

LAGRANGE AND NEWTON DIVIDED-DIFFERENCE INTERPOLATING
POLYNOMIALS

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GRUPO N°2 (32-38)

PROGRAMA DE INGENIERÍA MECÁNICA

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- 30. Given the data in the following table:

a. Interpolate at $x = 0.55$ using the second-degree Lagrange interpolating polynomial.

b. Confirm the results by executing the user-defined function LagrangeInterp.

x	0.2	0.4	0.9
y	-0.16	-0.24	-0.09

a)

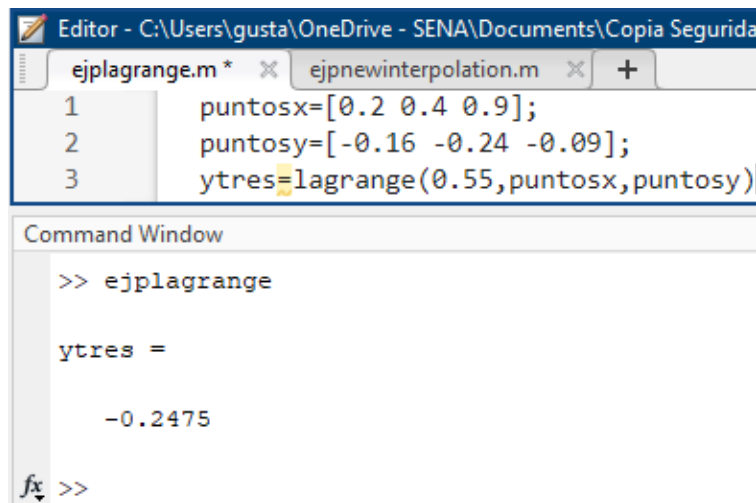
$$L(x) = -\frac{4}{25} \frac{(x - \frac{2}{5}) \cdot (x - \frac{9}{10})}{(\frac{1}{5} - \frac{2}{5}) \cdot (\frac{1}{5} - \frac{9}{10})} - \frac{6}{25} \frac{(x - \frac{1}{5}) \cdot (x - \frac{9}{10})}{(\frac{2}{5} - \frac{1}{5}) \cdot (\frac{2}{5} - \frac{9}{10})} - \frac{9}{100} \frac{(x - \frac{1}{5}) \cdot (x - \frac{2}{5})}{(\frac{9}{10} - \frac{1}{5}) \cdot (\frac{9}{10} - \frac{2}{5})}$$

$$L(x) = (-\frac{8}{7}x^2 + \frac{52}{35}x - \frac{72}{175}) + (\frac{12}{5}x^2 - \frac{66}{25}x + \frac{54}{125}) + (-\frac{9}{35}x^2 + \frac{27}{175}x - \frac{18}{875})$$

$$L(x) = x^2 - x$$

$$L(x) = (0,55)^2 - (0,55) = \boxed{-0,2475}$$

b)



The screenshot shows a MATLAB Editor window with the following code in `ejplagrange.m`:

```
1 puntosx=[0.2 0.4 0.9];
2 puntosy=[-0.16 -0.24 -0.09];
3 ytres=lagrange(0.55,puntosx,puntosy);
```

The Command Window shows the execution of the function:

```
>> ejplagrange

ytres =

    -0.2475

fx >>
```

Figura 1: Confirmación B

■ 31. Given the data in the following table:

- a. Interpolate at $x = 3$ with a first-degree Lagrange polynomial using two most suitable data points.
- b. Interpolate at $x = 3$ with a second-degree Lagrange polynomial using three most suitable data points.
- c. Compare the results of (a) and (b), and discuss.

x	0	1	2	4
y	1	0.96	1.68	1.82

a)

$$L(x) = \frac{42}{25} \frac{(x-4)}{(2-4)} + \frac{91}{50} \frac{(x-2)}{(4-2)}$$

$$L(x) = \frac{7}{100}x + \frac{77}{50}$$

$$L(x) = \frac{7}{100}(3) + \frac{77}{50} = \boxed{1,75}$$

b)

$$L(x) = \frac{24}{25} \frac{(x-2) \cdot (x-4)}{(1-2) \cdot (1-4)} + \frac{42}{25} \frac{(x-1) \cdot (x-4)}{(2-1) \cdot (2-4)} + \frac{91}{50} \frac{(x-1) \cdot (x-2)}{(4-1) \cdot (4-2)}$$

$$L(x) = -\frac{13}{60}x^2 + \frac{137}{100}x - \frac{29}{150}$$

$$L(x) = -\frac{13}{60}(3)^2 + \frac{137}{100}(3) - \frac{29}{150} = \boxed{1,9666}$$

c) La estimación proporcionada en b) es claramente superior porque los datos se utilizaron de manera más efectiva para la interpolación. Se acerca más al valor del polinomio de tercer grado $\boxed{2,265}$

