Lagrange and Newton Divided-Difference Interpolating Polynomials

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Grupo N°2 (32-38)

Programa de Ingeniería Mecánica

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- 30. Given the data in the following table:

  a. Interpolate at x = 0.55 using the second-degree Lagrange interpolating polynomial.

  b. Confirm the results by executing the user-defined function LagrangeInterp.

x	0.2	0.4	0.9
y	-0.16	-0.24	-0.09

a)

$$L(x) = -\frac{4}{25}\frac{(x-\frac{2}{5})\cdot(x-\frac{9}{10})}{(\frac{1}{5}-\frac{2}{5})\cdot(\frac{1}{5}-\frac{9}{10})} - \frac{6}{25}\frac{(x-\frac{1}{5})\cdot(x-\frac{9}{10})}{(\frac{2}{5}-\frac{1}{5})\cdot(\frac{2}{5}-\frac{9}{10})} - \frac{9}{100}\frac{(x-\frac{1}{5})\cdot(x-\frac{2}{5})}{(\frac{9}{10}-\frac{1}{5})\cdot(\frac{9}{10}-\frac{2}{5})}$$

$$L(x) = (-\frac{8}{7}x^2 + \frac{52}{35}x - \frac{72}{175}) + (\frac{12}{5}x^2 - \frac{66}{25}x + \frac{54}{125}) + (-\frac{9}{35}x^2 + \frac{27}{175}x - \frac{18}{875})$$

$$L(x) = x^2 - x$$

$$L(x) = (0{,}55)^2 - (0{,}55) = \boxed{-0{,}2475}$$

b)

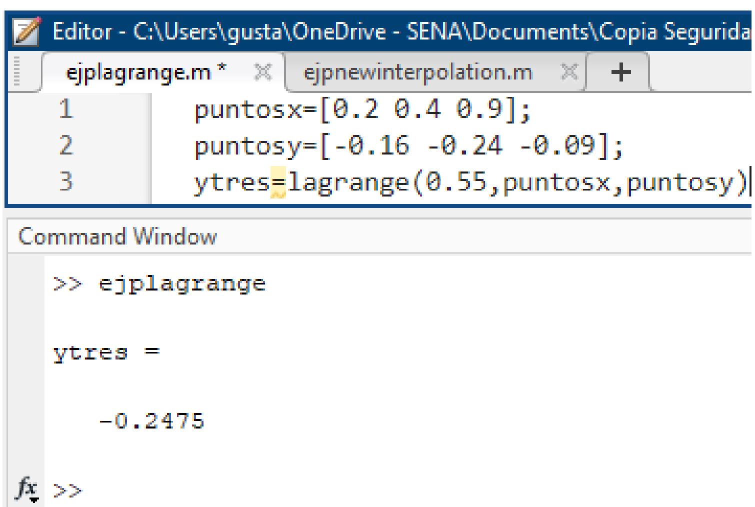

Figura 1: Confirmación B

- 31. Given the data in the following table:

  a. Interpolate at x = 3 with a first-degree Lagrange polynomial using two most suitable data points.

  b. Interpolate at x = 3 with a second-degree Lagrange polynomial using three most suitable data points.

  c. Compare the results of (a) and (b), and discuss.

x	0	1	2	4
y	1	0.96	1.68	1.82

a)

$$L(x) = \frac{42}{25}\frac{(x-4)}{(2-4)} + \frac{91}{50}\frac{(x-2)}{(4-2)}$$

$$L(x) = \frac{7}{100}x + \frac{77}{50}$$

$$L(x) = \frac{7}{100}(3) + \frac{77}{50} = \boxed{1{,}75}$$

b)

$$L(x) = \frac{24}{25}\frac{(x-2)\cdot(x-4)}{(1-2)\cdot(1-4)} + \frac{42}{25}\frac{(x-1)\cdot(x-4)}{(2-1)\cdot(2-4)} + \frac{91}{50}\frac{(x-1)\cdot(x-2)}{(4-1)\cdot(4-2)}$$

$$L(x) = -\frac{13}{60}x^2 + \frac{137}{100}x - \frac{29}{150}$$

$$L(x) = -\frac{13}{60}(3)^2 + \frac{137}{100}(3) - \frac{29}{150} = \boxed{1{,}9666}$$

c) La estimación proporcionada en b) es claramente superior porque los datos se utilizaron de manera más efectiva para la interpolación. Se acerca más al valor del polinomio de tercer grado $\boxed{2{,}265}$

