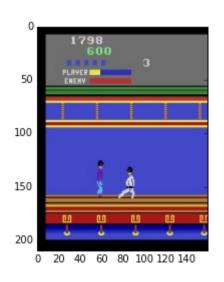


Decision process in the wild



Decision process in the wild











Model-free setting:

We don't know actual P(s',r|s,a)

Whachagonnado?

Model-free setting:

We don't know actual P(s',r|s,a)

Learn it?
Get rid of it?

More new letters

- $V_{\pi}(s)$ expected G from state s if you follow π
- $V^*(s)$ expected G from state s if you follow π^*

More new letters

- $V_{\pi}(s)$ expected G from state s if you follow π
- $V^*(s)$ expected G from state s if you follow π^*

- $Q_{\pi}(s,a)$ expected G from state s
 - if you start by taking action a
 - and follow π from next state on

• **Q*(s,a)** – guess what it is :)

More new letters

- $V_{\pi}(s)$ expected G from state s if you follow π
- $V^*(s)$ expected G from state s if you follow π^*

- $Q_{\pi}(s,a)$ expected G from state s
 - if you start by taking action a
 - and follow π from next state on

• $Q^*(s,a)$ – same as $Q_{\pi}(s,a)$ where $\pi = \pi^*$

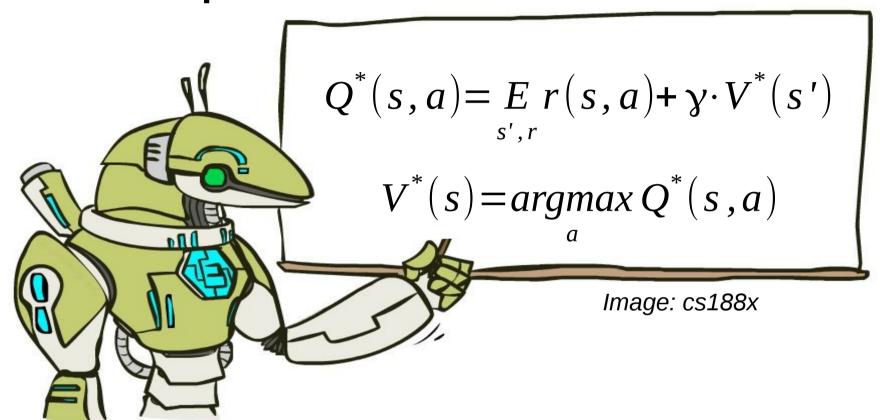
Any ideas?

- Assuming you know Q*(s,a),
 - how do you compute π*

- how do you compute V*(s)?

- Assuming you know V(s)
 - how do you compute Q(s,a)?

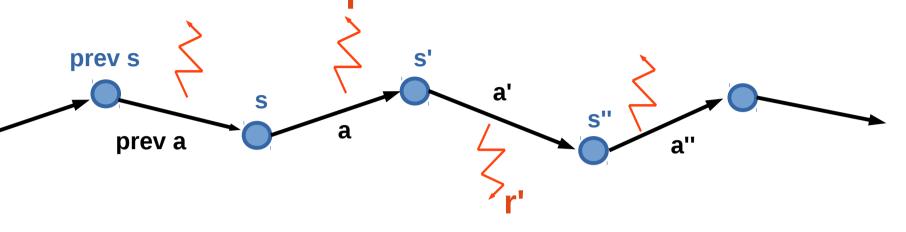
To sum up



Action value $Q_{\pi}(s,a)$ is the expected total reward G agent gets from state s by taking action a and following policy π from next state.

$$\pi(s)$$
: $argmax_a Q(s,a)$

Learning from trajectories



Model-based: you know P(s'|s,a)

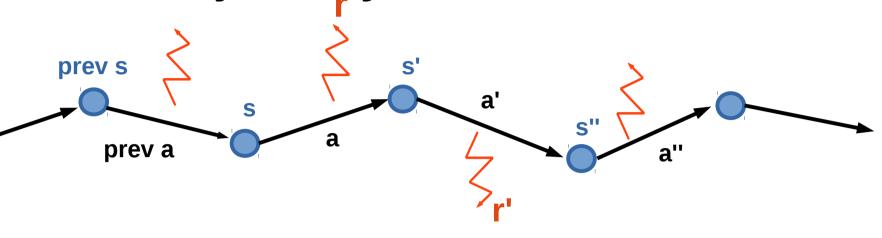
- can apply dynamic programming
- can plan ahead

Model-free: you can sample trajectories

- can try stuff out
- insurance not included

- Trajectory is a sequence of
 - states (s)
 - actions (a)
 - rewards (r)
- We can only sample trajectories