- 32. Given the data in the following table:

- Interpolate at $x = 2.5$ with a first-degree Lagrange polynomial using two most suitable data points.
- Interpolate at $x = 2.5$ with a second-degree Lagrange polynomial using three most suitable data points.
- Compare the results of (a) and (b), and discuss.

x	1	1.5	2	3	5
y	0	0.1761	0.3010	0.4771	0.6990

a)

$$L(x) = \frac{301}{1000} \frac{(x-3)}{(2-3)} + \frac{4771}{10000} \frac{(x-2)}{(3-2)}$$

$$L(x) = \frac{1761}{10000}x - \frac{32}{625}$$

$$L(x) = \frac{1761}{10000}(2,5) - \frac{32}{625} = \boxed{0,38905}$$

b)

$$L(x) = \frac{1761}{10000} \frac{(x-2) \cdot (x-3)}{(\frac{3}{2}-2) \cdot (\frac{3}{2}-3)} + \frac{301}{1000} \frac{(x-\frac{3}{2}) \cdot (x-3)}{(2-\frac{3}{2}) \cdot (2-3)} + \frac{4771}{10000} \frac{(x-\frac{3}{2}) \cdot (x-2)}{(3-\frac{3}{2}) \cdot (3-2)}$$

$$L(x) = -\frac{737}{15000}x^2 + \frac{12653}{30000}x - \frac{173}{500}$$

$$L(x) = -\frac{737}{15000}(2,5)^2 + \frac{12653}{30000}(2,5) - \frac{173}{500} = \boxed{0,401333}$$

- c) La estimación proporcionada en b) es claramente superior porque los datos se utilizaron de manera más efectiva para la interpolación. Se acerca más al valor del polinomio de cuarto grado $\boxed{0,3964375}$

- 33. Using format long and the user-defined function Lagrange Interp, given the data in the following table:
 - a. Interpolate at $x = 0.6$ with a second-degree Lagrange polynomial using three most suitable data points.
 - b. Interpolate at $x = 0.6$ with a third-degree Lagrange polynomial using four most suitable data points.
 - c. Compare the results of (a) and (b).

x	0.2	0.4	0.5	0.8	1.1	1.3
y	0.9048	0.8187	0.7788	0.6703	0.5769	0.5220

a)

The screenshot shows a MATLAB Editor window with two tabs: 'ejplagrange.m' and 'ejpnewinterpolation.m'. The 'ejplagrange.m' tab is active and contains the following code:

```

1 puntosx=[0.4 0.5 0.8];
2 puntosy=[0.8187 0.7788 0.6703];
3 ytres=lagrange(0.6,puntosx,puntosy)

```

Below the editor is the Command Window, which displays the following output:

```

>> format long
>> ejplagrange

ytres =

    0.740766666666667
fx >>

```

Figura 2: Polinomio segundo grado

b)

The image shows a MATLAB Editor window with a script named 'ejplagrange.m'. The script contains three lines of code: `puntosx=[0.4 0.5 0.8 1.1];`, `puntosy=[0.8187 0.7788 0.6703 0.5769];`, and `ytres=lagrange(0.6,puntosx,puntosy)`. Below the editor is the Command Window, which shows the execution of `format long` and `ejplagrange`, resulting in `ytres = 0.740820634920635`.

Figura 3: Polinomio tercer grado

c) La estimación proporcionada en b) es claramente superior porque los datos se utilizaron de manera más efectiva para la interpolación. Se acerca más al valor de quinto grado 0,740829629629630

