UM-SJTU Joint Institute

Physics Laboratory
(Vp141)

Laboratory Report

Excercise 5

Damped and Driven Oscillations
Mechanical Resonance

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Date: October 9, 2024

[rev4.1]

1 Introduction

The objective of this exercise is to understand the physics of alternating-current circuits, in particular the process of charging and discharging of capacitors, the phenomenon of electromagnetic induction in inductive elements, and other dynamic processes in RC, RL, and RLC series circuits. This experiment also measures the amplitude-frequency and the phase-frequency characteristics of RC, RL, and RLC series circuits.

2 Experimental setup

2.1 Equipments used in the experiment

Three basic elements of electric circuits include resistors, capacitors, and inductors. In this experiment, we will be using a alternating electric power source with a fixed frequency of 1000Hz, an inductor with a fixed inductance of 0.01H, a capacitor with a fixed capacitance of about 125nF, a fixed resistance of 100 Ω, and fixed electromotive force of 4 Vpp.

2.2 Measurements used in the experiment

In the first two circuits, i.e. the RC, RL circuits, our objective is to measure the half life of the capacitor's charging and discharging. During a complete period, the capacitor will first be charged and then be discharged, which means its voltage will first reach its peak and then drop back to zero. Here, the half life refers to the time it takes for the capacitor's voltage to increase a half from 0 or drop a half from its peak. The equations for calculating the voltage of the capacitor is shown as following:

$$RC\frac{dU_c}{dt} + U_c = E \tag{1}$$

$$RC\frac{dU_c}{dt} + U_c = 0 \tag{2}$$

We will first obtain the measured half life using an oscilloscope, then calculate the theoretical value of the half life using the following equations:

$$T_{1/2} = \tau ln(2) = RC * ln2 \tag{3}$$

$$T_{1/2} = \tau ln(2) = \frac{L}{R} * ln2 \tag{4}$$

After obtaining both the experimental and the theoretical values of the half life, we can compare them and derive some onclusions.

For the RLC circuits, it becomes a bit more complicated. We will first also deal with the half life. Here, we introduce a new constant β which is given by:

$$\beta = \frac{R}{2L} \tag{5}$$

This constant has the following relationship to the half life with:

$$\beta T_{1/2} = 1.68 \tag{6}$$

and we also have:

$$\tau = \frac{1}{\beta} = \frac{T_{1/2}}{1.68} \tag{7}$$

With all these relationships, we should also come up with a theoretical and experimental value of the time constant τ. We then compare them and draw some conclusions.

After that, we pay attention to the resonance in the RLC circuit. Resonance happens in the RLC circuit only when the frequency of the power satisfy the following equation:

$$f = \frac{1}{2\pi\sqrt{LC}} \tag{8}$$

We also would like to calculate the quality factor Q of the circuit using the following equation:

$$Q = \frac{1}{\omega_0 RC} \tag{9}$$

where f_0 is the resonance frequency, and f_1 and f_2 are two frequencies that satisfy: $I(f_1) = I(f_2) = I_m/\sqrt{2}$.

3 Measurements and Results

3.1 RC circuit half life

Below is the data table of the RC circuit: From the data collected we see that the experimental value of the half life is $8.0\mu s$, which means the experimental time constant τ is about $11.542\mu s$. The theoretical value can be derived using the equation 3, and we have the result that is should be $12.646\mu s$.

R 100 [Ω] ± 1 [Ω], f 1.000000 [kHz] ± 0.000001 [kHz], $\mathcal{E}$ 4.000 [Vpp] ± 0.001 [Vpp]	
C 126.46 [nF] ± 0.01 [nF]	$T_{1/2}$ 8.0 [μs] ± 0.1 [μs]

Table 1. $T_{1/2}$ measurement data for a RC series circuit.

Figure 1: Data table for RC circuit

R 100 [Ω] ± 1 [Ω], f 1.000000 [kHz] ± 0.000001 [kHz], $\mathcal{E}$ 4.000 [Vpp] ± 0.001 [Vpp]	
L 0.01 [H] ± 0.001 [H]	$T_{1/2}$ 55.0 [μs] ± 0.1 [μs]

Table 2. $T_{1/2}$ measurement data for a RL series circuit.

