Digital Signal Processing Laboratory (EE39203)

Experiment 3: Frequency Analysis

Introduction

In this experiment, we will examine continuous-time and discrete-time signals and systems using Fourier series and Fourier transformations. The signal is broken down into a series of intricate exponential functions for the Fourier representation of signals. Given that an LTI system responds to a complex exponential input by producing a complex exponential of the same frequency, these decompositions are crucial in the understanding of LTI systems. Only the input signal's amplitude and phase are altered. In order to fully understand the behaviour of an LTI system, its frequency response must be examined.

Part 1

Background exercises

Each signal given below represents one period of a periodic signal with period T0.
1. Period T0 = 2. For $t \in [0, 2]$:

$$s(t) = rect\left(t - \frac{1}{2}\right)$$

2. Period T0 = 1. For $t \in [-1/2 , 1/2]$:

$$s(t) = rect(2t) - \frac{1}{2}$$

Sketch the signal on the interval [0, T0]

1st signal given is $s(t) = rect(t - 1/2)$
for period $T_0 = 2$ and $t \in [0, 2]$

By fourier series expansion,

$$s(t) = a_0 + \sum_{k=1}^{\infty} a_K \cos(w_0 k t + \theta_K) + \sum_{K=1}^{\infty} b_K \sin(w_0 k t + \theta_K)$$

here $\theta_K = 0$,

$$a_0 = \frac{1}{T_0} \int_0^{T_0} s(t)\, dt = \frac{1}{2} \int_0^2 rect(t - 1/2) = \frac{1}{2} \int_0^1 dt$$

$$= 0.5$$

$$a_K = \frac{2}{T_0} \int_0^{T_0} \cos(\pi k t)\, rect(t - 1/2)\, dt$$

$$= \frac{1}{\pi K} (\sin \pi k t) \Big|_0^1 = 0$$

$$b_K = \frac{2}{T_0} \int_0^{T_0} \sin(\pi k t)\, rect(t - 1/2)\, dt$$

$$= \frac{2}{2} \int_0^1 \sin(\pi k t)\, dt$$

$$= \frac{-1}{\pi k} (\cos \pi k t) \Big|_0^1$$

$$= \frac{1 - \cos \pi k}{\pi k} \quad \begin{cases} \text{if } k \text{ is even } b_K = 0 \\ \text{if } k \text{ is odd } b_K = 2/\pi k \end{cases}$$

$$s(t) = 0.5 + \sum_{k=1}^{\infty} \frac{2}{\pi k} \sin(\pi k t) \quad \text{when } k \text{ is odd number}$$

For a 2nd signal

$s(t) = rect\left(2t - \frac{1}{2}\right)$

$T_0 = 1$ and $t = \left[-\frac{1}{2}, \frac{1}{2}\right]$

$$s(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega t) + \sum_{n=1}^{\infty} b_n \sin(n\omega_0 t)$$

$$a_0 = \frac{1}{T}\int_0^{T_0} s(t).\,dt = \frac{1}{1}\int_{-1/2}^{-1/4} -\frac{1}{2}dt + \int_{-1/4}^{0} \frac{1}{2}\,dt + \int_0^{1/4} \frac{1}{2}dt + \int_{1/2}^{1/4} -\frac{1}{2}\,dt$$

Since the curve is even, so b_n must be 0.

Now,

$$a_n = \frac{2}{T_0}\int_{<T_0>} x(t)\cos(n\omega_0 t)\,dt$$

$$= \frac{2}{1}\int_{-1/2}^{1/2} x(t)\cos(n\omega_0 t)\,dt$$

$$= 2\left\{\int_{-1/2}^{-1/4} -\frac{1}{2}\cos(n\omega_0 t)\,dt + \int_{-1/4}^{1/4} \frac{1}{2}\cos(n\omega_0 t)\,dt + \int_{1/4}^{1/2} -\frac{1}{2}\cos(n\omega_0 t)\,dt\right\}$$

$$= \frac{2\sin(n\pi/2)}{\pi n}$$

Therefore,

$$s(t) = \sum_{n=1}^{\infty} \frac{2}{n\pi}\sin\left(\frac{n\pi}{2}\right)\cos(n2\pi t)$$

Submit these background exercises with the lab report.