MDP trajectory

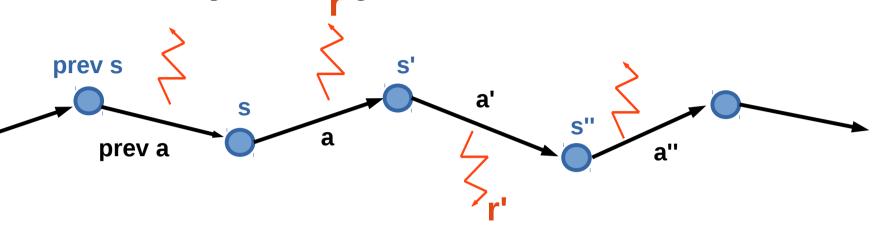


- Trajectory is a sequence of
 - states (s)
 - actions (a)
 - rewards (r)

Q: What to learn? V(s) or Q(s,a)

We can only sample trajectories

MDP trajectory



- Trajectory is a sequence of
 - states (s)
 - actions (a)
 - rewards (r)

Q: What to learn? V(s) or Q(s,a)

V(s) is useless without P(s'|s,a)

We can only sample trajectories

Remember we can improve Q(s,a) iteratively!

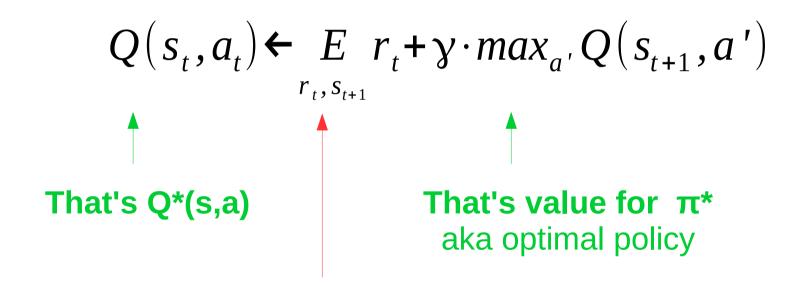
$$Q(s_t, a_t) \leftarrow E_{r_t, s_{t+1}} r_t + \gamma \cdot max_{a'} Q(s_{t+1}, a')$$

Remember we can improve Q(s,a) iteratively!

$$Q(s_t, a_t) \leftarrow E r_t + \gamma \cdot max_{a'} Q(s_{t+1}, a')$$

$$\uparrow \qquad \qquad \uparrow$$
That's Q*(s,a)
That's value for π *
aka optimal policy

Remember we can improve Q(s,a) iteratively!



That's something we don't have

What do we do?



Replace expectation with sampling

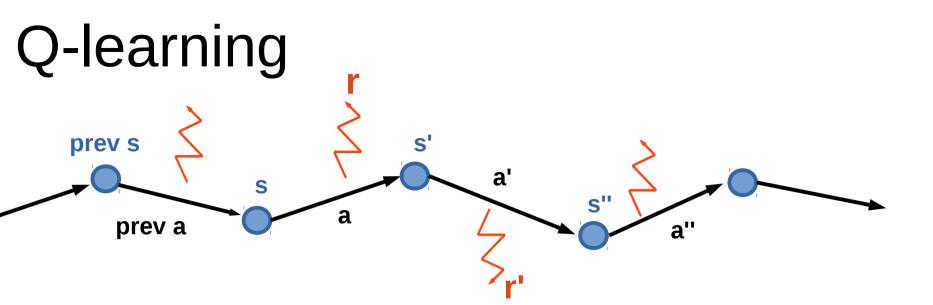
$$E_{r_t,s_{t+1}} r_t + \gamma \cdot \max_{a'} Q(s_{t+1},a') \approx \frac{1}{N} \sum_{i} r_i + \gamma \cdot \max_{a'} Q(s_i^{next},a')$$

Replace expectation with sampling

$$E_{r_t,s_{t+1}} r_t + \gamma \cdot \max_{a'} Q(s_{t+1},a') \approx \frac{1}{N} \sum_{i} r_i + \gamma \cdot \max_{a'} Q(s_i^{next},a')$$

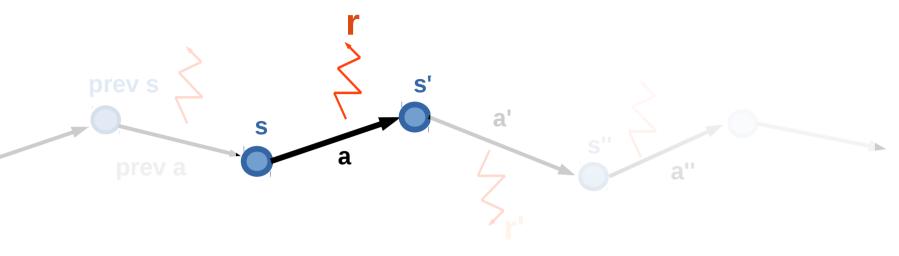
Use moving average with just one sample!

$$Q(s_t, a_t) \leftarrow \alpha \cdot (r_t + \gamma \cdot max_{a'} Q(s_{t+1}, a')) + (1 - \alpha) Q(s_t, a_t)$$



- Works on a sequence of
 - states (s)
 - actions (a)
 - rewards (r)

