SCHOOL OF SCIENCE AND TECHNOLOGY

DEPARTMENT OF DATA SCIENCE AND ANALYTICS

FALL 2024 – QUIZ 3

COURSE CODE: STA 4030A

UNIT NAME: BAYESIAN INFERENCE ND DECISION THEORY

DATE: 25TH OCTOBER 2024 **TOTAL MARKS**: 10 MARKS

INSTRUCTIONS:

For this exercise:

1. ANSWER ALL QUESTIONS
2. Do all your working in the Rmarkdown (.rmd).
3. Submissions should be in a **`.rmd` file**
4. NO SUBMISSIONS SHOULD BE DONE VIA EMAIL

QUESTIONS:

1. In previous years, students in this course collected data on people's preferences in the two Allais Gambles.

   Gamble 1:

A:	\$2500 with probability 0.33 \$2400 with probability 0.66 \$0 with probability 0.01	B:	\$2400 with certainty

   Gamble 2:

C:	\$2500 with probability 0.33 \$0 with probability 0.67	D:	\$2400 with probability 0.34 \$0 with probability 0.66

   For this problem, we will assume that responses are independent and identically distributed, and the probability is θ that a person chooses both B in the first gamble and C in the second gamble.

   a. Assume that the prior distribution for θ is Beta(1, 3).
      i. Find the prior mean and standard deviation for θ.
      ii. Find a 95% symmetric tail area credible interval for the prior probability that a person would choose B and C.
      iii. Do you think this is a reasonable prior distribution to use for this problem? Why or why not?

   b. In 2009, 19 out of 47 respondents chose B and C.
      i. Find the posterior distribution for the probability θ that a person in this population would choose B and C.
      ii. Name the distribution and the posterior hyperparameters.

   c. Find the posterior mean and standard deviation. Find a 95% symmetric tail area credible interval for θ.

   d. Make a triplot of the prior distribution, normalized likelihood and posterior distribution.

   e. Comment on your results.

2. Times were recorded at which vehicles passed a fixed point on the M1 motorway in Bedfordshire, England on March 23, 1985.[2] The total time was broken into 21 intervals of length 15 seconds. The number of cars passing in each interval was counted. The result was: 2, 2, 1, 1, 0, 4, 3, 0, 2, 1, 1, 1, 4, 0, 2, 2, 3, 2, 4, 3, 2. This can be summarized in the following table, that shows 3 intervals with zero cars, 5 intervals with 1 car, 7 intervals with 2 cars, 3 intervals with 3 cars and 3 intervals with 4 cars.

Number of Cars	Number of Occurrences
0	3
1	5
2	7
3	3
4	3
5 or more	0

Assume that counts of vehicles per 15-second interval are independent and identically distributed by Poisson random variables with unknown mean Λ.

a. Assume that Λ, the rate parameter of the Poisson distribution for counts, has a continuous gamma prior distribution for Λ with shape 1 and scale 10^6. (The gamma distribution with shape 1 tends to a uniform distribution as the scale tends to ∞, so this prior distribution is "almost" uniform.)
    i. Find the posterior distribution of Λ.
    ii. State the distribution type and hyperparameters.
b. Find the posterior mean and standard deviation of Λ.
c. Find a 95% symmetric tail area posterior credible interval for Λ.
d. Find a 95% symmetric tail area posterior credible interval for Θ, the mean time between vehicle arrivals.