Code:

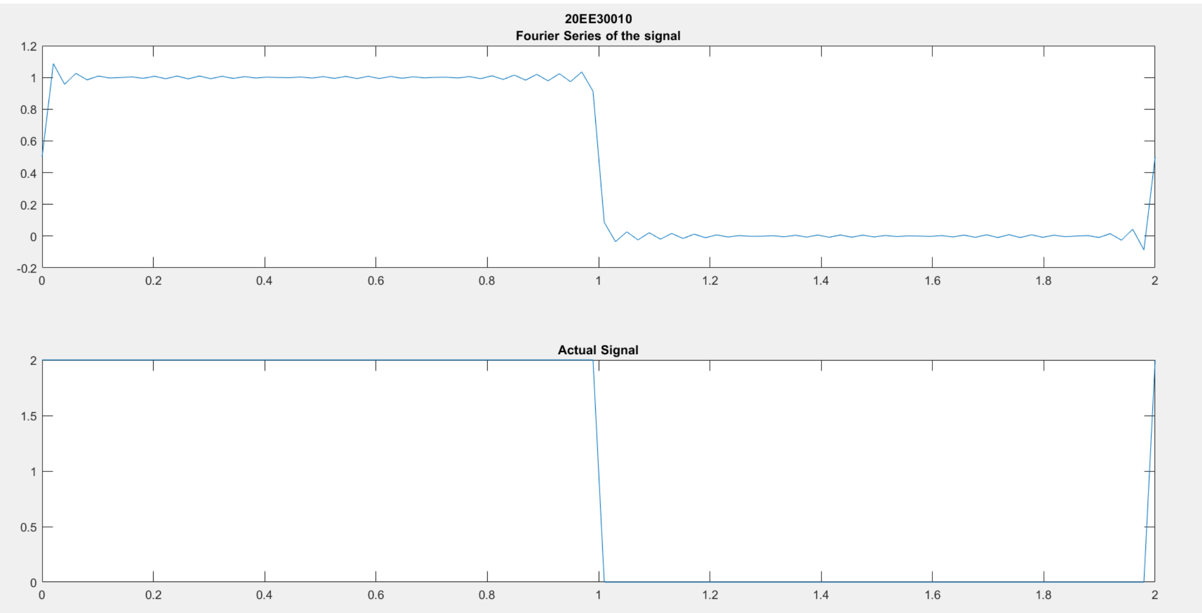
```
function [y,phi]=part1(k) % put the coefficients together s(t)=rect(t-0.5)

    f0=0.5;

    a=@(t) exp(-(1i*2*pi*f0*k*t)).*((square(t*pi)+1));
    y=f0*0.5*integral(a,0,1/f0);

    if(k==0)
        phi=angle(y)+pi/2;
        y=abs(y);
    else
        phi=angle(y)+pi/2;
        y=2*abs(y);
    end

end
```

```
function [y,phi]=p11(k) % put the coefficients together s(t)=rect(2t)-0.5

    f0=1;

    a=@(t) exp(-(1i*2*pi*f0*k*t)).*(square((t+0.25)*2*pi));
    y=f0*0.5*integral(a,0,1/f0);

    if(k==0)
        phi=angle(y)+pi/2;
        y=abs(y);
    else
        phi=angle(y)+pi/2;
        y=2*abs(y);
    end
end
```

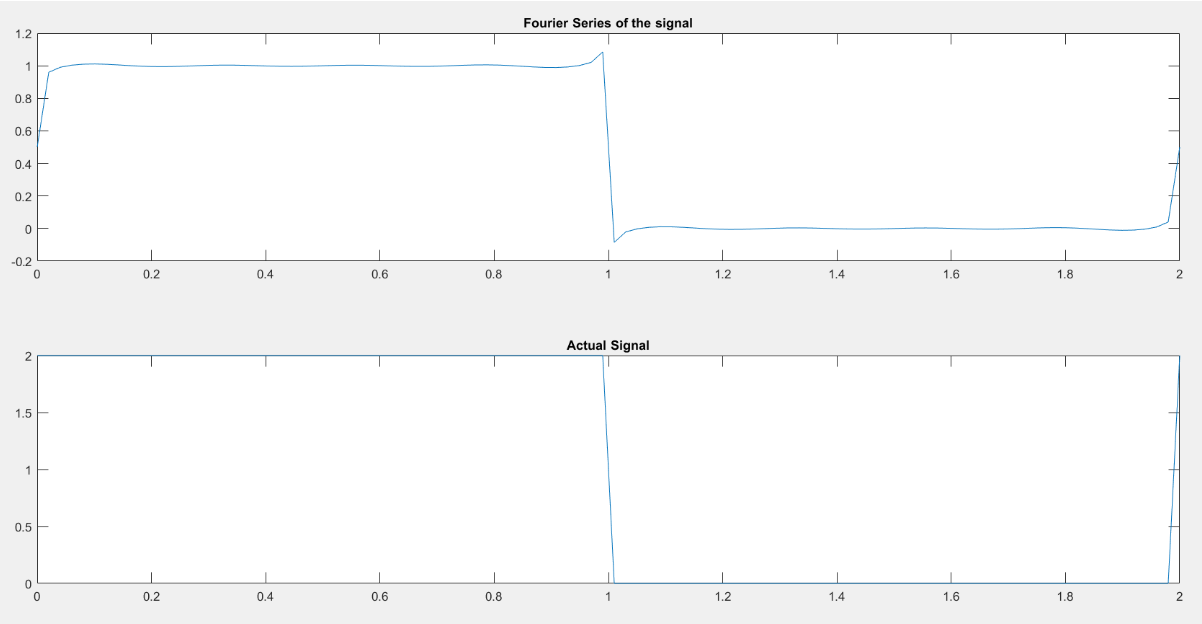
```
function p1(k)
    t=linspace(0,2,100);
    y1=square(t*pi)+1;

    y=zeros(1,length(t));
    p=0:k;
    phi=zeros(1,length(p));
    a=zeros(1,length(p));
    l=zeros(1,length(p));
    for i=0:k
        [a(i+1),phi(i+1)]=part1(i);

    end

    for i=1:length(t)
        for j=0:k

            l=sin((2*pi*j*t(i)*0.5)+phi(j+1));
            y(i)=y(i)+a(j+1)*l;
        end

    end
    tiledlayout(2,1)
    nexttile
    plot(t,y);
    title("Fourier Series of the signal")
    nexttile
    plot(t,y1);
    title("Actual Signal")
end
```

