CS 151: Mathematical Foundations of Computing
Homework Assignment 05
Fall 2023

Instructions

This assignment is due Tuesday, November 21, at 11:59PM (Central Time).

This assignment must be submitted on *Gradescope*. Handwritten submissions are allowed as long as they are legible. Submissions typed in LaTeX or Word are preferred. Each answer must be clearly labeled (1, 2, etc.) and matched to the corresponding question on *Gradescope*. A 5-point penalty will be applied to submissions that do not follow these guidelines.

For more instructions on how to submit assignments on *Gradescope* see this guide.

Late submissions will be accepted within 0-12 hours after the deadline with a 5-point penalty and within 12-24 hours after the deadline with a 20-point penalty. No late submissions will be accepted more than 24 hours after the deadline.

This assignment is individual. Offering or receiving any kind of unauthorized or unacknowledged assistance (including searching for solutions online) is a violation of the University's academic integrity policies, will result in a grade of zero for the assignment, and will be subject to disciplinary action.

Part I: Functions (25 pt.)

1. **(25 pt., 5 pt. each) For each of the following functions, determine whether the function is:**
   - ***Injective* (*one-to-one*).**
   - ***Surjective* (*onto*).**
   - ***Bijective.***

   Justify your answers.

   a. $f: \mathbb{Z} \to \mathbb{Z}$ **such that** $f(x) = 3x$.

   b. $f: \mathbb{Z} \to \mathbb{Z}$ **such that** $f(x) = 10 - x$.

   c. $f: \mathbb{Z} \to \mathbb{Z}$ **such that** $f(x) = \lfloor x/3 \rfloor$ **(i.e., floor of** $x/3$**).**

   d. $f: \mathbb{R} \to \mathbb{R}^+$ **such that** $f(x) = 2^x + 1$.

   e. $f: \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ **such that** $f(x, y) = x - y$.

Part II: Counting (75 pt.)

For each of the following problems, you should write your answer as an expression. You do not need to give the final numeric value. For example, if your answer is the number of 6-permutations of a set with 10 elements, you should write $P(10,6)$ or $10!/4!$ or $10 \times 9 \times 8 \times 7 \times 6 \times 5$, but not 151200.

2. **(20 pt., 5 pt. each) How many ways are there to choose 5 items from 11 distinct items if…**
    a. **…the order of the items matters and repetition of items is not allowed?**
    b. **…the order of the items matters and repetition of items is allowed?**
    c. **…the order of the items does not matter and repetition of items is not allowed?**
    d. **…the order of the items does not matter and repetition of items is allowed?**

3. **(25 pt., 5 pt. each) A cookie shop sells 14 different types of cookies. Assume that it has an unlimited supply of each type. How many ways are there to choose 9 cookies from these 14 types if…**
    a. **…there are no restrictions?**
    b. **…5 of the cookies are of one type and 4 are of another type (for example, 5 are chocolate chip and 4 are snickerdoodles)?**
    c. **…all the cookies are of different types?**
    d. **…at least 3 cookies are chocolate chip and at least 2 are snickerdoodles?**
    e. **…at most 2 are sugar cookies?**

4. **(15 pt., 5 pt. each) How many different bit strings of length 10…**
    a. **…contain exactly three 0's or end with 0?**
    b. **…contain at most five 1's?**
    c. **…contain at least five consecutive 0's or at least five consecutive 1's?**

5. **(15 pt., 5 pt. each) Given the equation $a + b + c + d = 55$, where a, b, c, and d are integers. How many different solutions does this equation have if…**
    a. **…all the variables must be nonnegative?**
    b. **…all the variables must be positive?**
    c. **…$a \geq 2$, $b \geq 3$, $c \geq 4$, and $d \geq 5$?**