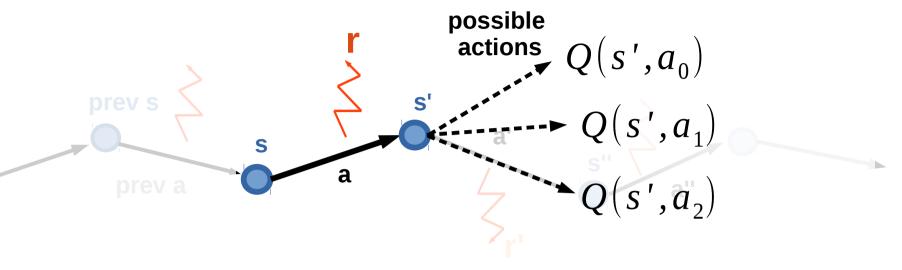
Q-learning



Initialize Q(s,a) with zeros

- Loop:
 - Sample <s,a,r,s'> from env

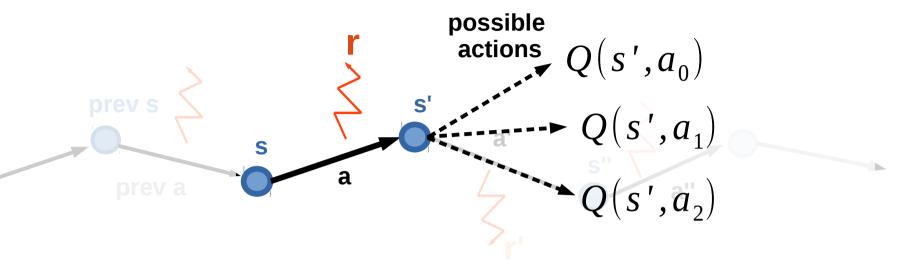
Q-learning



Initialize Q(s,a) with zeros

- Loop:
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 - Compute $\hat{Q}(s,a)=r(s,a)+\gamma \max_{a_i} Q(s',a_i)$

Q-learning

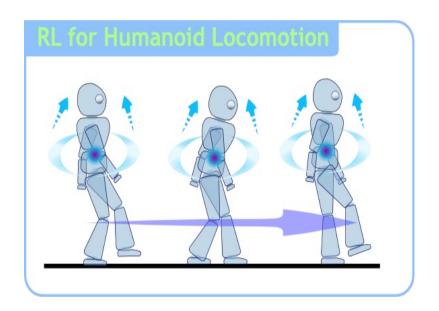


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 - Sample <s,a,r,s'> from env
 - Compute $\hat{Q}(s,a)=r(s,a)+\gamma \max_{a_i} Q(s',a_i)$
 - Update $Q(s,a) \leftarrow \alpha \cdot \hat{Q}(s,a) + (1-\alpha)Q(s,a)$

What could possibly go wrong?

Our mobile robot learns to walk.

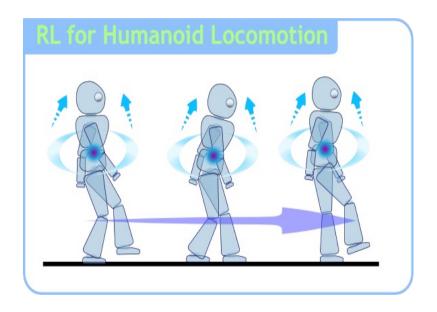


Initial Q(s,a) are zeros robot uses argmax Q(s,a)

He has just learned to crawl with positive reward! 60

What could possibly go wrong?

Our mobile robot learns to walk.



Initial Q(s,a) are zeros robot uses argmax Q(s,a)

Too bad, now he will never learn to walk upright $= {\ell}^{1}$

What could possibly go wrong?

New problem:

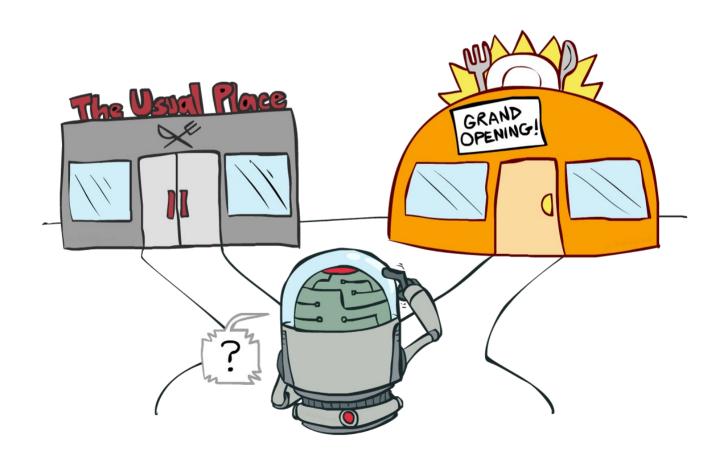
If our agent always takes "best" actions from his current point of view,

How will he ever learn that other actions may be better than his current best one?

Ideas?

Exploration Vs Exploitation

Balance between using what you learned and trying to find something even better



Exploration Vs Exploitation

Strategies:

- · ε-greedy
 - · With probability ε take random action; otherwise take optimal action.

