Computational Physics HW5

Siyuan Chen

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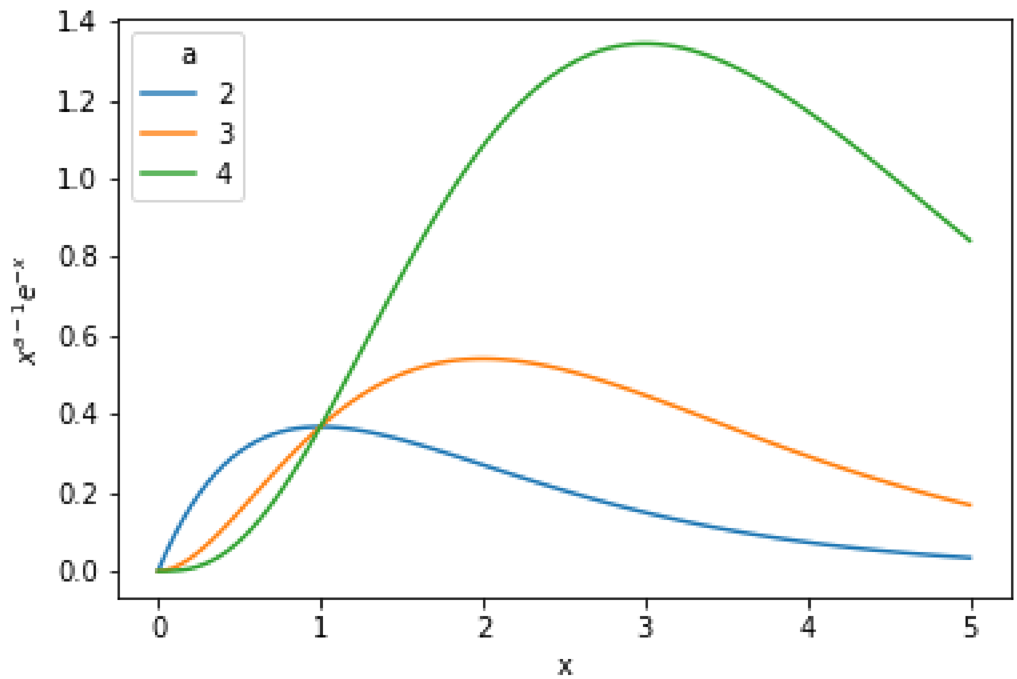

Figure 1: **P1(a): Integrand of Gamma function with sample a's**

P1:

P1(b): Given integrand function of gamma function, to find its maximum we look for the point when derivative is zero. Applying product rule, we get the derivative of integrand function is $(a-1)x^{a-2}e^{-x} - x^{a-1}e^{-x}$. $x = a-1$ and $x = 0$ both give zero derivative. But for higher order derivatives we observe that x to some power is always in every term and $x = 0$ gives zero second order derivative. Therefore, we find maximum at $x = a-1$.

P1(c): To get an accurate integration, we want to change the integration limits to be finite while the peak, which is the most significant part of integration, to be at the center. We do the change of variable $z = \frac{x}{c+x}$. When $z = \frac{1}{2}$, $x = c$. We want this to be where the maximum falls, that is $x = a-1$, thus set $c = x = a-1$.

P1(d): Take $x^{a-1} = e^{(a-1)lnx}$, then the integrand becomes $e^{(a-1)lnx-x}$. This is better than the x^{a-1} version since we put the very large power and very small exponential evaluations together into first order $(a-1)lnx - x$, which has much less opportunities to overflow or underflow.

P1(e): $z = \frac{x}{c+x}$, then $x = \frac{cz}{1-z}$, $dx = dz * c/(1-z)^2$, with our choice of $c = a-1$. Put all these into the integrand to get our integral in the new variable and do the fixedquad integral. Run gamma(3/2) for a unit test and we get 0.886226961308722 which agrees with expectation.

P1(f): Run gamma(3), gamma(6), gamma(10), we get 2.0000000000000018, 120.00000000000009, 362880.00000000023 which agree with factorial with only $10^(- 15)$ error.