- 32. Given the data in the following table:

  a. Interpolate at x = 2.5 with a first-degree Lagrange polynomial using two most suitable data points.

  b. Interpolate at x = 2.5 with a second-degree Lagrange polynomial using three most suitable data points.

  c. Compare the results of (a) and (b), and discuss.

x	1	1.5	2	3	5
y	0	0.1761	0.3010	0.4771	0.6990

a)

$$L(x) = \frac{301}{1000}\frac{(x-3)}{(2-3)} + \frac{4771}{10000}\frac{(x-2)}{(3-2)}$$

$$L(x) = \frac{1761}{10000}x - \frac{32}{625}$$

$$L(x) = \frac{1761}{10000}(2{,}5) - \frac{32}{625} = \boxed{0{,}38905}$$

b)

$$L(x) = \frac{1761}{10000}\frac{(x-2)\cdot(x-3)}{(\frac{3}{2}-2)\cdot(\frac{3}{2}-3)} + \frac{301}{1000}\frac{(x-\frac{3}{2})\cdot(x-3)}{(2-\frac{3}{2})\cdot(2-3)} + \frac{4771}{10000}\frac{(x-\frac{3}{2})\cdot(x-2)}{(3-\frac{3}{2})\cdot(3-2)}$$

$$L(x) = -\frac{737}{15000}x^2 + \frac{12653}{30000}x - \frac{173}{500}$$

$$L(x) = -\frac{737}{15000}(2{,}5)^2 + \frac{12653}{30000}(2{,}5) - \frac{173}{500} = \boxed{0{,}401333}$$

c) La estimación proporcionada en b) es claramente superior porque los datos se utilizaron de manera más efectiva para la interpolación. Se acerca más al valor del polinomio de cuarto grado $\boxed{0{,}3964375}$

- 33. Using format long and the user-defined function Lagrange Interp, given the data in the following table:

  a. Interpolate at x = 0.6 with a second-degree Lagrange polynomial using three most suitable data points.

  b. Interpolate at x = 0.6 with a third-degree Lagrange polynomial using four most suitable data points.

  c. Compare the results of (a) and (b).

x	0.2	0.4	0.5	0.8	1.1	1.3
y	0.9048	0.8187	0.7788	0.6703	0.5769	0.5220

a)

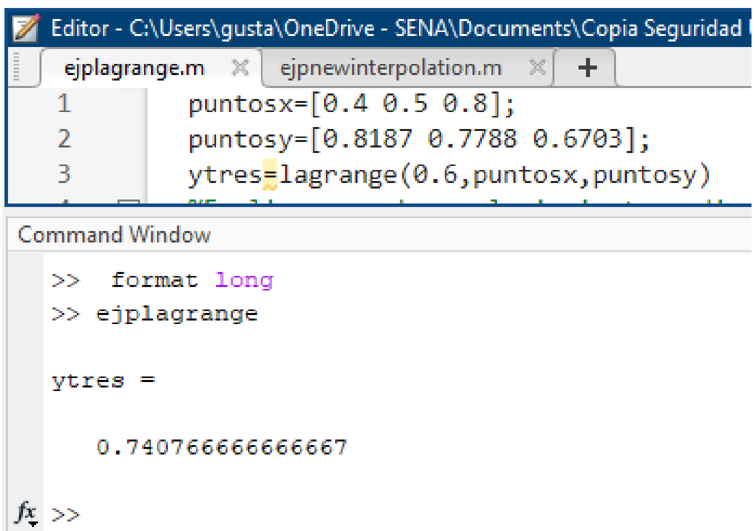

Figura 2: Polinomio segundo grado

b)

