

MTH3010A - Quiz 2

Solve all problems while demonstrating each step clearly. The Quiz is worth a total of 10 points.

Question 1

Suppose an asset has the following factor sensitivities: $\beta_1 = 0.5$, $\beta_2 = -0.3$, $\beta_3 = 0.7$. If the expected returns of the three factors are 8%, 6%, and 10% respectively, and the risk-free rate is 4%, calculate the expected return of the asset according to arbitrage pricing theory. (1 point)

The general formula for the expected return of an asset according to APT is:

$$E(R_i) = R_f + \sum_{j=1}^n \beta_{ij} (E(R_j) - R_f)$$

Where:

- $E(R_i)$ is the expected return of the asset
- R_f is the risk-free rate
- β_{ij} is the sensitivity of the asset to factor j
- $E(R_j)$ is the expected return of factor j

Given

- $\beta_1 = 0.5$
- $\beta_2 = -0.3$
- $\beta_3 = 0.7$
- $E(R_1) = 8\% = 0.08$
- $E(R_2) = 6\% = 0.06$
- $E(R_3) = 10\% = 0.10$
- $R_f = 4\% = 0.04$

Plugging these values into the formula, we get:

$$E(R_i) = 0.04 + 0.5 \times (0.08 - 0.04) + (-0.3) \times (0.06 - 0.04) + 0.7 \times (0.10 - 0.04) = 0.04 + 0.5 \times (0.04) + (-0.3) \times (0.02) + 0.7 \times (0.06) = 0.092$$

The expected return of the asset according to arbitrage pricing theory is 9.2%

Question 2

Consider a portfolio consisting of two assets: Asset A and Asset B. Asset A has an expected return of 10% and a standard deviation of 15%, while Asset B has an expected return of 8% and a standard deviation of 10%. If you allocate 60% of your portfolio to Asset A and 40% to Asset B, what is the expected return and variance of your portfolio?. (1 point)

The expected return of the portfolio, $E(R_p)$, is a weighted average of the expected returns of the individual assets:

2.1 Expected Return

$$E(R_p) = w_A E(R_A) + w_B E(R_B)$$

Where:

- w_A and w_B are the weights of Asset A and Asset B in the portfolio.
- $E(R_A)$ and $E(R_B)$ are the expected returns of Asset A and Asset B.

Given:

- $E(R_A) = 10\% = 0.10$
- $E(R_B) = 8\% = 0.08$
- $w_A = 60\% = 0.60$
- $w_B = 40\% = 0.40$

We can input this into the formula as follows:

$$E(R_p) = 0.60 \times 0.10 + 0.40 \times 0.08 = 0.06 + 0.032 = 0.092$$

So the expected return of the portfolio is 0.092 or 9.2%.

2.2 Variance

The variance of a portfolio, σ_p^2 is given by:

$$\sigma_p^2 = w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + 2w_A w_B \sigma_A \sigma_B \rho_{AB}$$

Where:

- σ_A and σ_B are the standard deviations of Asset A and Asset B.
- ρ_{AB} is the correlation coefficient between the returns of Asset A and Asset B.

If the correlation coefficient ρ_{AB} is not given, we assume it is zero (i.e., the assets are uncorrelated).

Given:

- $\sigma_A = 15\% = 0.15$
- $\sigma_B = 10\% = 0.10$
- $\rho_{AB} = 0$

Inputting these values into our formula we get:

$$\sigma_p^2 = (0.60)^2(0.15)^2 + (0.40)^2(0.10)^2 + 2(0.60)(0.40)(0.15)(0.10)(0) = 0.36 \times 0.0225 + 0.16 \times 0.01 + 0 = 0.0081 + 0.0016 = 0.0097$$

The standard deviation of the portfolio, σ_p , is: $\sigma_p = \sqrt{0.0097} = 0.0985$

So, the variance of the portfolio is 0.0097%, and the standard deviation is 0.0985 or 9.85%.

Question 3

Given a corporate bond with a yield of 6% and a similar maturity risk-free rate of 3%, calculate the credit spread. (1 point)

The credit spread of a bond is the difference between the yield of the corporate bond and the yield of a similar maturity risk-free bond. It represents the additional yield that investors require to compensate for the credit risk of the corporate bond compared to a risk-free bond.

Given:

- Yield of the corporate bond: 6% (or 0.06)
- Yield of the risk-free bond: 3% (or 0.03)

The credit spread is calculated as follows:

$$\text{Credit Spread} = \text{Yield of Corporate Bond} - \text{Yield of Risk-Free Bond} \\ \text{Credit Spread} = 0.06 - 0.03 \\ \text{Credit Spread} = 0.03$$

So, the credit spread is 0.03 or 3%.