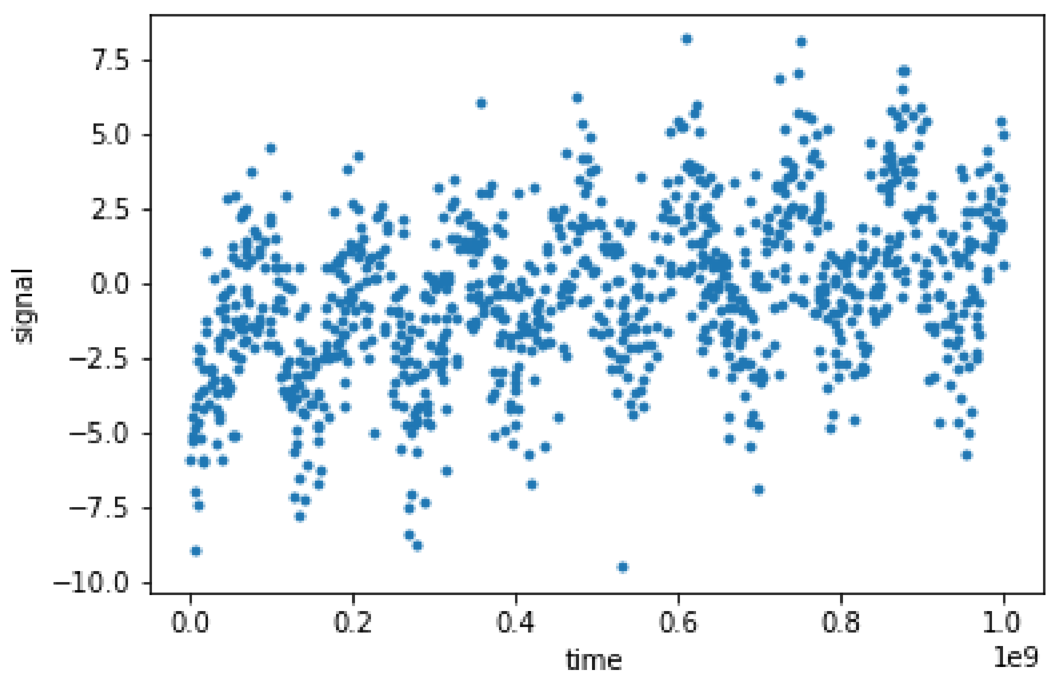

Figure 2: **P2(a): Plot of raw data**

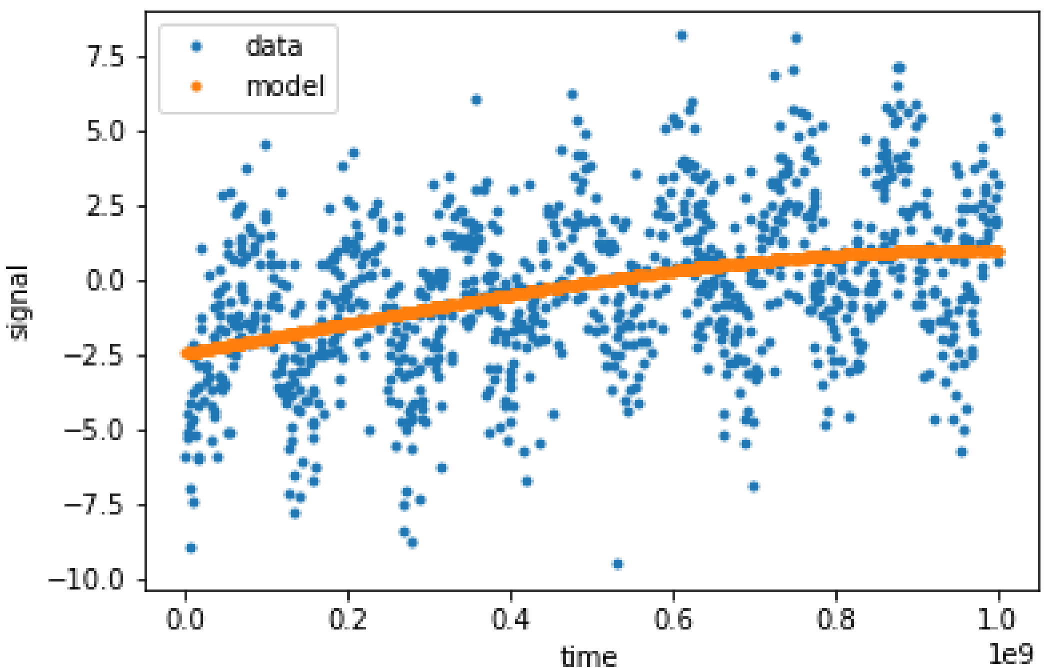

Figure 3: **P2(b):Third order fit.** Construct a matrix of time from the zero to the third orders, get its inversion by SVD then dot with signal to get the model coefficients.