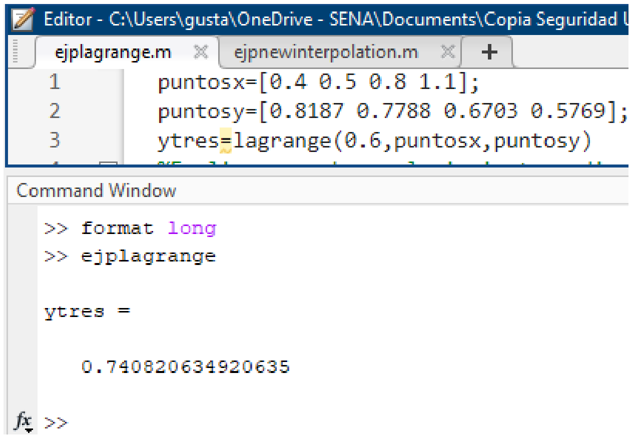

Figura 3: Polinomio tercer grado

c) La estimación proporcionada en b) es claramente superior porque los datos se utilizaron de manera más efectiva para la interpolación. Se acerca más al valor de quinto grado $\boxed{0{,}740829629629630}$

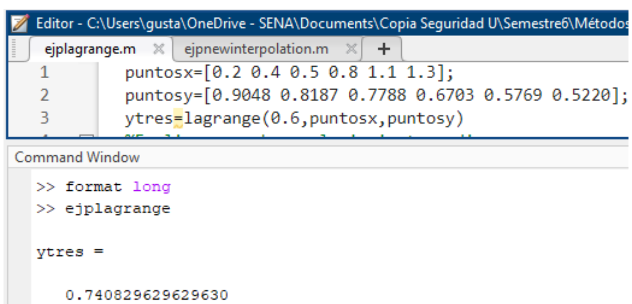

Figura 4: Polinomio quinto grado

- 34. Using format long and the user-defined function Lagrange Interp, given the data in the following table:

  a. Interpolate at x = 0.6 with a second-degree Lagrange polynomial using three most suitable data points.

  b. Interpolate at x = 0.6 with a third-degree Lagrange polynomial using four most suitable data points.

  c. Compare the results of (a) and (b).

x	0.1	0.2	0.4	0.7	0.9	1.0
y	0.9330	0.8706	0.7579	0.6156	0.5359	0.5000

a)

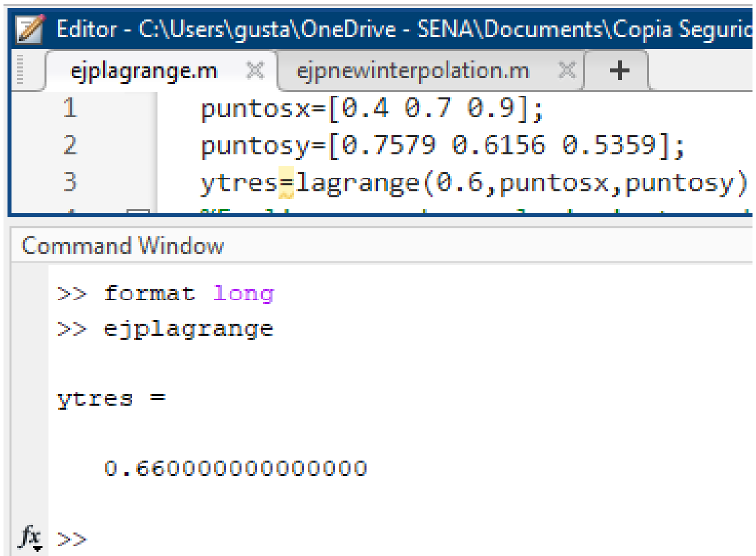

Figura 5: Polinomio segundo grado

b)