Plots:

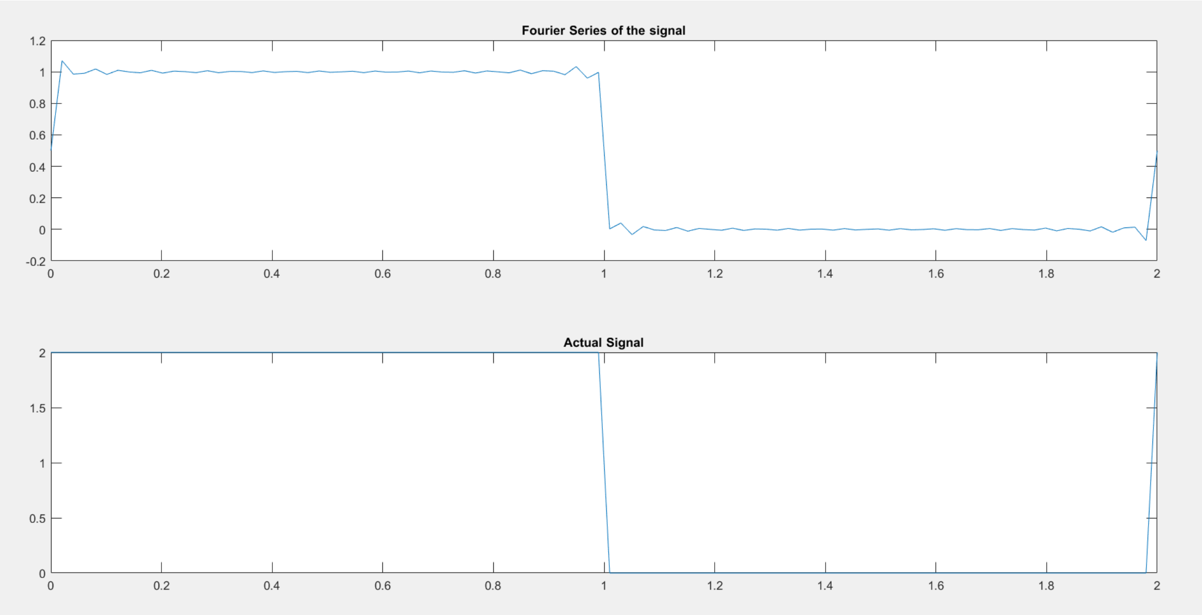
For k = 45



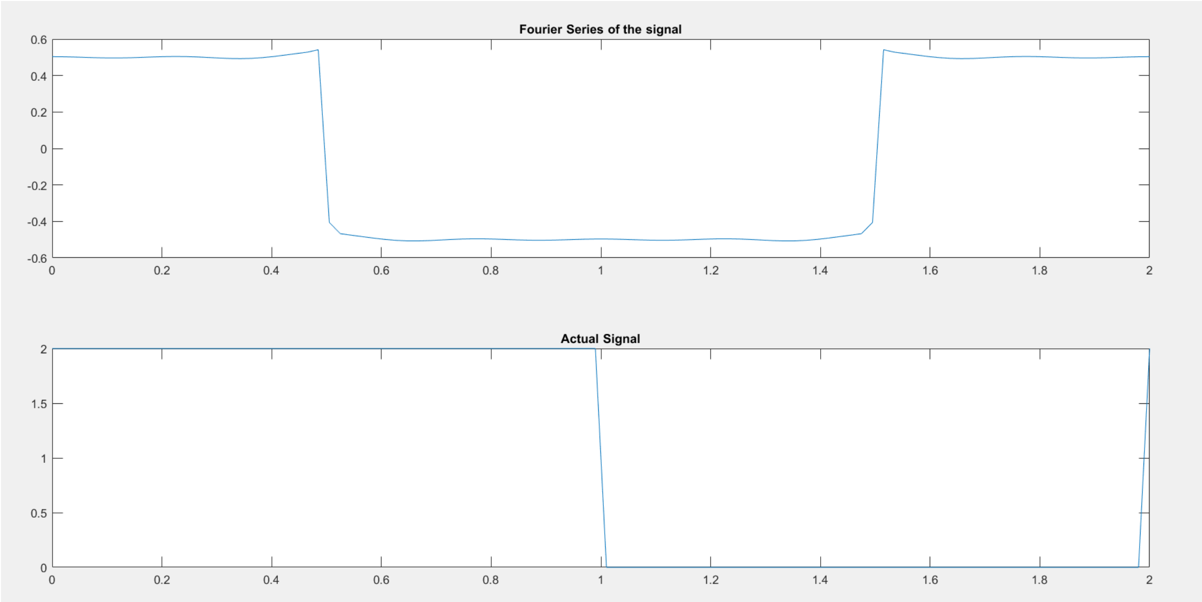
For k = 90



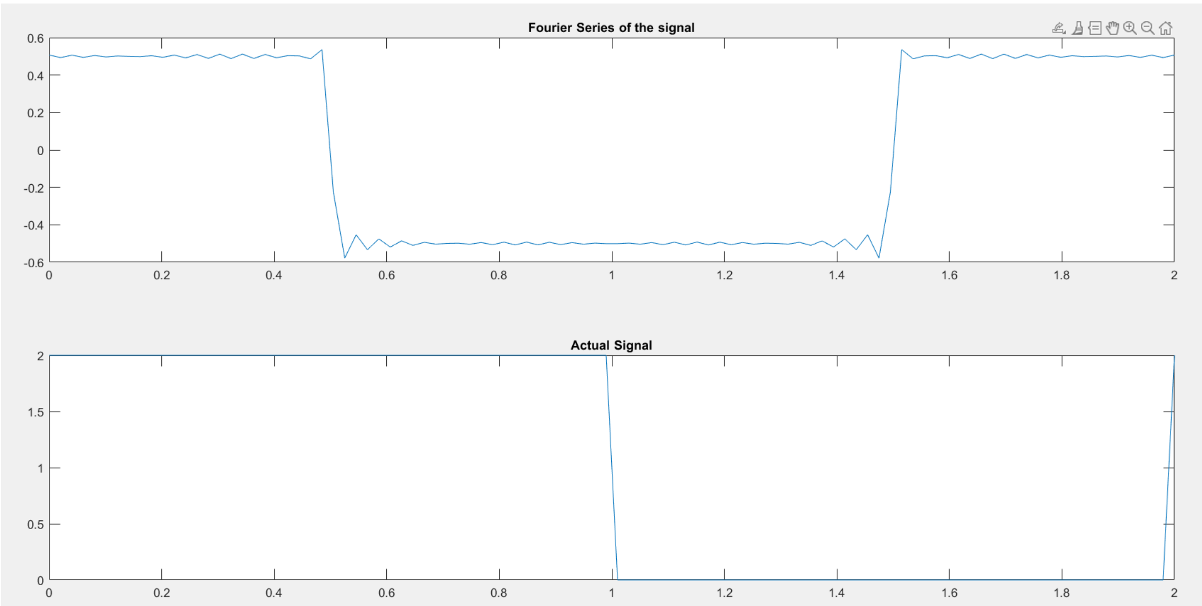
For k = 60



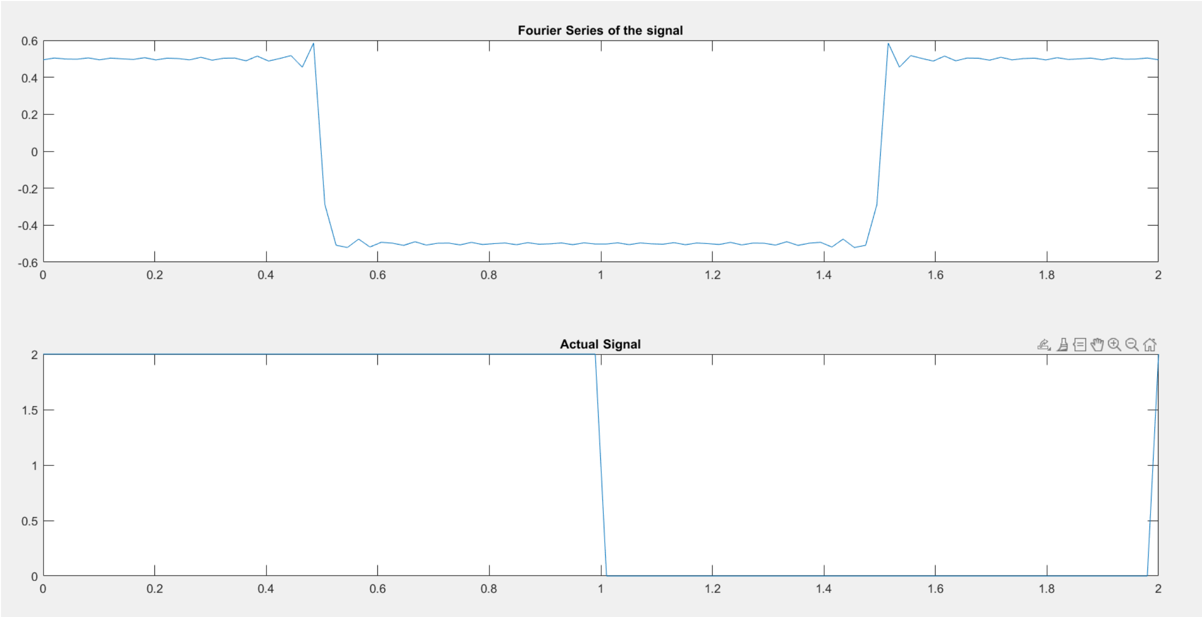
For k = 90



For k = 45



For k = 60



Part 2