Figure 2: Data table for RL circuit

3.2 RL circuit half life

Above is the data table of the RC circuit:

From the data collected we see that the experimental value of the half life is $55.0\mu s$, which means the experimental time constant τ is about $79.348\mu s$. The theoretical value can be derived using the equation 4, and we have the result that is should be $81\mu s$.

3.3 RLC circuit half life

Below is the data table of the RLC circuit:

L 0.01 [H] ± 0.001 [H], C 126.46 [nF] ± 0.01 [nF], $\mathcal{E}$ 4.000 [Vpp] ± 0.001 [Vpp], f 1.000000 [kHz] ± 0.000001 [kHz]	
$\beta t = 1.68$	$T_{1/2} =$ 53.0 [μs] ± 0.1 [μs]

Table 3. $T_{1/2}$ measurement data for a critically damped RLC series circuit.

Figure 3: Data table for RLC circuit

From the data collected we see that the experimental value of the half life is $353.0\mu s$ (here should be a mistake: missing a 3 for the first digit), which means the experimental time constant τ is about $210\mu s$. The theoretical value can be derived using the equation 7, and we have the result that is should be $200\mu s$.

3.4 RLC circuit resonance frequency

Below is the data table of the RLC resonance circuit:

R 100 [Ω] ± 1 [Ω], L 0.01 [H] ± 0.001 [H], C 490.9 [nF] ± 0.1 [nF]		
f_0 2.271094 [kHz] ± 0.000001 [kHz], $\mathcal{E}$ 4.000 [Vpp] ± 0.001 [Vpp]		
	U_R [V] ± 0.001 [V]	f [kHz] ± 0.000001 [kHz].
1	1.767	1.000000
2	1.933	1.100000
3	2.167	1.200000
4	2.333	1.300000
5	2.567	1.400000
6	2.833	1.500000
7	3.067	1.600000
8	3.233	1.700000
9	3.467	1.800000
10	3.733	1.900000
11	3.967	2.000000
12	4.100	2.100000
13	4.167	2.200000
14	4.233	2.300000
15	4.167	2.400000
16	4.033	2.500000
17	3.900	2.600000
18	3.733	2.700000
19	3.667	2.800000
20	3.467	2.900000
21	3.333	3.000000

Table 4. Measurement data for the U_R vs. f dependence for a RLC resonant circuit.

Figure 4: Data table for RLC circuit

We see from the data table that the theoretical value of f_0 is 2271.064 Hz, which is derived using the equation 8. Here we need 2 plots: I vs f and φ vs f. I and φ are given using the following equations:

$$I = \frac{U_R}{R} \tag{10}$$

$$\varphi = tan^{-1}(\frac{2\pi f L - \frac{1}{2\pi f C}}{R}) \tag{11}$$

After calculation, we have the following 2 graphs:

From the figures we can see that the resonance frequency is about 2.27-2.28kHz. And for the quality factor, we use equation 9 and get Q to be about 1.46