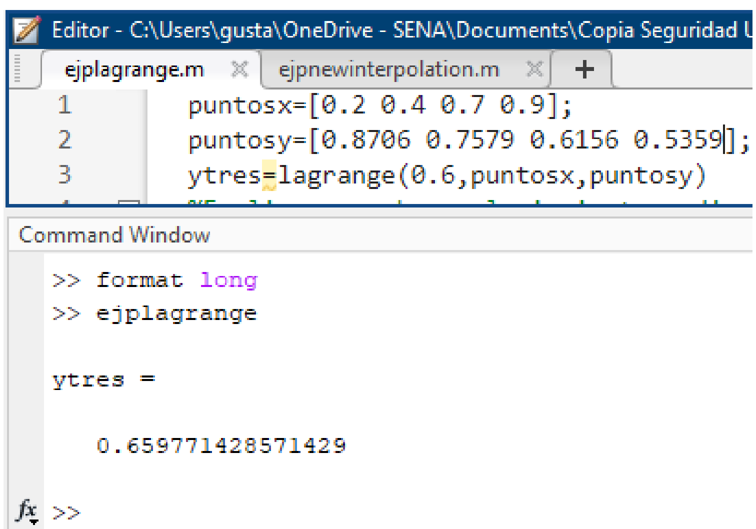

Figura 6: Polinomio tercer grado

c) La estimación proporcionada en b) es claramente superior porque los datos se utilizaron de manera más efectiva para la interpolación. Se acerca más al valor de quinto grado $\boxed{0{,}6597804232804 23}$

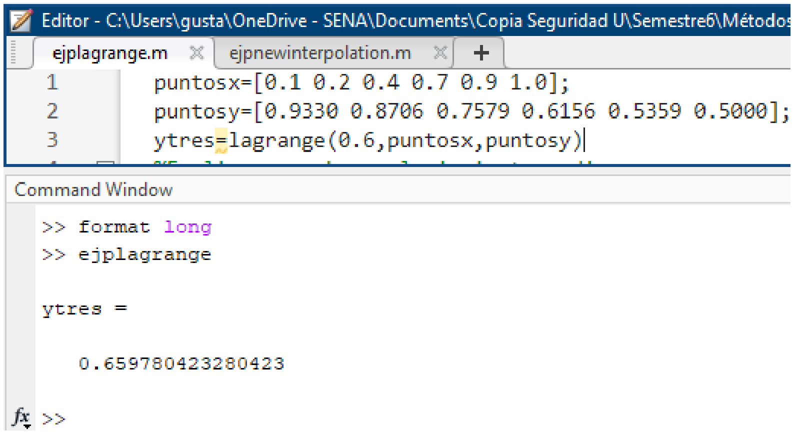

Figura 7: Polinomio quinto grado

- 35. Using the user-defined function LagrangeInterp, given the data in the following table:

  a. Interpolate at x = 1.7 with a second-degree Lagrange polynomial using three most suitable data points.

  b. Interpolate at x = 9 with a third-degree Lagrange polynomial using four most suitable data points.

x	0	1	3	7	12
y	0	1	1.44	1.91	2.29

a)

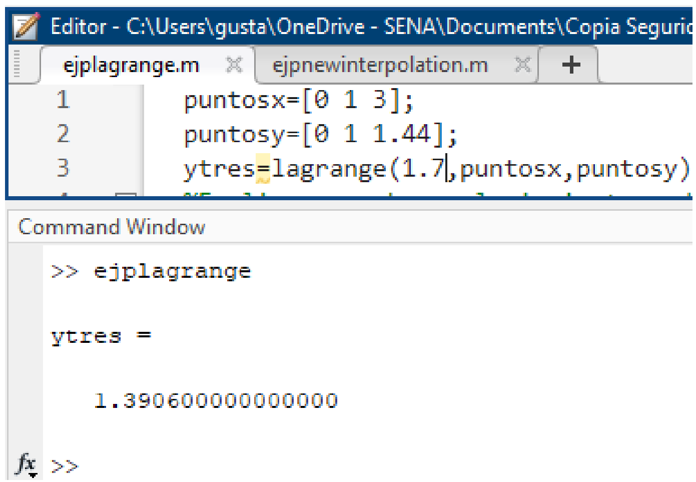

Figura 8: Polinomio segundo grado

b)