Exploration Vs Exploitation

Strategies:

- · ε-greedy
 - · With probability ε take random action; otherwise take optimal action.
- · Softmax

Pick action proportional to softmax of shifted normalized Q-values.

$$\pi(a|s) = softmax(\frac{Q(s,a)}{\tau})$$

More cool stuff coming later

Exploration over time

Idea:

If you want to converge to optimal policy, you need to gradually reduce exploration

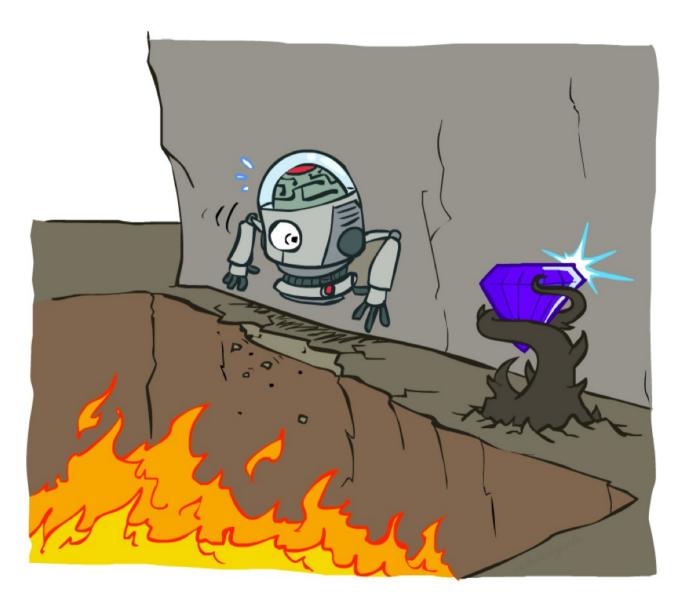
Example:

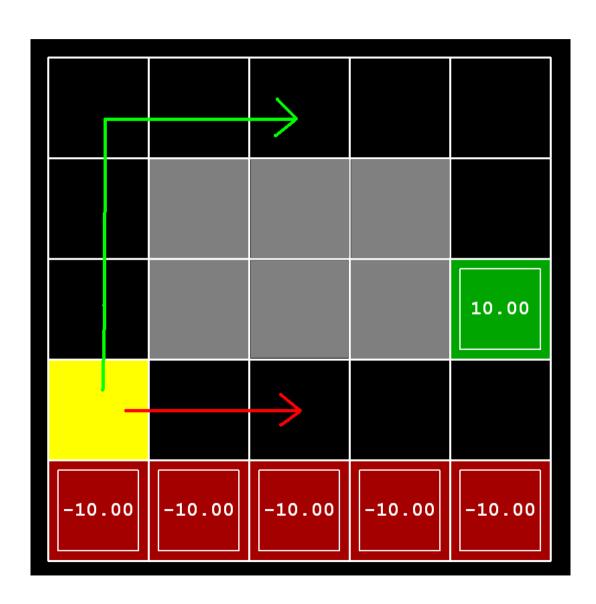
Initialize ε -greedy ε = 0.5, then gradually reduce it

- If $\epsilon \to 0$, it's greedy in the limit
- · Be careful with non-stationary environments

A popular vote

More lecture Vs some seminar?





Conditions

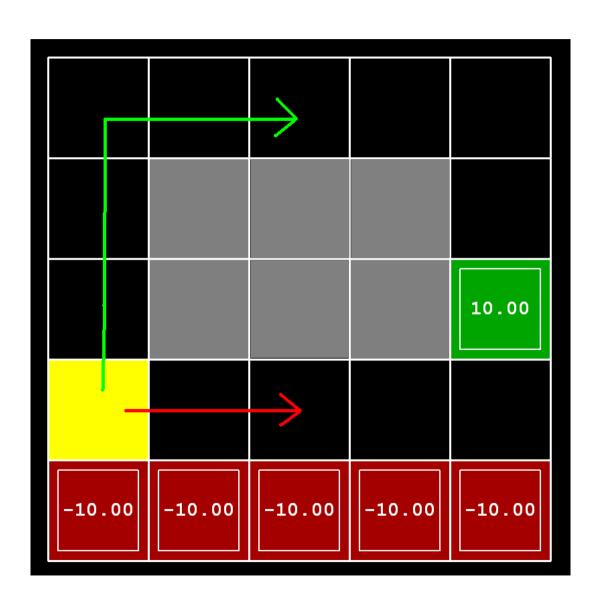
· Q-learning

$$\gamma = 0.99 \ \epsilon = 0.1$$

· no slipping

Trivia:

What will q-learning learn?



