# Homework 4

March 10, 2024

## 1: One More Grammar

$$
\begin{aligned}
S &\to TU \mid cSd \\
T &\to \epsilon \mid cTa \\
U &\to aXd \mid aUd \\
X &\to bc \mid bXc
\end{aligned}
$$

## 2: Push It All Down

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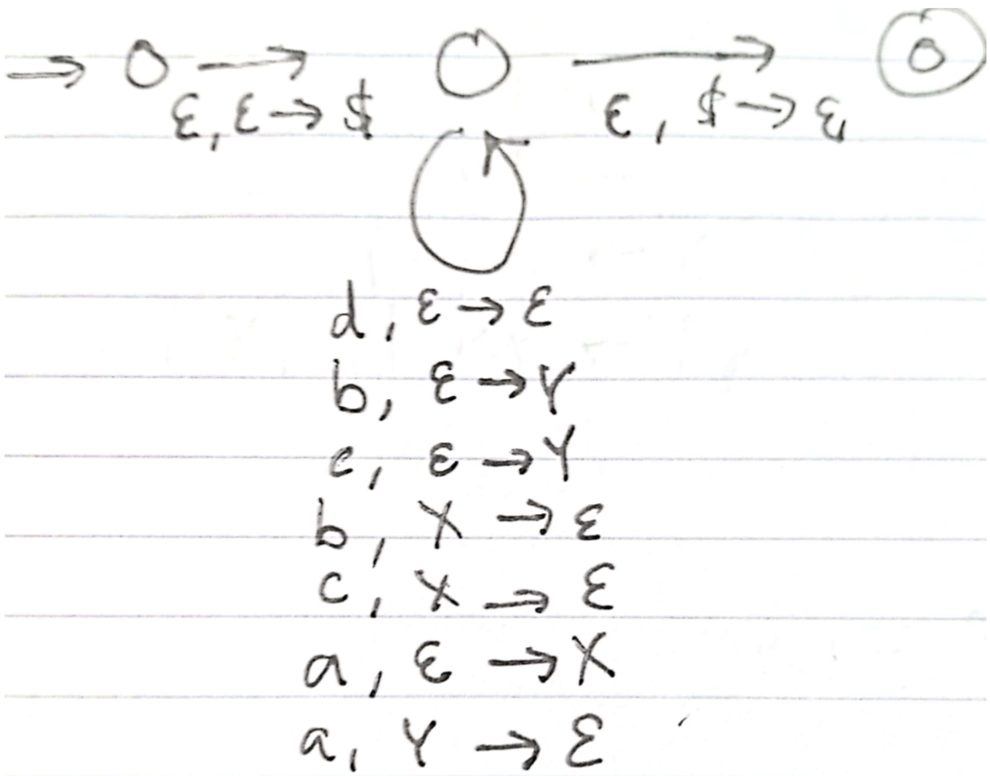

## 3: This One Might Need Context

Suppose, for the sake of contradiction, that C is context-free. Therefore, there is a CFG that describes it with a pumping length p.

Let $s = a^p b^p c^{p+1}$. By the pumping lumma, we can divide s into uvxyz such that $|vy| > 0$ and $|vxy| \leq p$. We proceed by cases on v and y:

Case 1 (all a's): Pumping up, $uv^2xy^2z \notin C$ because the number of a's $\geq$ the number of c's.

Case 2 (all b's): Pumping up, $uv^2xy^2z \notin C$ because the number of b's $\geq$ the number of c's.

Case 3 (all c's): Pumping down, $uxz \notin C$ because the number of c's $\leq$ the number of a's or the number of b's.

Case 4 (mix of a's and b's): Pumping up, $uv^2xy^2z \notin C$ because the number of a's $\geq$ the number of c's and the number of b's $\geq$ the number of c's.

Case 5 (mix of b's and c's): Pumping down, $uxz \notin C$ because the number of c's $\leq$ the number of a's.

All cases result in a contradiction of the pumping lumma, thus L is not context-free.