# Homework 6

April 24, 2024

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**1: I Think I Recognize This One**

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Suppose, for the sake of contradiction, that $TR_{TM}$ is decidable. Therefore a TM E decides it.
A = "On input $< M, w >$,
1. Construct M' = "On input x,
    1. If the input is not in the correct format, reject.
    2. Run M on w, if M accepts w, accept. Otherwise, reject.
2. Simulate E with M' as an input. If E accepts, accept. Otherwise, reject."

A accepts exactly when M accepts w and rejects otherwise. This is a contradiction because we know that $A_{TM}$ is undecidable. Thus, $TR_{TM}$ must be undecidable.

## 2: A LENGTH-y Decision

Suppose, for the sake of contradiction, that $LENGTH_{TM}$ is decidable. Therefore a TM E decides it.

A = "On input $< M, w >$,

1. Construct M' = "On input x,
    1. If the input doesn't have a length, reject.
    2. Run M on w, if M accepts w, accept. Otherwise, reject.
2. Simulate E with M' as an input. If E accepts, accept. Otherwise, reject."

A accepts exactly when M accepts w and rejects otherwise. This is a contradiction because we know that $A_{TM}$ is undecidable. Thus, $LENGTH_{TM}$ must be undecidable.

**3: Sort This One Out**

M = "On input $< L, K >$,
1. We will initialize a variable that keeps track of the maximum element.
2. Iterate through each element in L.
3. For every element in L, if the element is greater than the maximum element, update the max element to that.
4. Once you reach the end of the list, return the max."

The loop only iterates one through the list so the time complexity is O(N). Therefore, M runs in polynomial time relative to the size of L.