Conditions

· Q-learning

$$\gamma = 0.99 \ \epsilon = 0.1$$

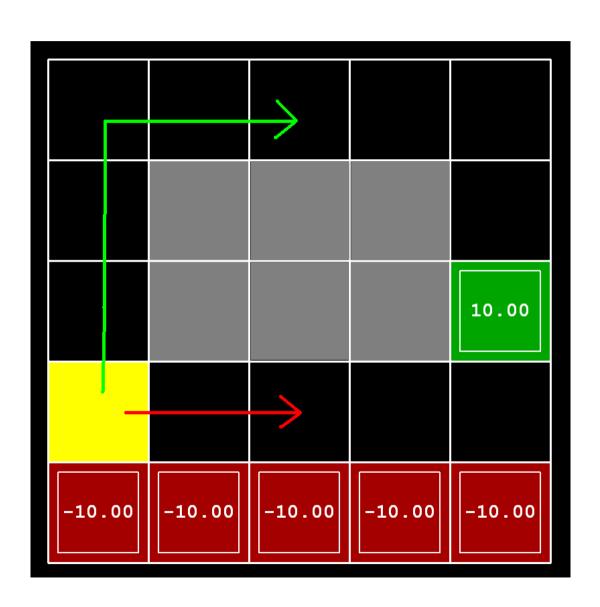
no slipping

Trivia:

What will q-learning learn?

follow the short path

Will it maximize reward?



Conditions

· Q-learning

$$\gamma = 0.99 \ \epsilon = 0.1$$

no slipping

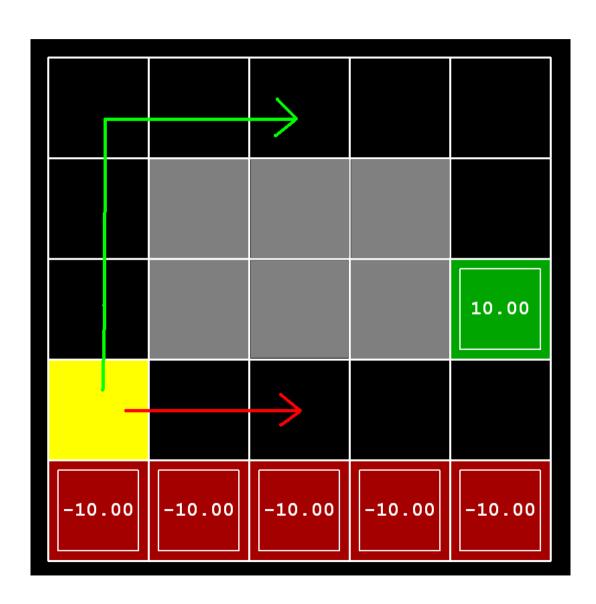
Trivia:

What will q-learning learn?

follow the short path

Will it maximize reward?

no, robot will fall due to epsilon-greedy "exploration"



Conditions

Q-learning

$$\gamma = 0.99 \ \epsilon = 0.1$$

no slipping

Decisions must account for actual policy!

e.g. ε-greedy policy

Generalized update rule

Update rule (from Bellman eq.)

$$Q(s_t, a_t) \leftarrow \alpha \cdot \hat{Q}(s_t, a_t) + (1 - \alpha)Q(s_t, a_t)$$
"better Q(s,a)"

Q-learning VS SARSA

Update rule (from Bellman eq.)

$$Q(s_t, a_t) \leftarrow \alpha \cdot \hat{Q}(s_t, a_t) + (1 - \alpha)Q(s_t, a_t)$$

Q-learning

$$\hat{Q}(s,a) = r(s,a) + \gamma \cdot \max_{a'} Q(s',a')$$

Q-learning VS SARSA

Update rule (from Bellman eq.)

$$Q(s_t, a_t) \leftarrow \alpha \cdot \hat{Q}(s_t, a_t) + (1 - \alpha)Q(s_t, a_t)$$

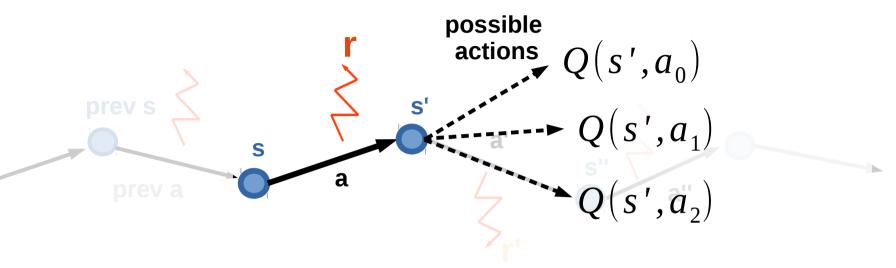
Q-learning

$$\hat{Q}(s,a) = r(s,a) + \gamma \cdot \max_{a'} Q(s',a')$$

SARSA

$$\hat{Q}(s,a) = r(s,a) + \gamma \cdot E_{a' \sim \pi(a'|s')} Q(s',a')$$

Recap: Q-learning

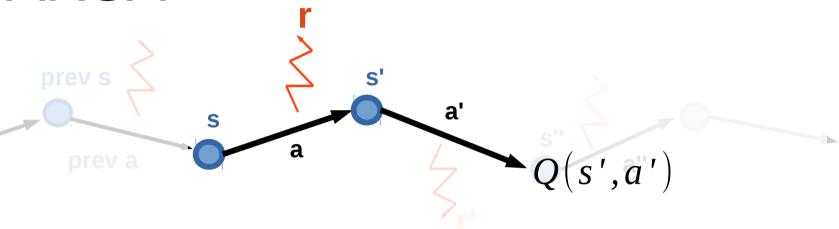


$$\forall s \in S, \forall a \in A, Q(s,a) \leftarrow 0$$

Loop:

