Lecture Assignment 1 (Extra Credit)

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September 13, 2023

1 Given definitions:

- An integer n is even if and only if there exists an integer k such that $n = 2k$.
- An integer n is odd if and only if there exists an integer k such that $n = 2k + 1$.
- A real number r is rational if and only if there exist integers a and b with $b \neq 0$ such that $r = \frac{a}{b}$.

2 Objective:

1. If $m + n$ and $n + p$ are odd integers, where m, n, and p are integers then $m + p$ is even.
2. The product of any two rational numbers is rational.

3 Proof:

1. Let m, n, p be integers and $m + n$ and $n + p$ are odd integers, we will show that $m + p$ is even.
   Since $m + n$ is odd, then $m + n = 2k + 1$ (1) for some integer k. And since $n + p$ is odd, then $n + p = 2j + 1$ (2) for some integer j.
   Adding both equations together, we get $m + n + n + p = 2k + 1 + 2j + 1$, simplifying to $m + p = 2k + 2j + 2 - 2n$ or $m + p = 2(k + j + 1 - n)$.
   Since k, j, n are integers, the sum of them are also integers.
   Since $m + p$ is equal to $2m$, where $m = (k + j + 1 - n)$, $m + p$ is even.

2. Let a and b be two rational numbers, which can also be expressed as $a = c/d$ and $b = e/f$, where c, d, e, f are both integers, and $d \neq 0$ and $f \neq 0$, we will prove that their product is also rational.
   The product of two rational numbers is $a * b$ which can also be expressed as $\frac{c}{d} * \frac{e}{f} = \frac{c*e}{d*f}$
   Since c and e are both integers, then their product is also an integer, giving us $\frac{k}{d*f}$ where k is the product of c and e.
   Since d and f are both nonzeroes, then their product j will be a nonzero integer.
   Now we have $a * b = \frac{k}{j}$, where k, j are both integers and $j \neq 0$. Hence the product of a and b is also rational.