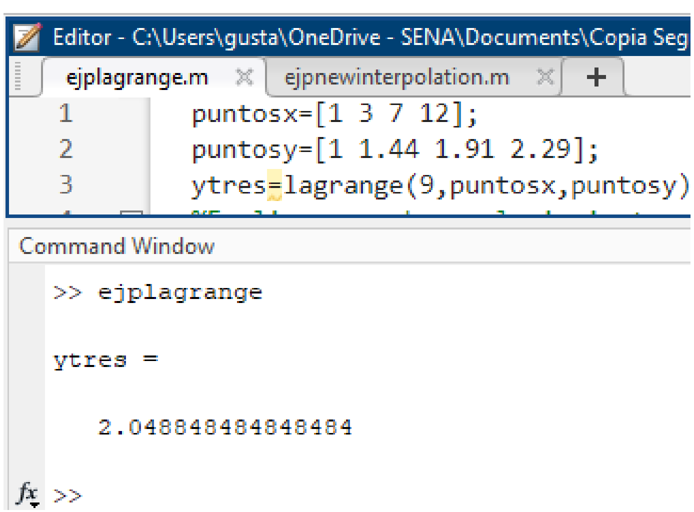

Figura 9: Polinomio tercer grado

- 36. Using the user-defined function LagrangeInterp, given the data in the following table:

  a. Interpolate at x = 1.5 with a second-degree Lagrange polynomial using three most suitable data points.

  b. Interpolate at x = 3 with a third-degree Lagrange polynomial using four most suitable data points.

x	0	1	2	2.5	4
y	1	1	0.6	0.48	0.29

a)

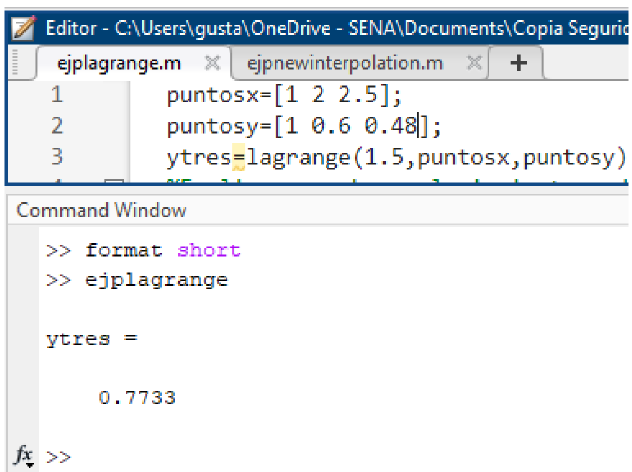

Figura 10: Polinomio segundo grado

b)

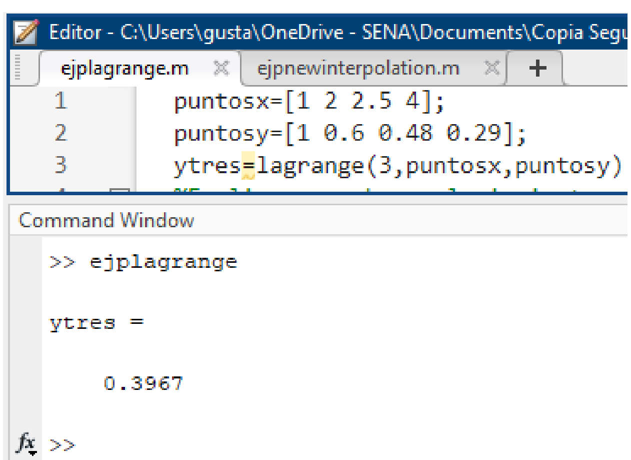

Figura 11: Polinomio tercer grado

- 37. For the data in the following table, construct a divided differences table and interpolate at x = 0.25 using Newton interpolating polynomials p1(x), p2(x), and p3(x).