- Sample <s,a,r,s'> from env
- Compute $\hat{Q}(s,a)=r(s,a)+\gamma \max_{a_i} Q(s',a_i)$
- Update $Q(s,a) \leftarrow \alpha \cdot \hat{Q}(s,a) + (1-\alpha)Q(s,a)$

SARSA

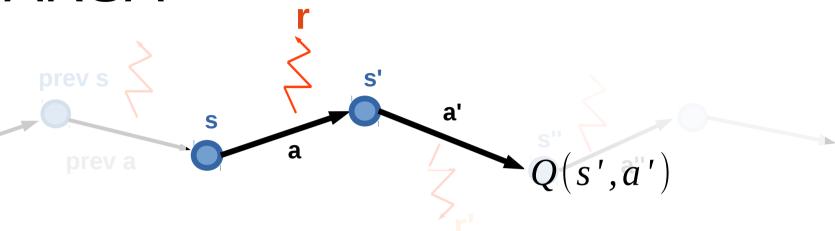


$$\forall s \in S, \forall a \in A, Q(s,a) \leftarrow 0$$

Loop:

- Sample <s,a,r,s',a'> from env
- Compute $\hat{Q}(s,a)=r(s,a)+\gamma Q(s',a')$
- Update $Q(s,a) \leftarrow \alpha \cdot \hat{Q}(s,a) + (1-\alpha)Q(s,a)$

SARSA



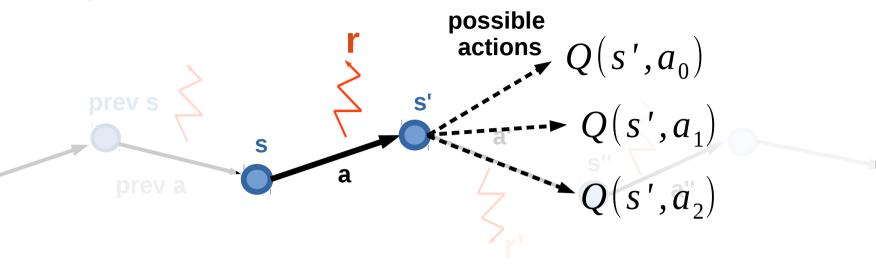
$$\forall s \in S, \forall a \in A, Q(s,a) \leftarrow 0$$

Loop:

hence "SARSA"

- Sample <s,a,r,s',a'> from env
- Compute $\hat{Q}(s,a)=r(s,a)+\gamma Q(s',a')$ next action (not max)
- Update $Q(s,a) \leftarrow \alpha \cdot \hat{Q}(s,a) + (1-\alpha)Q(s,a)$

Expected value SARSA



$$\forall s \in S, \forall a \in A, Q(s,a) \leftarrow 0$$

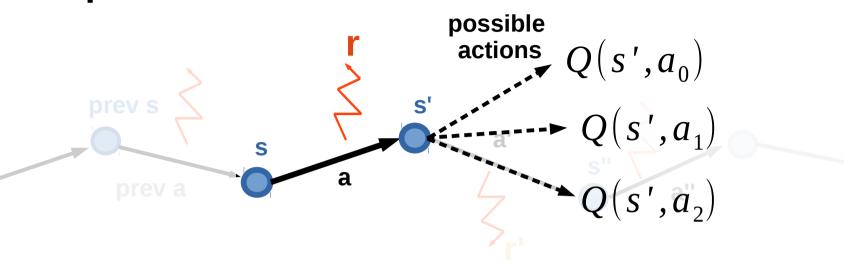
Loop:

Sample <s,a,r,s'> from env

- Compute
$$\hat{Q}(s,a)=r(s,a)+\gamma \mathop{E}_{a_i\sim\pi(a|s')}Q(s',a_i)$$

- Update
$$Q(s,a) \leftarrow \alpha \cdot \hat{Q}(s,a) + (1-\alpha)Q(s,a)$$

Expected value SARSA



$$\forall s \in S, \forall a \in A, Q(s,a) \leftarrow 0$$

Loop:

Sample <s,a,r,s'> from env

Expected value

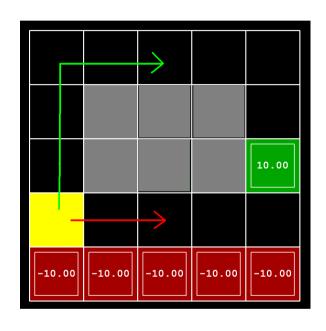
- Compute
$$\hat{Q}(s,a)=r(s,a)+\gamma \mathop{E}_{a_i\sim\pi(a|s')} Q(s',a_i)$$

- Update
$$Q(s,a) \leftarrow \alpha \cdot \hat{Q}(s,a) + (1-\alpha)Q(s,a)$$

Difference

 SARSA gets optimal rewards under current policy

 Q-learning policy would be optimal under





Two problem setups

on-policy

off-policy

Agent **can** pick actions

Most obvious setup :)

 Agent always follows his own policy

- Learning with exploration,
 playing without exploration
- Learning from expert (expert is imperfect)
- Learning from sessions (recorded data)

Two problem setups

on-policy

off-policy

Agent **can** pick actions

Agent can't pick actions

On-policy algorithms can't learn off-policy

Off-policy algorithms can learn on-policy

learn optimal policy even if agent takes random actions

Q: which of Q-learning, SARSA and exp. val. SARSA will **only** work on-policy?