Question 4

Calculate the credit valuation adjustment (CVA) for a portfolio of derivative contracts with a total notional value of 10 million dollars, a default probability of 1%, and an LGD of 40%. (1 point)

The Credit Valuation Adjustment (CVA) is the expected loss on a derivative portfolio due to the counterparty's default. It is calculated using the formula:

$$\text{CVA} = \text{Notional Value} \times \text{Probability of Default} \times \text{Loss Given Default}$$

Given:

- Notional Value = 10,000,000
- Probability of Default (PD) = 1% = 0.01
- Loss Given Default (LGD) = 40% = 0.40

Plugging these values into the formula:

$$\text{CVA} = 10,000,000 \times 0.01 \times 0.40 \\ \text{CVA} = 10,000,000 \times 0.004 \\ \text{CVA} = 40,000$$

So, the Credit Valuation Adjustment (CVA) for the portfolio is 40,000 dollars

Question 5

Calculate the Sharpe ratio for a portfolio with an average annual return of 10% and a standard deviation of returns of 15%. The risk-free rate is 3%. (1 point)

The Sharpe ratio measures the risk-adjusted return of a portfolio. It is calculated using the formula:

$$\text{Sharpe Ratio} = \frac{E(R_p) - R_f}{\sigma_p}$$

where:

- $E(R_p)$ is the average annual return of the portfolio.
- R_f is the risk-free rate.
- σ_p is the standard deviation of the portfolio's returns.

Given:

- $E(R_p) = 10\% = 0.10$
- $R_f = 3\% = 0.03$
- $\sigma_p = 15\% = 0.15$

Plugging these values into the formula:

$$\text{Sharpe Ratio} = \frac{0.10 - 0.03}{0.15} \quad \text{Sharpe Ratio} = \frac{0.07}{0.15} \quad \text{Sharpe Ratio} = 0.4667$$

So, the Sharpe ratio for the portfolio is approximately 0.47.

Question 6

Calculate the beta of a stock with an average return of 12% and a standard deviation of returns of 18%. The average return of the market is 8%, and its standard deviation of returns is 12%. (1 point)

To calculate the beta of a stock, we use the formula:

$$\beta = \frac{\text{Cov}(R_i, R_m)}{\sigma_m^2}$$

where:

- $\text{Cov}(R_i, R_m)$ is the covariance of the stock's returns with the market returns.
- σ_m^2 is the variance of the market returns.

However, we need additional information to directly compute the covariance. Instead, we can use the correlation between the stock's returns and the market returns if it's given, but since we don't have the correlation here, we can approach this using the Capital Asset Pricing Model (CAPM) and the fact that beta can also be estimated using the formula:

$$\beta = \frac{\sigma_i}{\sigma_m} \cdot \rho_{i,m}$$

where:

- σ_i is the standard deviation of the stock's returns.
- σ_m is the standard deviation of the market returns.
- $\rho_{i,m}$ is the correlation coefficient between the stock's returns and the market returns.

To find the beta, we typically need the correlation coefficient ($\rho_{i,m}$). Since we don't have it, we'll assume a general form of the formula where we need to estimate (β).

Given:

- Average return of the stock $E(R_i)$: $12\% = 0.12$
- Standard deviation of the stock's returns σ_i : $18\% = 0.18$
- Average return of the market $E(R_m)$: $8\% = 0.08$
- Standard deviation of the market's returns σ_m : $12\% = 0.12$

If we don't have the correlation coefficient, we can't directly compute the exact beta. If you have historical data for the stock and the market, you could calculate the covariance and then compute beta.

Assuming the correlation coefficient $\rho_{i,m}$ is given or is available, beta can be estimated as follows:

$$\beta = \frac{\sigma_i}{\sigma_m} \cdot \rho_{i,m}$$

If we assume $\rho_{i,m} = 1$ (perfect correlation) for simplicity:

$$\beta = \frac{0.18}{0.12} \cdot 1 = 1.5$$

So, under the assumption of perfect correlation (which is an upper bound), the beta of the stock is approximately 1.5.

To get a precise beta, you would need the actual correlation coefficient between the stock's returns and the market's returns. If the correlation is less than 1, the beta will be proportionally lower.

Question 7

Consider a hypothetical portfolio with the following thirty historical returns data:

Table 1: Historical Returns Data (Days 1-15)

Day	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Return (%)	-1.2	0.5	1.0	-0.7	-1.5	2.3	-0.4	0.6	-2.1	1.2	0.8	-1.1	1.5	-0.3	2.0

Table 2: Historical Returns Data (Days 16-30)

Day	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Return (%)	-1.7	0.4	-0.5	1.8	-2.2	0.9	1.1	-1.6	2.4	-0.9	1.3	-0.8	0.7	-1.9	1.4

Calculate the volatility of the returns and determine both the VaR and CVaR of the portfolio using the historical method on returns data, the parametric method, and Monte Carlo simulation. Comment on the results by the three methods and give a convincing justification. (4 points)

7.1 Calculate Volatility

Volatility is calculated as the standard deviation of the returns.