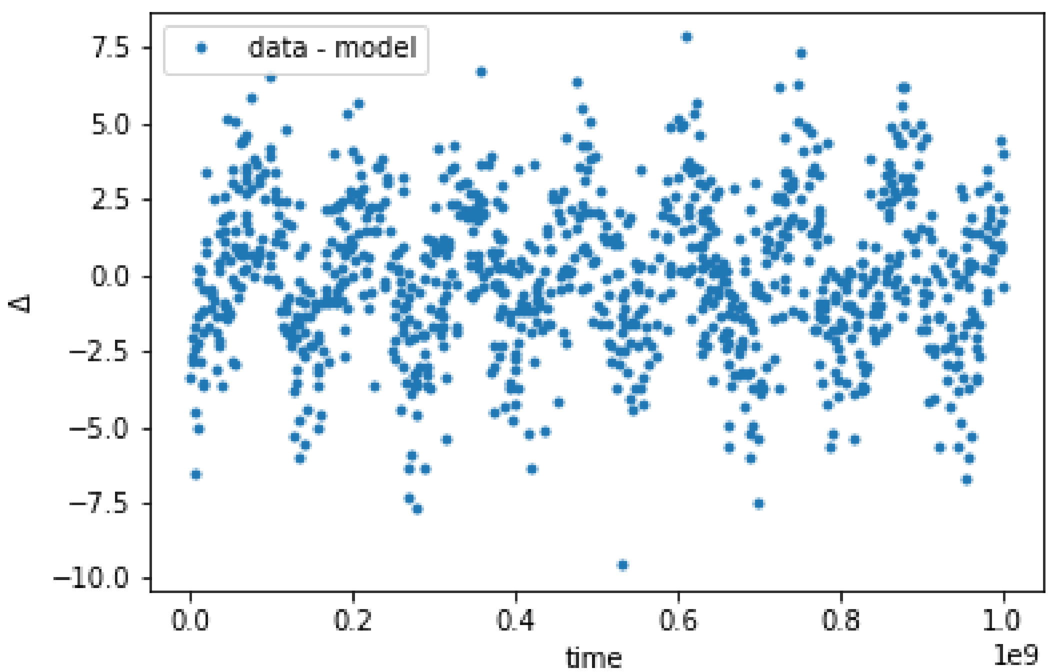

Figure 4: **P2(c):Residual comparing with third order fit** The magnitude of residual is much larger than the uncertainty, and there is obviously an oscillating trend that our model didn't capture suggesting our third order model is not a good explanation.