Discrete-Time Frequency Analysis

2.1 Discrete-Time Fourier Transform

For the following signals use your DTFT function to (i) Compute X(w) , and (ii) Plot the magnitude and the phase of X(w) in a single plot using the subplot command. Hint: Use the abs() and angle() commands.

1. $x[n] = \delta[n]$
2. $x[n] = \delta[n-5]$
3. $x[n] = (0.5)^n u[n]$

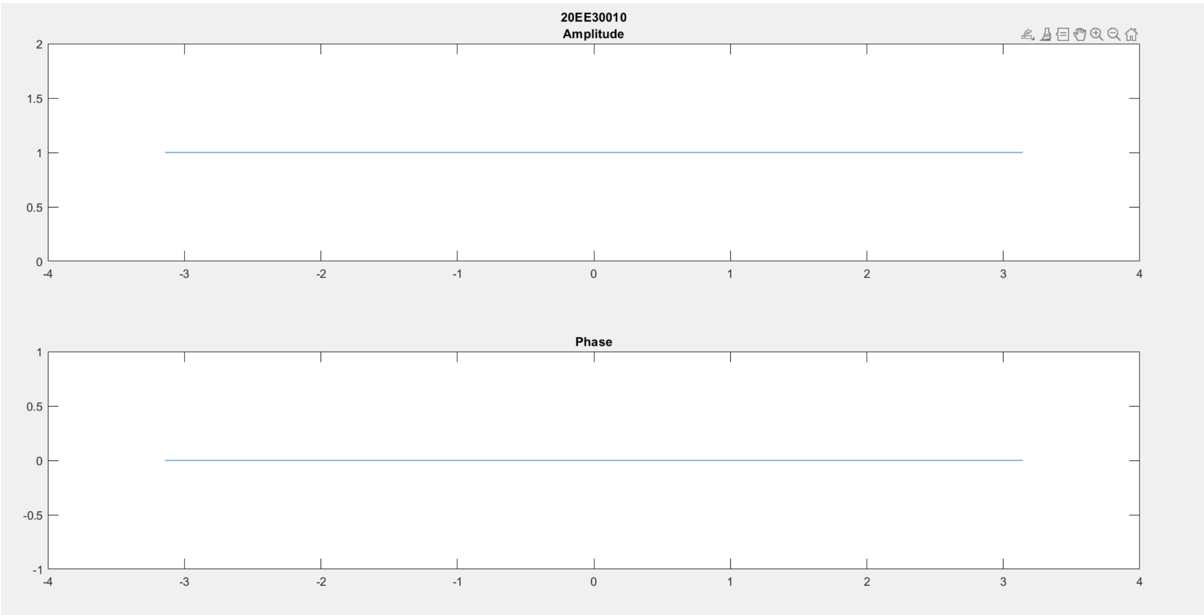
Hand in a printout of your Matlab function. Also, hand in plots of the DTFT's magnitude and phase for each of the three signals

