

# Homework 4

April 14, 2024

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## 1: One More Grammar

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$$S \rightarrow TU \mid cSd$$

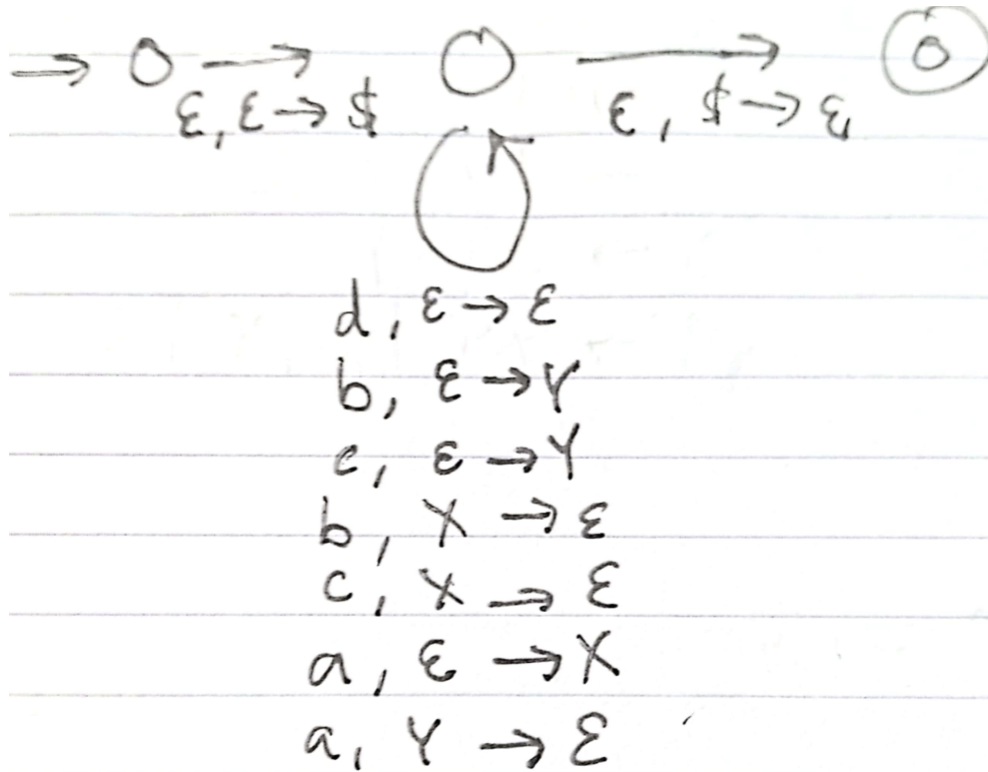
$$T \rightarrow \epsilon \mid cTa$$

$$U \rightarrow aXd \mid aUd$$

$$X \rightarrow bc \mid bXc$$

## 2: Push It All Down

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### 3: This One Might Need Context

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Suppose, for the sake of contradiction, that  $C$  is context-free. Therefore, there is a CFG that describes it with a pumping length  $p$ .

Let  $s = a^p b^p c^{p+1}$ . By the pumping lemma, we can divide  $s$  into  $uvxyz$  such that  $|vy| > 0$  and  $|vxy| \leq p$ . We proceed by cases on  $v$  and  $y$ :

Case 1 (all a's): Pumping up,  $uv^2xy^2z \notin C$  because the number of a's  $\geq$  the number of c's.

Case 2 (all b's): Pumping up,  $uv^2xy^2z \notin C$  because the number of b's  $\geq$  the number of c's.

Case 3 (all c's): Pumping down,  $uxz \notin C$  because the number of c's  $\leq$  the number of a's or the number of b's.

Case 4 (mix of a's and b's): Pumping up,  $uv^2xy^2z \notin C$  because the number of a's  $\geq$  the number of c's and the number of b's  $\geq$  the number of c's.

Case 5 (mix of b's and c's): Pumping down,  $uxz \notin C$  because the number of c's  $\leq$  the number of a's.

All cases result in a contradiction of the pumping lemma, thus  $L$  is not context-free.