Two problem setups

on-policy

off-policy

Agent **can** pick actions

- On-policy algorithms can't learn off-policy
- SARSA
- more later

- Off-policy algorithms can learn on-policy
- Q-learning
- Expected Value SARSA

Two problem setups

on-policy

off-policy

Agent **can** pick actions

- On-policy algorithms can't learn off-policy
- SARSA
- more coming soon

- Off-policy algorithms can learn on-policy
- Q-learning
- Expected Value SARSA

Two problem setups

on-policy

off-policy

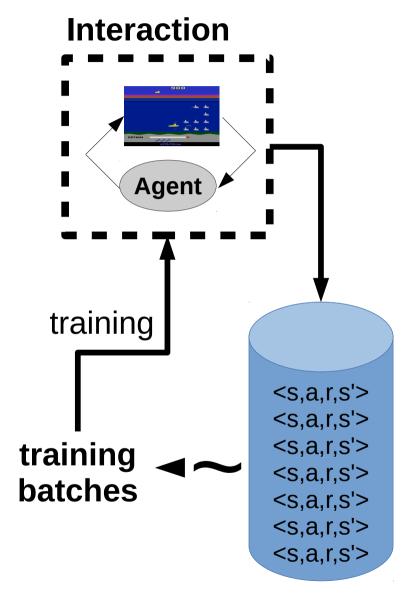
Agent **can** pick actions

- On-policy algorithms can't learn off-policy
- SARSA
- more coming soon

- Off-policy algorithms can learn on-policy
- Q-learning
- Expected Value SARSA

Experience replay

Idea: store several past interactions <s,a,r,s'>
Train on random subsamples



Replay buffer

Experience replay

Idea: store several past interactions <s,a,r,s'>
Train on random subsamples

Training curriculum:

- play 1 step and record it
- pick N random transitions to train

Profit: you don't need to re-visit same (s,a) many times to learn it.

Interaction **Agent** training <s,a,r,s'> <s,a,r,s'> <s,a,r,s'> training <s,a,r,s'> **batches** <s,a,r,s'> <s,a,r,s'> <s,a,r,s'>

Only works with off-policy algorithms!

Btw, why only them?

Replay buffer

Experience replay

Idea: store several past interactions <s,a,r,s'>
Train on random subsamples

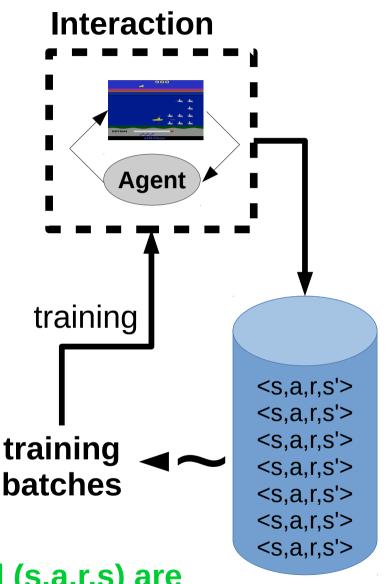
Training curriculum:

- play 1 step and record it
- pick N random transitions to train

Profit: you don't need to re-visit same (s,a) many times to learn it.

Only works with off-policy algorithms!

Old (s,a,r,s) are from older/weaker version of policy!



New stuff we learned

• Anything?

New stuff we learned

V(s), V*(s), Q(s,a), Q*(s,a)

Value iteration (model-based)

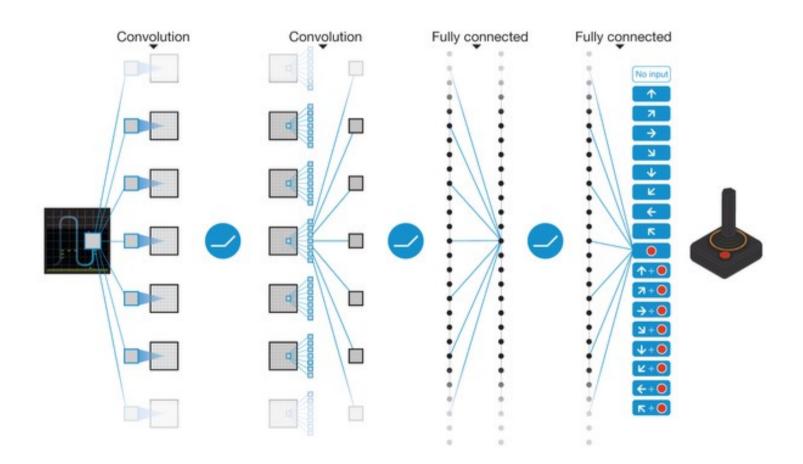
Q-learning, SARSA (model-free)