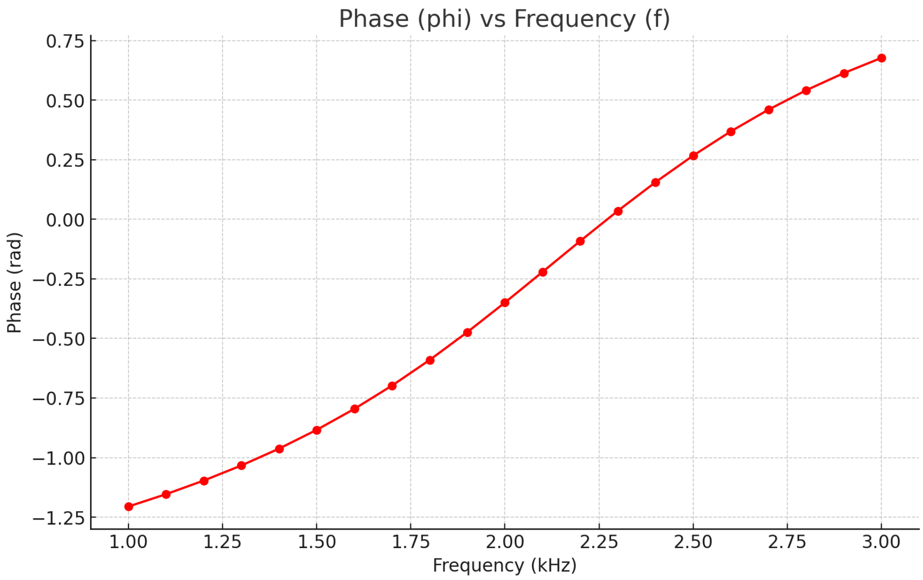


Figure 5: Data table for RLC circuit

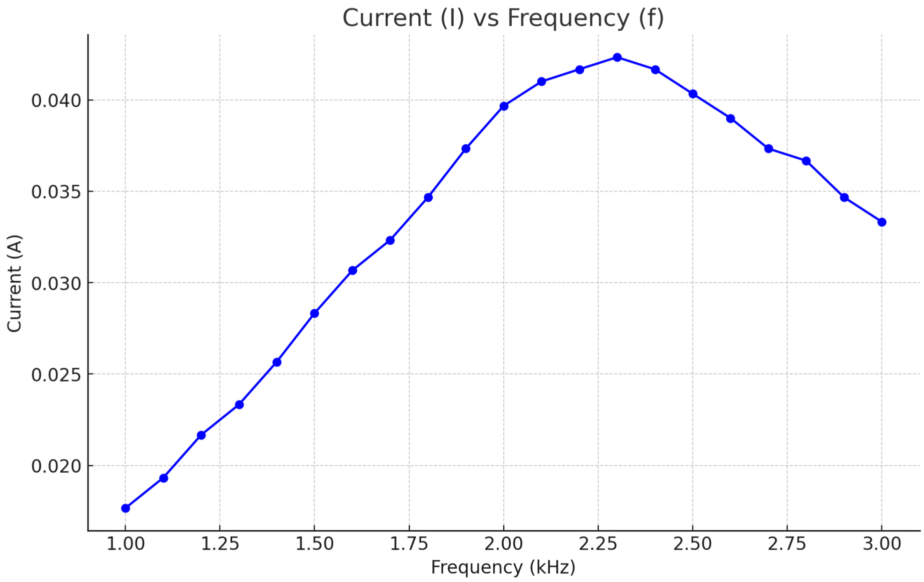


Figure 6: Data table for RLC circuit

4 Conclusions and discussion

As mentioned above, the objective of this experiment is to check the theories about RC, RL and RLC circuits to deepen our understanding, and have a better understanding about the theories. In the RC circuit and the RL circuit, both time constants, the theoretical and the experimental ones, are very close to each other. If we calculate the mistake percentage, we will see that they are all smaller than the 10% expectation and is within our tolerance. For the RLC circuit, there should be a little mistake in the data table as mentioned before, which is missing a 3 for the most significant digit, but after fixing this little mistake, the value is also within the range of 10%, so it is also acceptable. Then there is the resonance frequency derived in the RLC circuit. The theoretical resonance frequency of this circuit is 2271.064Hz, and the experiment data reveals the experimental resonance frequency to be about from 2.27kHz to 2.28kHz, which is accurate enough.

However, although some results are within our tolerance of error, the percentage of error is a bit high, or in other words close to the boundary of tolerance. We propose that these mistakes may have originated from the huge amount of usage of the equipments. In the first 2 experiments, the capacity is expected to be at 100nF, but the result of measurement is 126.46nF, and we see that the error is over 25%. So we guess that this is a reasonable explanation of the errors.