x	0	0.5	0.9	1.2
y	1	0.9098	0.7725	0.6626

Para los puntos las diferencias divididas quedan de la forma:

j	X	**1st Divided Diff**	**2nd Divided Diff**	**3rd Divided Diff**	**4th Divided Diff**
0	0	1	xxxxxxxxxxxx	xxxxxxxxxxxx	xxxxxxxxxxxx
1	0.5	0.9098	-0.1804	xxxxxxxxxxxx	xxxxxxxxxxxx
2	0.9	0.7726	-0.3430	-1.3085	xxxxxxxxxxxx
3	1.2	0.6626	-0.6667	-1.0767	-7.9507

$$P_1(x) = 1 + \left(\frac{1}{0-\frac{1}{2}} + \frac{\frac{4549}{5000}}{\frac{1}{2}-0}\right)(x-0)$$

$$P_1(x) = -\frac{451}{2500}x + 1$$

$$P_1(0{,}25) = -\frac{451}{2500}(0{,}25) + 1 = \boxed{0{,}9549}$$

$$P_2(x) = 1 + \left(\frac{1}{0-\frac{1}{2}} + \frac{\frac{4549}{5000}}{\frac{1}{2}-0}\right)(x-0) + \left(\frac{1}{\left(0-\frac{1}{2}\right)\left(0-\frac{9}{10}\right)} + \frac{\frac{4549}{5000}}{\left(\frac{1}{2}-0\right)\left(\frac{1}{2}-\frac{9}{10}\right)} + \frac{\frac{309}{400}}{\left(\frac{9}{10}-0\right)\left(\frac{9}{10}-\frac{1}{2}\right)}\right)\left((x-0)\left(x-\frac{1}{2}\right)\right)$$

$$P_2(x) = -\frac{3257}{18000}x^2 - \frac{16187}{180000}x + 1$$

$$P_2(0{,}25) = -\frac{3257}{18000}(0{,}25)^2 - \frac{16187}{180000}(0{,}25) + 1 = \boxed{0{,}9662090277777778}$$

$$P_3(x) = 1 + \left(\frac{1}{0-\frac{1}{2}} + \frac{\frac{4549}{5000}}{\frac{1}{2}-0}\right)(x-0) + \left(\frac{1}{\left(0-\frac{1}{2}\right)\left(0-\frac{9}{10}\right)} + \frac{\frac{4549}{5000}}{\left(\frac{1}{2}-0\right)\left(\frac{1}{2}-\frac{9}{10}\right)} + \frac{\frac{309}{400}}{\left(\frac{9}{10}-0\right)\left(\frac{9}{10}-\frac{1}{2}\right)}\right)$$

$$\left((x-0)\left(x-\frac{1}{2}\right)\right) + \left(\frac{1}{\left(\left(0-\frac{1}{2}\right)\left(0-\frac{9}{10}\right)\right)\left(0-\frac{6}{5}\right)} + \frac{\frac{4549}{5000}}{\left(\left(\frac{1}{2}-0\right)\left(\frac{1}{2}-\frac{9}{10}\right)\right)\left(\frac{1}{2}-\frac{6}{5}\right)} + \frac{\frac{309}{400}}{\left(\left(\frac{9}{10}-0\right)\left(\frac{9}{10}-\frac{1}{2}\right)\right)\left(\frac{9}{10}-\frac{6}{5}\right)}\right)$$

$$+\frac{\frac{3313}{5000}}{\left(\left(\frac{6}{5}-0\right)\left(\frac{6}{5}-\frac{1}{2}\right)\right)\left(\frac{6}{5}-\frac{9}{10}\right)}\left(\left((x-0)\left(x-\frac{1}{2}\right)\right)\left(x-\frac{9}{10}\right)\right)$$

$$P_3(x)=\frac{4661}{37800}x^3-\frac{19093}{54000}x^2-\frac{21697}{630000}x+1$$

$$P_3(0{,}25)=\frac{4661}{37800}(0{,}25)^3-\frac{19093}{54000}(0{,}25)^2-\frac{21697}{630000}(0{,}25)+1=\boxed{0{,}9712183697089947}$$