Given the returns for 30 days, we first list them:

Returns(%) = [-1.2, 0.5, 1.0, -0.7, -1.5, 2.3, -0.4, 0.6, -2.1, 1.2, 0.8, -1.1, 1.5, -0.3, 2.0, -1.7, 0.4, -0.5, 1.8, -2.2, 0.9, 1.1, -1.6, 2.4, -0.9, 1.3, -0.8, 0.7, -1.9, 1.4]

Convert these returns to decimals (divide by 100):

Returns = [-0.012, 0.005, 0.01, -0.007, -0.015, 0.023, -0.004, 0.006, -0.021, 0.012, 0.008, -0.011, 0.015, -0.003, 0.02, -0.017, 0.004, -0.005, 0.018, -0.022, 0.009, 0.011, -0.016, 0.024, -0.009, 0.013, -0.008, 0.007, -0.019, 0.014]

Using Python for these calculations:

```
In [2]: import numpy as np

# Given returns data
returns = [-0.012, 0.005, 0.01, -0.007, -0.015, 0.023, -0.004, 0.006, -0.021, 0.012,
           0.008, -0.011, 0.015, -0.003, 0.02, -0.017, 0.004, -0.005, 0.018, -0.022,
           0.009, 0.011, -0.016, 0.024, -0.009, 0.013, -0.008, 0.007, -0.019, 0.014]

# Calculate volatility (standard deviation)
volatility = np.std(returns, ddof=1)
print('Volatility: ', volatility)

Volatility: 0.013883753839997855
```

7.2 Historical VaR

To calculate the VaR at a 95% confidence level using the historical method, we:

1. Sort the returns in ascending order.
2. Find the 5th percentile (since $1 - 0.95 = 0.05$).

Using Python:

```
In [3]: # Calculate 5th percentile (VaR at 95% confidence level)
historical_var = np.percentile(returns, 5)
print('historical VaR: ', historical_var)

historical VaR: -0.0201
```

7.3 Historical CVaR

To calculate the CVaR, we take the average of the returns that are below the VaR threshold.

Using Python:

```
In [4]: # Calculate CVaR
historical_cvar = np.mean([r for r in returns if r <= historical_var])
print('historical CVaR: ', historical_cvar)

historical CVaR: -0.0215
```

7.4 Parametric VaR and CVaR

Assuming the returns are normally distributed, we can calculate the parametric VaR and CVaR using the mean (μ) and standard deviation (σ) of the returns. Parametric VaR

$$VaR_{\alpha} = \mu + z_{\alpha} \sigma$$

Where:

- z_{α} is the z-score for the 5th percentile (for 95% confidence level, $z_{\{0.05\}} = -1.645$)

Using Python:

```
In [5]: # Calculate mean return
mean_return = np.mean(returns)

# Calculate z-score for 95% confidence level
z_score = -1.645

# Calculate parametric VaR
parametric_var = mean_return + z_score * volatility
print('parametric VaR: ', parametric_var)
```

parametric VaR: -0.02183877506679647

$$CVaR_{\alpha} = \mu + \frac{\phi(z_{\alpha})}{\alpha} \sigma$$

Where: $\phi(z_{\alpha})$ is the probability density function of the normal distribution at z_{α} .

Using Python:

```
In [6]: from scipy.stats import norm

# Calculate phi(z_alpha)
phi = norm.pdf(z_score)

# Calculate parametric CVaR
parametric_cvar = mean_return + (phi / 0.05) * volatility
print('parametric CVaR: ', parametric_cvar)
```

parametric CVaR: 0.029631302388871808

7.5 Monte Carlo Simulation

To calculate the VaR and CVaR using Monte Carlo simulation:

1. Simulate a large number of potential returns based on the mean and standard deviation.
2. Calculate the 5th percentile of these simulated returns for VaR.
3. Calculate the average of the returns below this percentile for CVaR.

Using Python:

```
In [8]: # Set the number of simulations
num_simulations = 10000

# Simulate returns
simulated_returns = np.random.normal(mean_return, volatility, num_simulations)

# Calculate VaR from simulated returns
mc_var = np.percentile(simulated_returns, 5)

# Calculate CVaR from simulated returns
mc_cvar = np.mean([r for r in simulated_returns if r <= mc_var])

print('\n Monte Carlo VaR: ', mc_var, '\n Monte Carlo CVaR: ', mc_cvar)
```

Monte Carlo VaR: -0.021850103295117843
Monte Carlo CVaR: -0.027698380024799085

7.6 Comment on the Results

Comparing the results from the three methods:

1. Historical Method: Relies on actual past returns, which may not capture future volatility accurately.
2. Parametric Method: Assumes returns are normally distributed, which might not always be true, but provides a theoretical approach.
3. Monte Carlo Simulation: Provides a flexible method to model potential future returns, capturing more possible outcomes but requiring assumptions on return distribution.

Each method has its strengths and limitations, and the choice of method depends on the context and the availability of data. Historical and Monte Carlo methods might provide a more realistic view if returns are not normally distributed, while parametric methods are useful

for theoretical models.

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