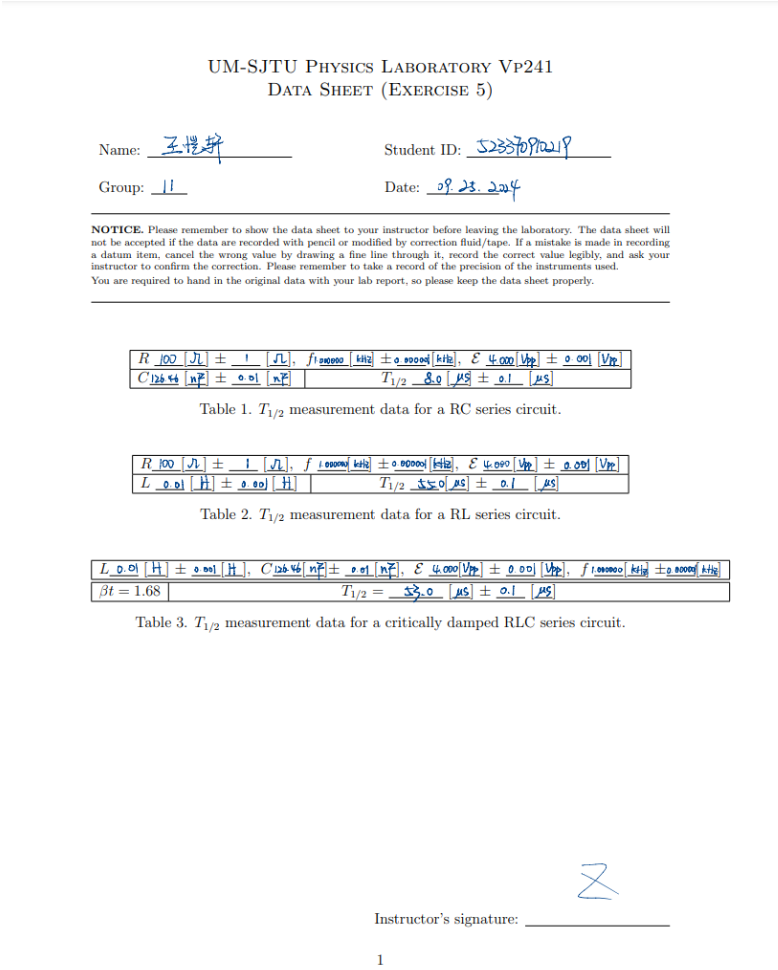
5 Works cited

Department of Physics, Shanghai Jiaotong University, Exercise 5 (RC, RL, and RLC Circuits) - lab manual [rev. 2.6], 2024

Python Software Foundation. (2020). Python Language Reference, version 3.9. Available at http://www.python.org

All the figures displayed in the article (excluding the appendix) are given using Python 3.9.

A Datasheet



UM-SJTU Physics Laboratory Vp241
Data Sheet (Exercise 5)

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Group: 11 Date: 09. 23. 2024

NOTICE. Please remember to show the data sheet to your instructor before leaving the laboratory. The data sheet will not be accepted if the data are recorded with pencil or modified by correction fluid/tape. If a mistake is made in recording a datum item, cancel the wrong value by drawing a fine line through it, record the correct value legibly, and ask your instructor to confirm the correction. Please remember to take a record of the precision of the instruments used.
You are required to hand in the original data with your lab report, so please keep the data sheet properly.

R 100 [Ω] ± 1 [Ω], f 1.000000 [kHz] ± 0.000001 [kHz], $\mathcal{E}$ 4.000 [Vpp] ± 0.001 [Vpp]	
C 126.46 [nF] ± 0.01 [nF]	$T_{1/2}$ 8.0 [μs] ± 0.1 [μs]

Table 1. $T_{1/2}$ measurement data for a RC series circuit.

R 100 [Ω] ± 1 [Ω], f 1.000000 [kHz] ± 0.000001 [kHz], $\mathcal{E}$ 4.000 [Vpp] ± 0.001 [Vpp]	
L 0.01 [H] ± 0.001 [H]	$T_{1/2}$ 55.0 [μs] ± 0.1 [μs]

Table 2. $T_{1/2}$ measurement data for a RL series circuit.

L 0.01 [H] ± 0.001 [H], C 126.46 [nF] ± 0.01 [nF], $\mathcal{E}$ 4.000 [Vpp] ± 0.001 [Vpp], f 1.000000 [kHz] ± 0.000001 [kHz]	
$\beta t = 1.68$	$T_{1/2}$ = 53.0 [μs] ± 0.1 [μs]

Table 3. $T_{1/2}$ measurement data for a critically damped RLC series circuit.

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Figure 7: Datasheet 1

R 100 [Ω] ± 1 [Ω], L 0.01 [H] ± 0.001 [H], C 490.9 [nF] ± 0.1 [nF]		
f_0 2.271094 [kHz] ± 0.000001 [kHz], $\mathcal{E}$ 4.000 [Vpp] ± 0.001 [Vpp]		
	U_R [V] ± 0.001 [V]	f [kHz] ± 0.000001 [kHz].
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Table 4. Measurement data for the U_R vs. f dependence for a RLC resonant circuit.

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Figure 8: Datasheet 2