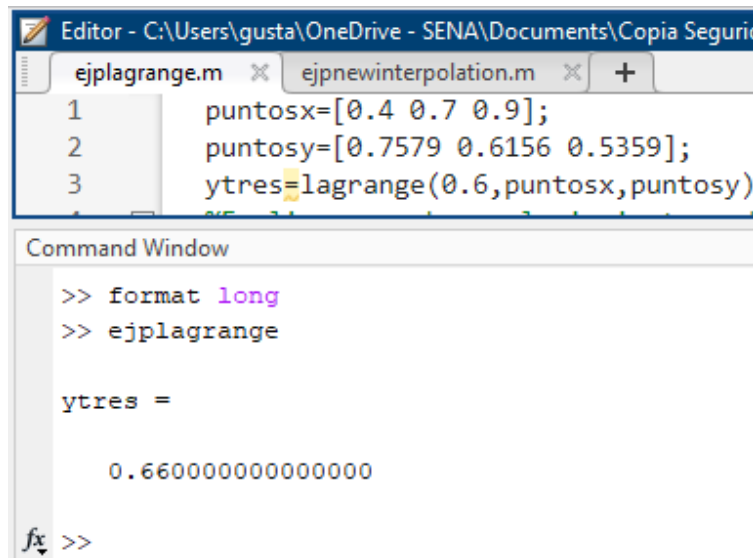
The image shows a MATLAB Editor window with a script named 'ejplagrange.m'. The script contains three lines of code: `puntosx=[0.2 0.4 0.5 0.8 1.1 1.3];`, `puntosy=[0.9048 0.8187 0.7788 0.6703 0.5769 0.5220];`, and `ytres=lagrange(0.6,puntosx,puntosy)`. Below the editor is the Command Window, which shows the execution of `format long` and `ejplagrange`, resulting in `ytres = 0.740829629629630`.

Figura 4: Polinomio quinto grado

- 34. Using format long and the user-defined function Lagrange Interp, given the data in the following table:
 - a. Interpolate at $x = 0.6$ with a second-degree Lagrange polynomial using three most suitable data points.
 - b. Interpolate at $x = 0.6$ with a third-degree Lagrange polynomial using four most suitable data points.
 - c. Compare the results of (a) and (b).

x	0.1	0.2	0.4	0.7	0.9	1.0
y	0.9330	0.8706	0.7579	0.6156	0.5359	0.5000

a)



The image shows a MATLAB Editor window with two tabs: 'ejplagrange.m' and 'ejpnewinterpolation.m'. The 'ejplagrange.m' tab is active, showing the following code:

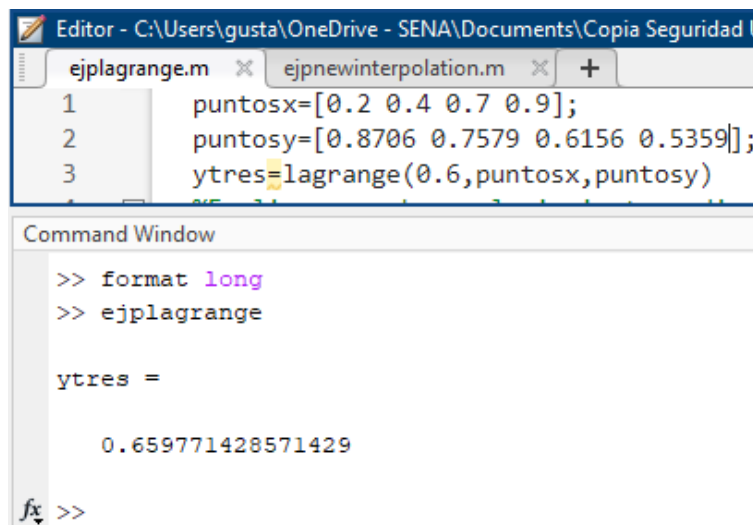
```
1 puntosx=[0.4 0.7 0.9];  
2 puntosy=[0.7579 0.6156 0.5359];  
3 ytres=lagrange(0.6,puntosx,puntosy)
```

Below the editor is the Command Window, which displays the following output:

```
>> format long  
>> ejplagrange  
  
ytres =  
  
0.6600000000000000  
  
fx >>
```

Figura 5: Polinomio segundo grado

b)



The image shows a MATLAB Editor window with two tabs: 'ejplagrange.m' and 'ejpnewinterpolation.m'. The 'ejplagrange.m' tab is active, showing the following code:

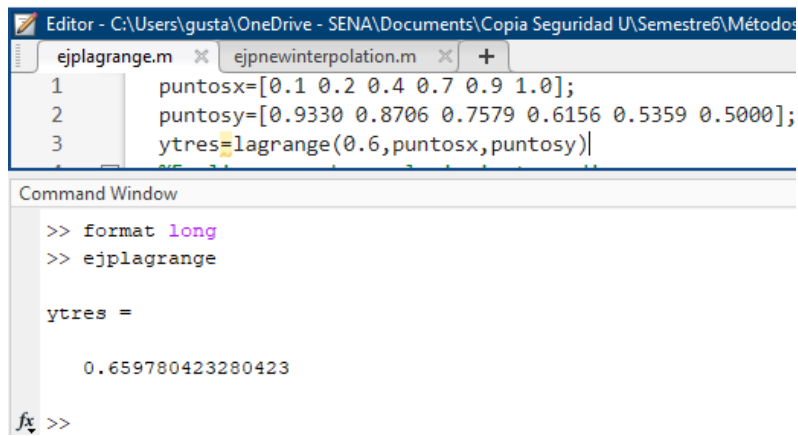
```
1 puntosx=[0.2 0.4 0.7 0.9];  
2 puntosy=[0.8706 0.7579 0.6156 0.5359];  
3 ytres=lagrange(0.6,puntosx,puntosy)
```

Below the editor is the Command Window, which displays the following output:

```
>> format long  
>> ejplagrange  
  
ytres =  
  
0.659771428571429  
  
fx >>
```

Figura 6: Polinomio tercer grado

c) La estimación proporcionada en b) es claramente superior porque los datos se utilizaron de manera más efectiva para la interpolación. Se acerca más al valor de quinto grado 0,659780423280423



The image shows a MATLAB environment. The Editor window displays a script named 'ejplagrange.m' with the following code:

```
1 puntosx=[0.1 0.2 0.4 0.7 0.9 1.0];  
2 puntosy=[0.9330 0.8706 0.7579 0.6156 0.5359 0.5000];  
3 ytres=lagrange(0.6,puntosx,puntosy);
```

The Command Window shows the execution of the script:

```
>> format long  
>> ejplagrange  
  
ytres =  
  
0.659780423280423  
fx >>
```

Figura 7: Polinomio quinto grado

- 35. Using the user-defined function LagrangeInterp, given the data in the following table:
 - a. Interpolate at $x = 1.7$ with a second-degree Lagrange polynomial using three most suitable data points.
 - b. Interpolate at $x = 9$ with a third-degree Lagrange polynomial using four most suitable data points.

x	0	1	3	7	12
y	0	1	1.44	1.91	2.29

a)

The image shows a MATLAB Editor window with two tabs: 'ejplagrange.m' and 'ejpnewinterpolation.m'. The 'ejplagrange.m' tab is active, showing the following code:

```
1 puntosx=[0 1 3];
2 puntosy=[0 1 1.44];
3 ytres=lagrange(1.7,puntosx,puntosy)
```

Below the editor is the Command Window, which shows the execution of the command 'ejplagrange' and the resulting output:

```
>> ejplagrange

ytres =

    1.3906000000000000
```

Figura 8: Polinomio segundo grado

b)

The image shows a MATLAB Editor window with two tabs: 'ejplagrange.m' and 'ejpnewinterpolation.m'. The 'ejplagrange.m' tab is active, showing the following code:

```
1 puntosx=[1 3 7 12];
2 puntosy=[1 1.44 1.91 2.29];
3 ytres=lagrange(9,puntosx,puntosy)
```

Below the editor is the Command Window, which shows the execution of the command 'ejplagrange' and the resulting output:

```
>> ejplagrange

ytres =

    2.048848484848484
```

Figura 9: Polinomio tercer grado

- 36. Using the user-defined function LagrangeInterp, given the data in the following table:
 - a. Interpolate at $x = 1.5$ with a second-degree Lagrange polynomial using three most suitable data points.
 - b. Interpolate at $x = 3$ with a third-degree Lagrange polynomial using four most suitable data points.

x	0	1	2	2.5	4
y	1	1	0.6	0.48	0.29

a)

The image shows a MATLAB Editor window with a file named 'ejplagrange.m'. The code in the editor is as follows:

```
1 puntosx=[1 2 2.5];
2 puntosy=[1 0.6 0.48];
3 ytres=lagrange(1.5,puntosx,puntosy)
```

Below the editor is the Command Window, which shows the execution of the code:

```
>> format short
>> ejplagrange

ytres =

    0.7733

fx >>
```

Figura 10: Polinomio segundo grado

b)

The image shows a MATLAB Editor window with a file named 'ejplagrange.m'. The code in the editor is as follows:

```
1 puntosx=[1 2 2.5 4];
2 puntosy=[1 0.6 0.48 0.29];
3 ytres=lagrange(3,puntosx,puntosy)
```