Code:

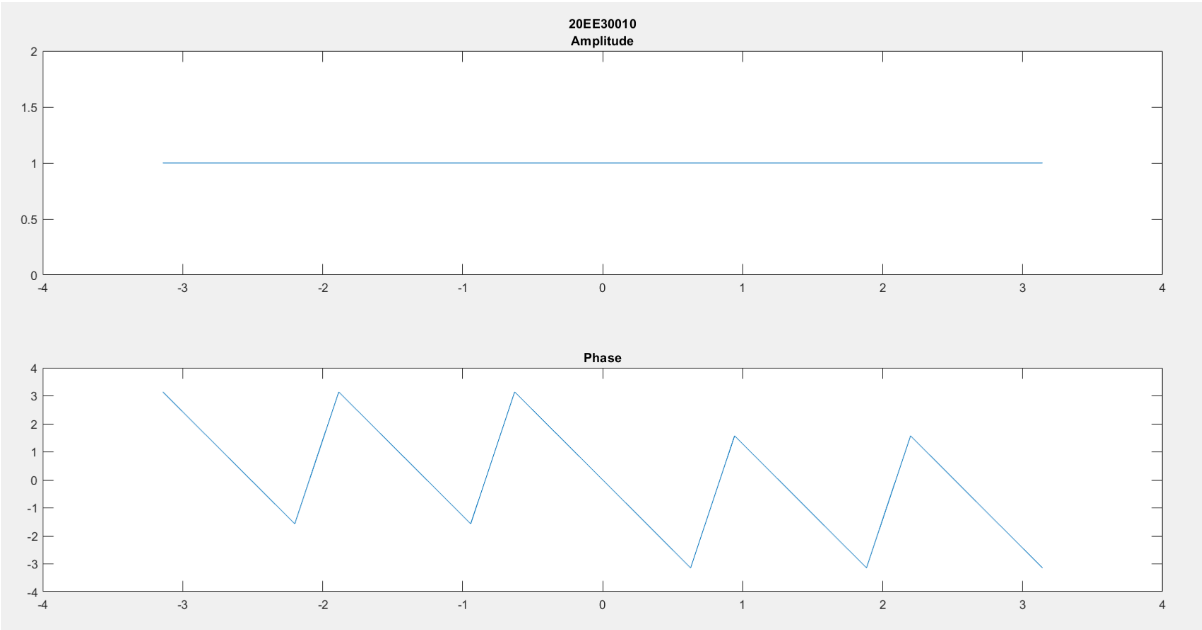
```
function X=p2(x,n0,dw)

    w=-pi:dw:pi;
    X=zeros(1,length(w));

    for p=1:length(w)
        for j =1:length(x)
            X(p)=X(p)+x(j)*exp(-(j+n0-1)*(1i*w(p)));
        end
    end


    tiledlayout(2,1)
    nexttile
    plot(w,abs(X));
    title("Amplitude")
    nexttile
    plot(w,angle(X));
    title("Phase")

end
```

```
function y=p3() %script to generate the Amplitude and Phase plot
    k=-10:50;
    x = 2* ((1/2).^k).*heaviside(k);

    X=p2(x,-10,pi/10);

end
```