Exploration vs exploitation (basics)

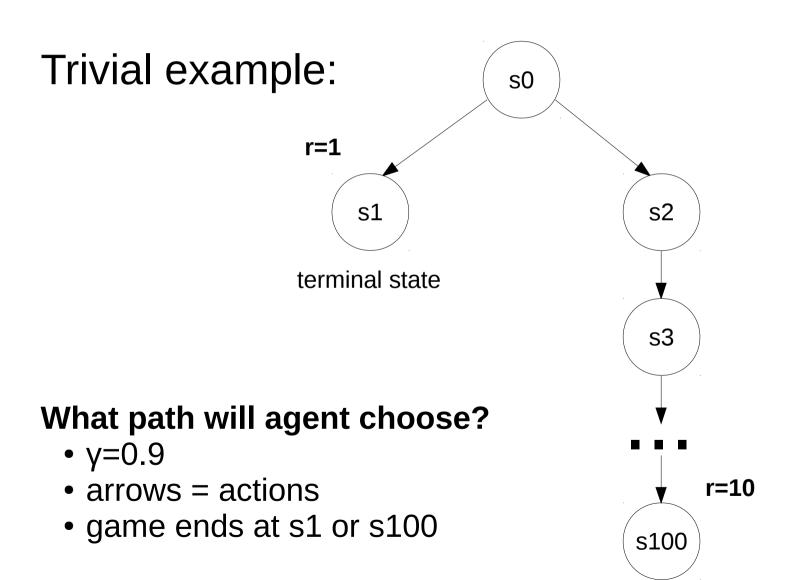
- Learning On-policy vs Off-policy
 - Using experience replay

Coming next...

- What if state space is large/continuous
 - Deep reinforcement learning



Remember discounted rewards?



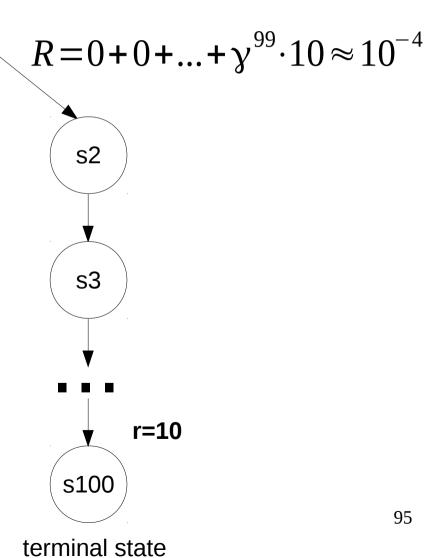
terminal state

Trivial example:

R=1 R=1 s1terminal state

What path will agent choose?

- y=0.9
- arrows = actions
- game ends at s1 or s100
- left action has higher R!



Deephack'17 qualification round, Atari Skiing



- You steer the red guy
- Session lasts ~5k steps
- You get -3~-7 reward each tick (faster game = better score)
- At the end of session, you get up to r=-30k (based on passing gates, etc.)
- Q-learning with gamma=0.99 fails it doesn't learn to pass gates

What's the problem?

Deephack'17 qualification round, Atari Skiing



- You steer the red guy
- Session lasts ~5k steps
- You get -3~-7 reward each tick (faster game = better score)
- At the end of session, you get up to r=-30k (based on passing gates, etc.)
 - Q-learning with gamma=0.99 fails

CoastRunner7 experiment (openAI)



- You control the boat
- Rewards for getting to checkpoints
- Rewards for collecting bonuses
- What could possibly go wrong?
- "Optimal" policy video: https://www.youtube.com/watch?v=tlOIHko8ySg

Nuts and bolts: MC vs TD

Monte-carlo

- Ignores intermediate rewards doesn't need γ (discount)
- Needs full episode to learn Infinite MDP are a problem
- Doesn't use Markov property
 Works with non-markov envs

Temporal Difference

- Uses intermediate rewards trains faster under right γ
- Learns from incomplete episode Works with infinite MDP
- Requires markov property
 Non-markov env is a problem



Nuts and bolts: discount

• Effective horizon $1+\gamma+\gamma^2+...=\frac{1}{(1-\gamma)}$

Heuristic: your agent stops giving a damn in *this many* turns.

Typical values:

- y=0.9, 10 turns
- y=0.95, 20 turns
- y=0.99, 100 turns
- γ=1, infinitely long

Higher γ = less stable algorithm. γ =1 only works for episodic MDP (finite amount of turns).

Nuts and bolts: discount

• Effective horizon $1+\gamma+\gamma^2+...=\frac{1}{(1-\gamma)}$

Heuristic: your agent stops giving a damn in this many turns.

- Atari Skiing, reward was delayed by in 5k steps
- y=0.99 is not enough
- γ=1 and a few hacks works better
- Or use a better reward function



#ShamelessSelfPromotion

More stuff

https://github.com/yandexdataschool/practical_rl/tree /fall17

- Videos lectures, more theory, more coding
 - Model-based week2
 - Mode-free week3

Let's write some code!