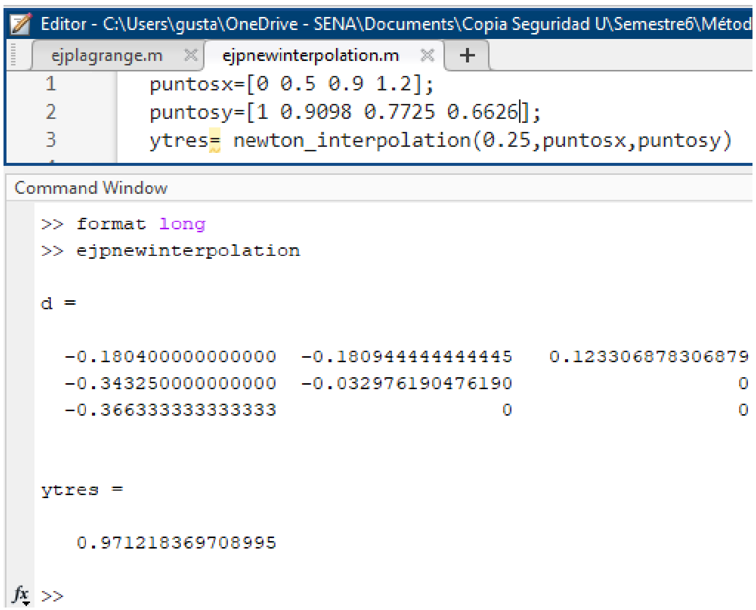

Figura 12: Confirmación con NewtonInterp

- 38. For the data in the following table, construct a divided differences table and interpolate at x = 0.3 using Newton interpolating polynomials p1(x), p2(x), and p3(x).

x	0	0.4	0.8	1
y	1	2.68	5.79	8.15

Para los puntos las diferencias divididas quedan de la forma:

j	X	**1st Divided Diff**	**2nd Divided Diff**	**3rd Divided Diff**	**4th Divided Diff**
0	0	1	xxxxxxxxxxx	xxxxxxxxxxx	xxxxxxxxxxx
1	0.4	2.68	4.2000	xxxxxxxxxxx	xxxxxxxxxxx
2	0.8	5.79	7.7750	8.9375	xxxxxxxxxxx
3	1	8.75	14.8000	35.1250	130.9375

$$P_1(x) = 1 + \left(\frac{1}{0-\frac{2}{5}} + \frac{\frac{67}{25}}{\frac{2}{5}-0}\right)(x-0)$$

$$P_1(x) = \frac{21}{5}x + 1$$

$$P_1(0{,}3) = \frac{21}{5}(0{,}3) + 1 = \boxed{2{,}26}$$

$$P_2(x) = 1 + \left(\frac{1}{0-\frac{2}{5}} + \frac{\frac{67}{25}}{\frac{2}{5}-0}\right)(x-0) + \left(\frac{1}{\left(0-\frac{2}{5}\right)\left(0-\frac{4}{5}\right)} + \frac{\frac{67}{25}}{\left(\frac{2}{5}-0\right)\left(\frac{2}{5}-\frac{4}{5}\right)} + \frac{\frac{579}{100}}{\left(\frac{4}{5}-0\right)\left(\frac{4}{5}-\frac{2}{5}\right)}\right)\left((x-0)\left(x-\frac{2}{5}\right)\right)$$

$$P_2(x) = \frac{143}{32}x^2 + \frac{193}{80}x + 1$$

$$P_2(0{,}3) = \frac{143}{32}(0{,}3)^2 + \frac{193}{80}(0{,}3) + 1 = \boxed{2{,}1259375}$$

$$P_3(x) = 1 + \left(\frac{1}{0-\frac{2}{5}} + \frac{\frac{67}{25}}{\frac{2}{5}-0}\right)(x-0) + \left(\frac{1}{\left(0-\frac{2}{5}\right)\left(0-\frac{4}{5}\right)} + \frac{\frac{67}{25}}{\left(\frac{2}{5}-0\right)\left(\frac{2}{5}-\frac{4}{5}\right)} + \frac{\frac{579}{100}}{\left(\frac{4}{5}-0\right)\left(\frac{4}{5}-\frac{2}{5}\right)}\right)\left((x-0)\left(x-\frac{2}{5}\right)\right)$$

$$+\left(\frac{1}{\left(\left(0-\frac{2}{5}\right)\left(0-\frac{4}{5}\right)\right)(0-1)} + \frac{\frac{67}{25}}{\left(\left(\frac{2}{5}-0\right)\left(\frac{2}{5}-\frac{4}{5}\right)\right)\left(\frac{2}{5}-1\right)} + \frac{\frac{579}{100}}{\left(\left(\frac{4}{5}-0\right)\left(\frac{4}{5}-\frac{2}{5}\right)\right)\left(\frac{4}{5}-1\right)} + \frac{\frac{163}{20}}{\left((1-0)\left(1-\frac{2}{5}\right)\right)\left(1-\frac{4}{5}\right)}\right)$$