Below the editor is the Command Window, which shows the execution of the code:

```
>> ejplagrange

ytres =

    0.3967

fx >>
```

Figura 11: Polinomio tercer grado

- 37. For the data in the following table, construct a divided differences table and interpolate at $x = 0.25$ using Newton interpolating polynomials $p_1(x)$, $p_2(x)$, and $p_3(x)$.

x	0	0.5	0.9	1.2
y	1	0.9098	0.7725	0.6626

Para los puntos las diferencias divididas quedan de la forma:

j	X	1st Divided Diff	2nd Divided Diff	3rd Divided Diff	4th Divided Diff
0	0	1	xxxxxxxxxxx	xxxxxxxxxxx	xxxxxxxxxxx
1	0.5	0.9098	-0.1804	xxxxxxxxxxx	xxxxxxxxxxx
2	0.9	0.7726	-0.3430	-1.3085	xxxxxxxxxxx
3	1.2	0.6626	-0.6667	-1.0767	-7.9507

$$P_1(x) = 1 + \left(\frac{1}{0 - \frac{1}{2}} + \frac{\frac{4549}{5000}}{\frac{1}{2} - 0} \right) (x - 0)$$

$$P_1(x) = -\frac{451}{2500}x + 1$$

$$P_1(0,25) = -\frac{451}{2500}(0,25) + 1 = \boxed{0,9549}$$

$$P_2(x) = 1 + \left(\frac{1}{0 - \frac{1}{2}} + \frac{\frac{4549}{5000}}{\frac{1}{2} - 0} \right) (x - 0) + \left(\frac{1}{(0 - \frac{1}{2})(0 - \frac{9}{10})} + \frac{\frac{4549}{5000}}{(\frac{1}{2} - 0)(\frac{1}{2} - \frac{9}{10})} + \frac{\frac{309}{400}}{(\frac{9}{10} - 0)(\frac{9}{10} - \frac{1}{2})} \right) \left((x - 0) \left(x - \frac{1}{2} \right) \right)$$

$$P_2(x) = -\frac{3257}{18000}x^2 - \frac{16187}{180000}x + 1$$

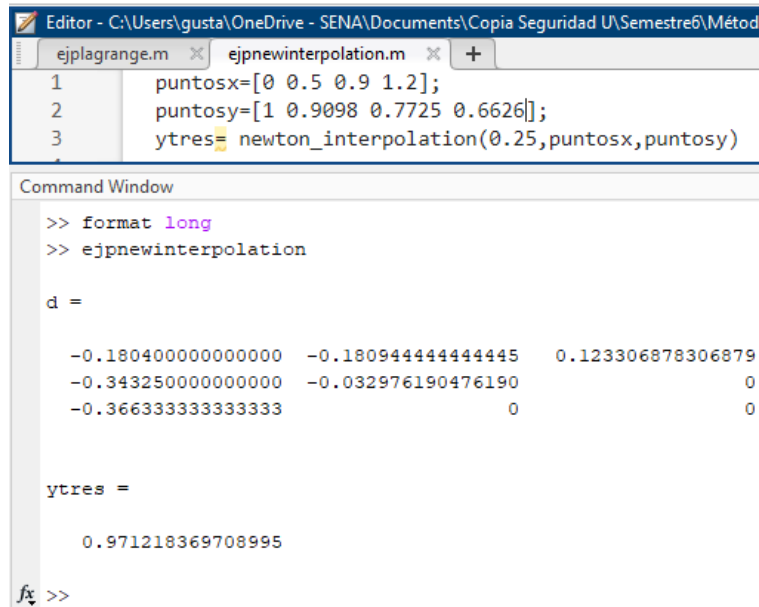
$$P_2(0,25) = -\frac{3257}{18000}(0,25)^2 - \frac{16187}{180000}(0,25) + 1 = \boxed{0,9662090277777778}$$