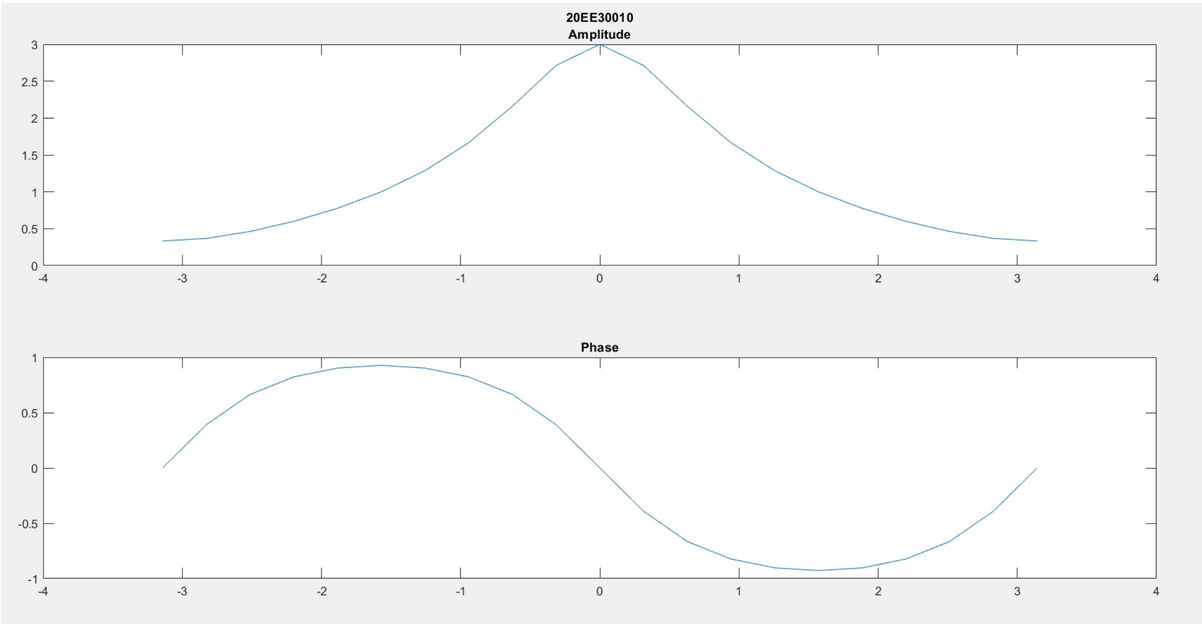
1.)



2.)



3.)



2.2 Magnitude and Phase of the Frequency Response of a Discrete Time Systems

Consider the discrete-time system described by the following difference equation:
y[n]= 0.9y[n-1] + 0.3x[n] + 0.24x[n-1]

Consider the system to be causal.
i. Create a system flowchart.
ii. In the equation above, swap out x[n] for [n] to get the system's impulse response.

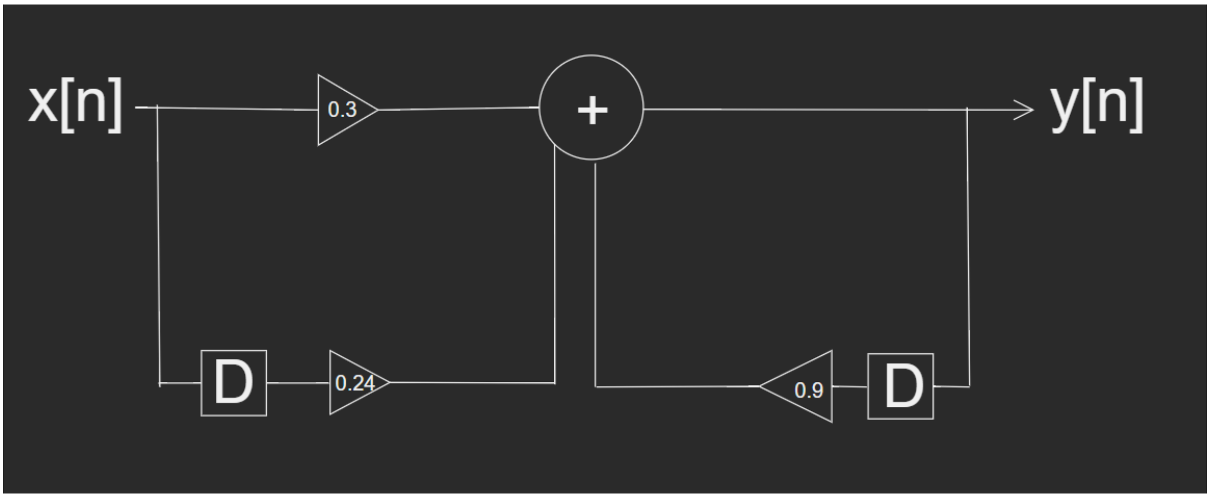
(Set up the initial conditions using causality.)

iii. To find the system's frequency response, use your response from (ii).

iv. Use a different approach to determine the system's frequency response.Specifically, take the DTFT of the left-hand-side and right-hand-side of the difference equation, and then use linearity and the time-shifting property of the DTFT along with the fact that