$$\left(\left((x-0)\left(x-\frac{2}{5}\right)\right)\left(x-\frac{4}{5}\right)\right)$$

$$P_3(x)=\frac{215}{96}x^3+\frac{57}{32}x^2+\frac{751}{240}x+1$$

$$P_3(0{,}3)=\frac{215}{96}(0{,}3)^3+\frac{57}{32}(0{,}3)^2+\frac{751}{240}x+1=\boxed{2{,}15953125}$$

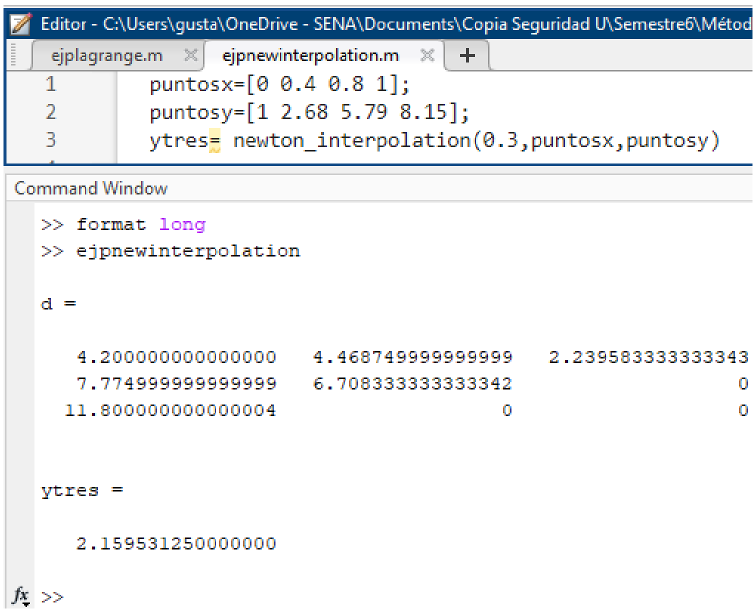

Figura 13: Confirmación con NewtonInterp

- 39. Consider the data in the following table.

  a. Construct a divided differences table and interpolate at x = 1.75 using the third-degree Newton interpolating polynomial p3(x).

  b. Suppose one more point (x = 3, y = 9.11) is added to the data. Update the divided-differences table from (a) and interpolate at x = 1.75 using the fourth-degree Newton interpolating polynomial p4(x).

x	1	1.5	2	2.5
y	1.22	2.69	4.48	6.59

- 40. Consider the data in the following table.

  a. Construct a divided differences table and interpolate at x = 4 using the third-degree Newton interpolating polynomial p3(x).

b. Suppose one more point (x = 7, y = 0.18) is added to the data. Update the divided-difference table from (a) and interpolate at x = 4 using the fourth-degree Newton interpolating polynomial p4(x).

x	1	3	5	6
y	1	0.45	0.26	0.21

- 41. Given the data in the following table

  a. Construct a divided differences table and interpolate at x = 2.4 and x = 4.2 using the fourth-degree Newton interpolating polynomial p4(x).

  b. Confirm the results by executing the user-defined function NewtonInterp.

x	1	2	3	4	6
y	0.69	1.10	1.39	1.61	1.95

- 42. Given the data in the following table

  a. Construct a divided differences table and interpolate at x = 2.5 and x = 5 using the fourth-degree Newton interpolating polynomial p4(x).

  b. Confirm the results by executing the user-defined function NewtonInterp.

x	1	2	3	4	6
y	0.89	1.81	2.94	4.38	8.72

- 43. Using the user-defined function NewtonInterp, given the data in the following table, interpolate at x = 4.5 via Newton interpolating polynomials of all possible degrees, and comment on accuracy.

x	1	2	3.5	5	7	9	10
y	1	1.4142	1.8708	2.2361	2.6458	3	3.1623