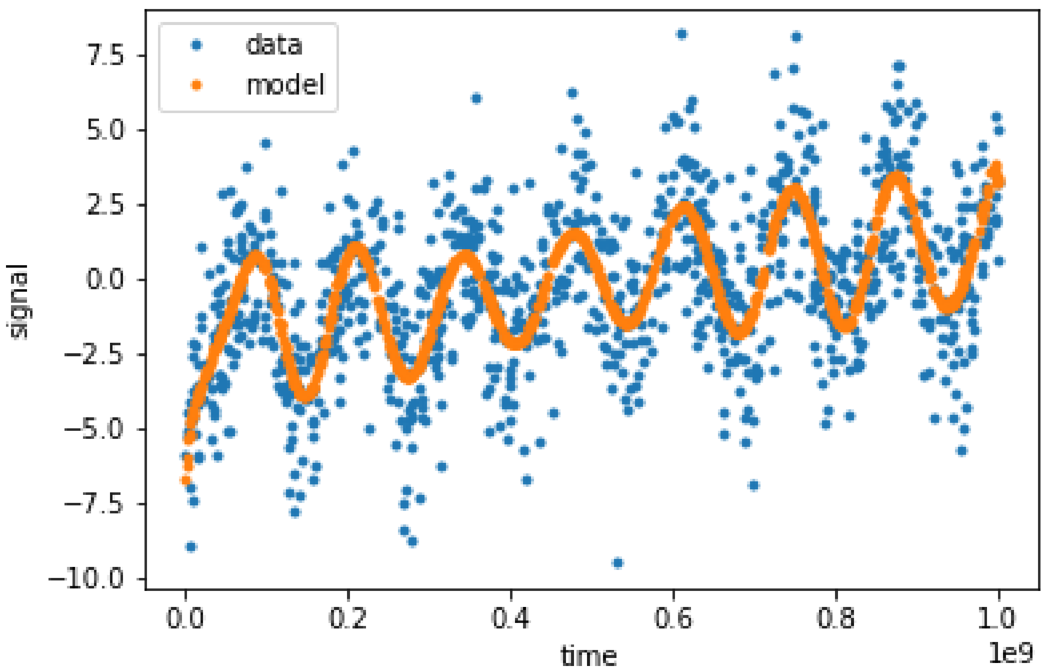

Figure 5: **P2(d):30th order polynomial fit** The condition number of the 30th order polynomial fit is about 1.4 wich is much better than 121 of the third order fit.

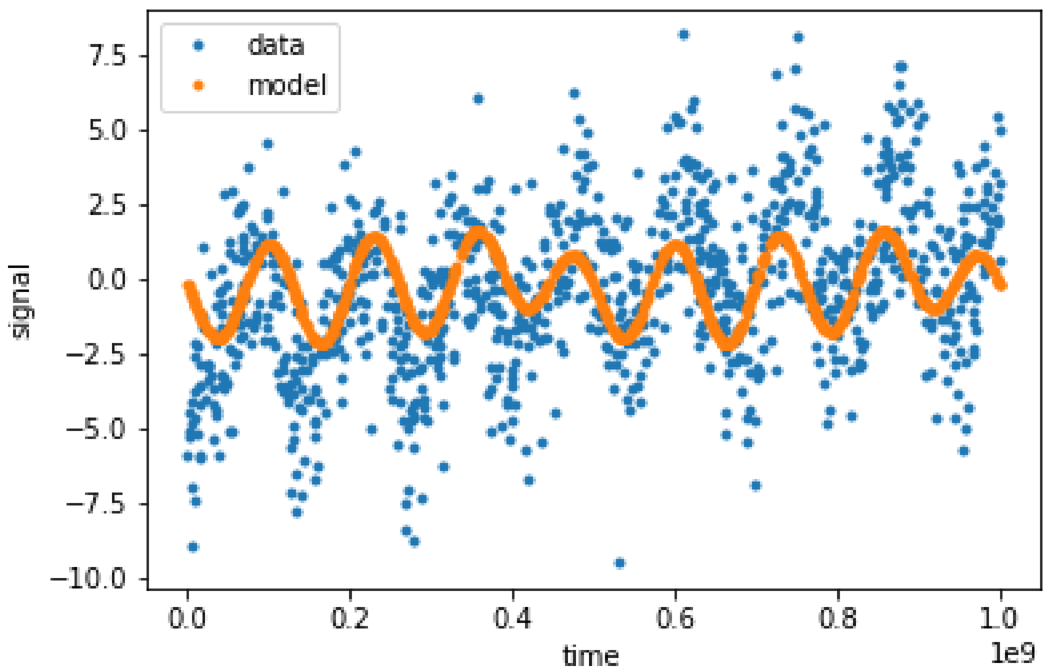

Figure 6: **P2(e):Fourier series fit** Following the same procedure and use fourier modes instead of polynomials, we obtain above fitting with the 4th fourier modes. The condition number is 1.5 which is close to 1. I intensionally chosen 4 fourier modes since from we can see about 8 periods over the time recorded from raw data. We chose the base frequency by half of the time covered, thus 4 times this frequency is 8 periods which describes the periodic behavior we eyeball. The period of this model is 1/8 time unit.