$$P_3(x) = 1 + \left(\frac{1}{0 - \frac{1}{2}} + \frac{\frac{4549}{5000}}{\frac{1}{2} - 0} \right) (x - 0) + \left(\frac{1}{(0 - \frac{1}{2})(0 - \frac{9}{10})} + \frac{\frac{4549}{5000}}{(\frac{1}{2} - 0)(\frac{1}{2} - \frac{9}{10})} + \frac{\frac{309}{400}}{(\frac{9}{10} - 0)(\frac{9}{10} - \frac{1}{2})} \right) \left((x - 0) \left(x - \frac{1}{2} \right) \right) + \left(\frac{1}{((0 - \frac{1}{2})(0 - \frac{9}{10}))(0 - \frac{6}{5})} + \frac{\frac{4549}{5000}}{((\frac{1}{2} - 0)(\frac{1}{2} - \frac{9}{10}))(\frac{1}{2} - \frac{6}{5})} + \frac{\frac{309}{400}}{((\frac{9}{10} - 0)(\frac{9}{10} - \frac{1}{2}))(\frac{9}{10} - \frac{6}{5})} \right) \left((x - 0) \left(x - \frac{1}{2} \right) \left(x - \frac{9}{10} \right) \right)$$

$$+ \frac{\frac{3313}{5000}}{\left(\left(\frac{6}{5} - 0\right)\left(\frac{6}{5} - \frac{1}{2}\right)\right)\left(\frac{6}{5} - \frac{9}{10}\right)} \left(\left((x-0) \left(x - \frac{1}{2} \right) \right) \left(x - \frac{9}{10} \right) \right)$$

$$P_3(x) = \frac{4661}{37800}x^3 - \frac{19093}{54000}x^2 - \frac{21697}{630000}x + 1$$

$$P_3(0,25) = \frac{4661}{37800}(0,25)^3 - \frac{19093}{54000}(0,25)^2 - \frac{21697}{630000}(0,25) + 1 = \boxed{0,9712183697089947}$$



The screenshot shows a MATLAB editor window with the following script in the main window:

```

1 puntosx=[0 0.5 0.9 1.2];
2 puntosy=[1 0.9098 0.7725 0.6626];
3 ytres= newton_interpolation(0.25,puntosx,puntosy)

```

The Command Window shows the following output:

```

>> format long
>> ejpnewinterpolation

d =

-0.1804000000000000 -0.1809444444444445 0.123306878306879
-0.3432500000000000 -0.032976190476190 0
-0.3663333333333333 0 0

ytres =

0.971218369708995
fx >>

```

Figura 12: Confirmación con NewtonInterp

- 38. For the data in the following table, construct a divided differences table and interpolate at $x = 0.3$ using Newton interpolating polynomials $p_1(x)$, $p_2(x)$, and $p_3(x)$.

x	0	0.4	0.8	1
y	1	2.68	5.79	8.15

Para los puntos las diferencias divididas quedan de la forma:

j	X	1st Divided Diff	2nd Divided Diff	3rd Divided Diff	4th Divided Diff
0	0	1	xxxxxxxxxxx	xxxxxxxxxxx	xxxxxxxxxxx
1	0.4	2.68	4.2000	xxxxxxxxxxx	xxxxxxxxxxx
2	0.8	5.79	7.7750	8.9375	xxxxxxxxxxx
3	1	8.75	14.8000	35.1250	130.9375

$$P_1(x) = 1 + \left(\frac{1}{0 - \frac{2}{5}} + \frac{\frac{67}{25}}{\frac{2}{5} - 0} \right) (x - 0)$$

$$P_1(x) = \frac{21}{5}x + 1$$

$$P_1(0,3) = \frac{21}{5}(0,3) + 1 = \boxed{2,26}$$

$$P_2(x) = 1 + \left(\frac{1}{0 - \frac{2}{5}} + \frac{\frac{67}{25}}{\frac{2}{5} - 0} \right) (x - 0) + \left(\frac{1}{\left(0 - \frac{2}{5}\right) \left(0 - \frac{4}{5}\right)} + \frac{\frac{67}{25}}{\left(\frac{2}{5} - 0\right) \left(\frac{2}{5} - \frac{4}{5}\right)} + \frac{\frac{579}{100}}{\left(\frac{4}{5} - 0\right) \left(\frac{4}{5} - \frac{2}{5}\right)} \right) \left((x - 0) \left(x - \frac{2}{5} \right) \right)$$

$$P_2(x) = \frac{143}{32}x^2 + \frac{193}{80}x + 1$$

$$P_2(0,3) = \frac{143}{32}(0,3)^2 + \frac{193}{80}(0,3) + 1 = \boxed{2,1259375}$$

