CS 151: Mathematical Foundations of Computing
Homework Assignment 06
Fall 2023

Instructions

This assignment is due Sunday, December 03, at 11:59PM (Central Time). No late submissions will be accepted for this assignment.

This assignment must be submitted on *Gradescope*. Handwritten submissions are allowed as long as they are legible. Submissions typed in LaTeX or Word are preferred. Each answer must be clearly labeled (1a, 1b, etc.) and matched to the corresponding question on *Gradescope*. A 5-point penalty will be applied to submissions that do not follow these guidelines.

For more instructions on how to submit assignments on *Gradescope* see this guide.

This assignment is individual. Offering or receiving any kind of unauthorized or unacknowledged assistance (including searching for solutions online) is a violation of the University's academic integrity policies, will result in a grade of zero for the assignment, and will be subject to disciplinary action.

Part I: Probability (100 pt.)

For each of the following problems, you should write your answer as an expression. You do not need to give the final numeric value. For example, you should write $C(4,2)/2^4$ instead of 0.375.

1. **(15 pt., 5 pt. each) A player in the Mega Millions lottery picks five different integers between 1 and 70, inclusive, and a sixth integer between 1 and 25, inclusive, which may duplicate one of the earlier five integers. The player wins the jackpot (currently $335 million) if the first five numbers picked match the first five numbers drawn (in any order) and the sixth number picked matches the sixth number drawn.**
    a. **What is the probability that a player wins the jackpot?**
    b. **What is the probability that a player wins the second prize ($1,000,000) by matching the first five numbers drawn (in any order) but not the sixth number?**
    c. **What is the probability that a player wins the third prize ($10,000) by matching exactly four of the first five numbers drawn (in any order) and the sixth number?**

2. **(15 pt., 5 pt. each) A coin is flipped 9 times in a row.**
    a. **What is the probability that it lands heads up exactly 3 times?**
    b. **What is the probability that it lands heads up at most 3 times?**
    c. **What is the probability that it lands heads up more than 3 times?**

3. **(15 pt., 5 pt. each) A standard deck of playing cards consist of 52 cards. Each card has a rank and a suit. There are 13 possible ranks (A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K), 4 possible suits (spades, clubs, hearts, diamonds), and 13 cards for each suit (one for each rank).**
    a. **What is the probability that a hand of 8 cards contains 8 cards of one suit (for example, 8 hearts)?**
    b. **What is the probability that a hand of 8 cards contains 4 cards of one suit and 4 cards of a second suit (for example, 4 hearts and 4 diamonds)?**
    c. **What is the probability that a hand of 8 cards contains 6 cards of one suit and 2 cards of a second suit (for example, 6 hearts and 2 diamonds)?**

4. **(10 pt.) A red die and a blue die are thrown. Both dice are loaded (that is, not all sides are equally likely). Rolling a 2 with the red die is three times as likely as rolling each of the other five numbers and rolling a 5 with the blue die is two times as likely as rolling each of the other five numbers.**
    a. **(2.5 pt.) What is the probability of each outcome of the red die?**
    b. **(2.5 pt.) What is the probability of each outcome of the blue die?**
    c. **(5 pt.) What is the probability that the sum of the numbers on the two dice is equal to 8?**

5. **(25 pt.) A red and a blue die are thrown. Both dice are fair (that is, all sides are equally likely). The events A, B, and C are defined as follows:**
   A: The number on the red die is 4.
   B: The sum of the numbers on the two dice is even.
   C: The sum of the numbers on the two dice is at least 10.
    a. **(9 pt.) Calculate the probability of each individual event; that is, $p(A)$, $p(B)$, and $p(C)$.**
    b. **(4 pt.) What is $p(C|A)$?**
    c. **(4 pt.) What is $p(B|A)$?**
    d. **(4 pt.) What is $p(B|C)$?**
    e. **(4 pt.) Consider all pairs of events: (A, B), (A, C), and (B, C). Which pairs of events are independent and which pairs of events are not independent? Justify your answer.**

6. **(20 pt., 5 pt. each) Suppose that 12% of the patients in a hospital are infected with a virus. Also suppose that when a test for this virus is administered, 95% of the patients infected with the virus test positive and 98% of the patients not infected with the virus test negative.**
    a. **What is the probability that a patient is infected if they test positive?**
    b. **What is the probability that a patient is not infected if they test positive?**
    c. **What is the probability that a patient is infected if they test negative?**
    d. **What is the probability that a patient is not infected if they test negative?**