$$P_3(x) = 1 + \left(\frac{1}{0 - \frac{2}{5}} + \frac{\frac{67}{25}}{\frac{2}{5} - 0} \right) (x - 0) + \left(\frac{1}{\left(0 - \frac{2}{5}\right) \left(0 - \frac{4}{5}\right)} + \frac{\frac{67}{25}}{\left(\frac{2}{5} - 0\right) \left(\frac{2}{5} - \frac{4}{5}\right)} + \frac{\frac{579}{100}}{\left(\frac{4}{5} - 0\right) \left(\frac{4}{5} - \frac{2}{5}\right)} \right) \left((x - 0) \left(x - \frac{2}{5} \right) \right)$$

$$+ \left(\frac{1}{\left(\left(0 - \frac{2}{5}\right) \left(0 - \frac{4}{5}\right)\right) \left(0 - 1\right)} + \frac{\frac{67}{25}}{\left(\left(\frac{2}{5} - 0\right) \left(\frac{2}{5} - \frac{4}{5}\right)\right) \left(\frac{2}{5} - 1\right)} + \frac{\frac{579}{100}}{\left(\left(\frac{4}{5} - 0\right) \left(\frac{4}{5} - \frac{2}{5}\right)\right) \left(\frac{4}{5} - 1\right)} + \frac{\frac{163}{20}}{\left(\left(1 - 0\right) \left(1 - \frac{2}{5}\right)\right) \left(1 - \frac{4}{5}\right)} \right)$$

$$\left(\left((x-0) \left(x - \frac{2}{5} \right) \right) \left(x - \frac{4}{5} \right) \right)$$

$$P_3(x) = \frac{215}{96}x^3 + \frac{57}{32}x^2 + \frac{751}{240}x + 1$$

$$P_3(0,3) = \frac{215}{96}(0,3)^3 + \frac{57}{32}(0,3)^2 + \frac{751}{240}x + 1 = \boxed{2,15953125}$$

```

Editor - C:\Users\gusta\OneDrive - SENA\Documents\Copia Seguridad U\Semestre6\Métod
ejplagrange.m  ejpnewinterpolation.m  +
1 puntosx=[0 0.4 0.8 1];
2 puntosy=[1 2.68 5.79 8.15];
3 ytres= newton_interpolation(0.3,puntosx,puntosy)

Command Window

>> format long
>> ejpnewinterpolation

d =

    4.200000000000000    4.468749999999999    2.239583333333343
    7.774999999999999    6.708333333333342             0
   11.800000000000004             0             0

ytres =

    2.159531250000000

fx >>

```

Figura 13: Confirmación con NewtonInterp

- 39. Consider the data in the following table.
 - a. Construct a divided differences table and interpolate at $x = 1.75$ using the third-degree Newton interpolating polynomial $p_3(x)$.
 - b. Suppose one more point ($x = 3, y = 9.11$) is added to the data. Update the divided-differences table from (a) and interpolate at $x = 1.75$ using the fourth-degree Newton interpolating polynomial $p_4(x)$.

x	1	1.5	2	2.5
y	1.22	2.69	4.48	6.59

- 40. Consider the data in the following table.
 - a. Construct a divided differences table and interpolate at $x = 4$ using the third-degree Newton interpolating polynomial $p_3(x)$.

b. Suppose one more point ($x = 7$, $y = 0.18$) is added to the data. Update the divided-difference table from (a) and interpolate at $x = 4$ using the fourth-degree Newton interpolating polynomial $p_4(x)$.

x	1	3	5	6
y	1	0.45	0.26	0.21

- 41. Given the data in the following table

a. Construct a divided differences table and interpolate at $x = 2.4$ and $x = 4.2$ using the fourth-degree Newton interpolating polynomial $p_4(x)$.

b. Confirm the results by executing the user-defined function `NewtonInterp`.

x	1	2	3	4	6
y	0.69	1.10	1.39	1.61	1.95

- 42. Given the data in the following table

a. Construct a divided differences table and interpolate at $x = 2.5$ and $x = 5$ using the fourth-degree Newton interpolating polynomial $p_4(x)$.

b. Confirm the results by executing the user-defined function `NewtonInterp`.

x	1	2	3	4	6
y	0.89	1.81	2.94	4.38	8.72

- 43. Using the user-defined function `NewtonInterp`, given the data in the following table, interpolate at $x = 4.5$ via Newton interpolating polynomials of all possible degrees, and comment on accuracy.

x	1	2	3.5	5	7	9	10
y	1	1.4142	1.8708	2.2361	2.6458	3	3.1623