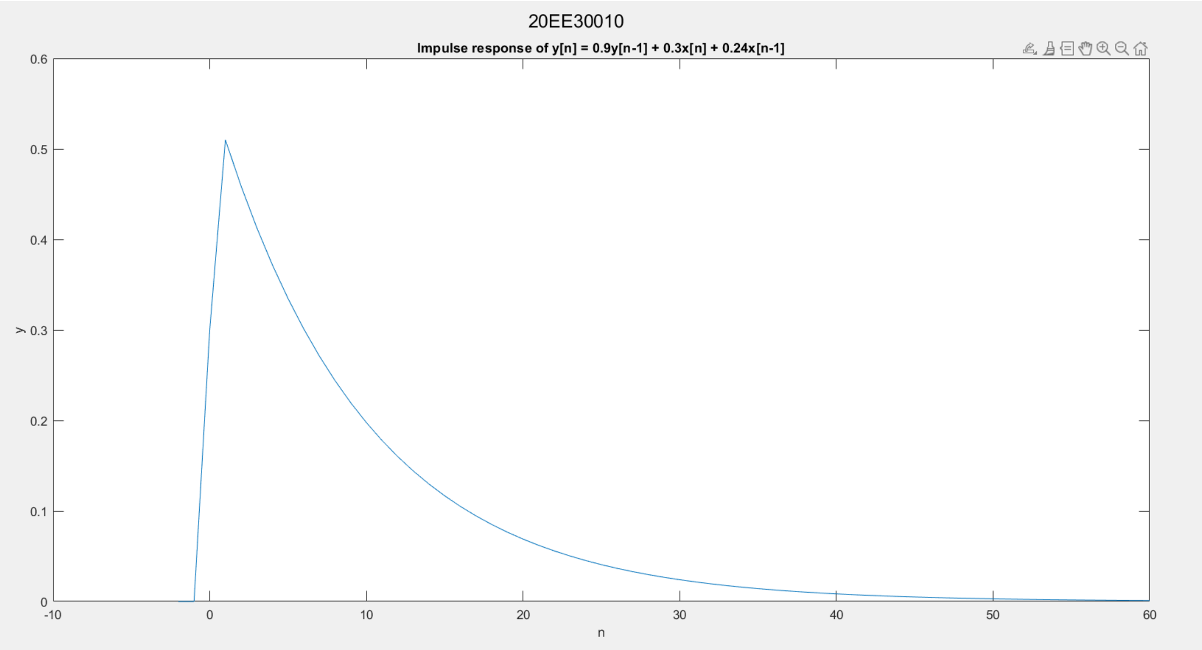
$$\mathrm{H(w)} = \frac{Y(w)}{X(w)}$$

Submit these exercises with the lab report

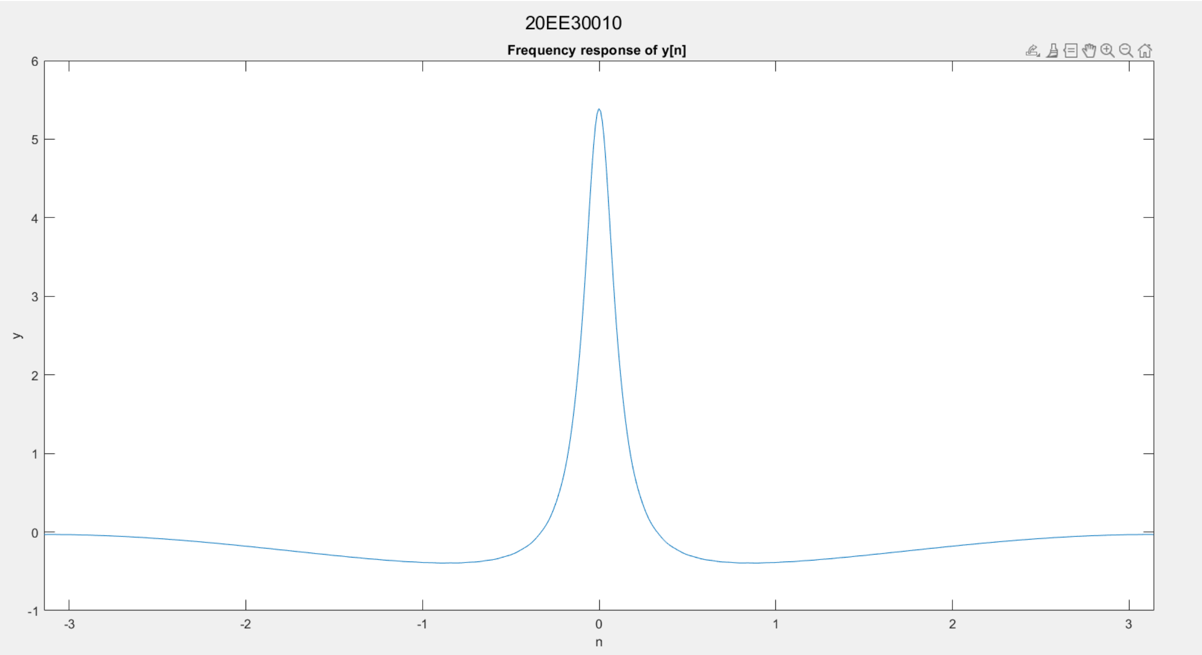


Code :

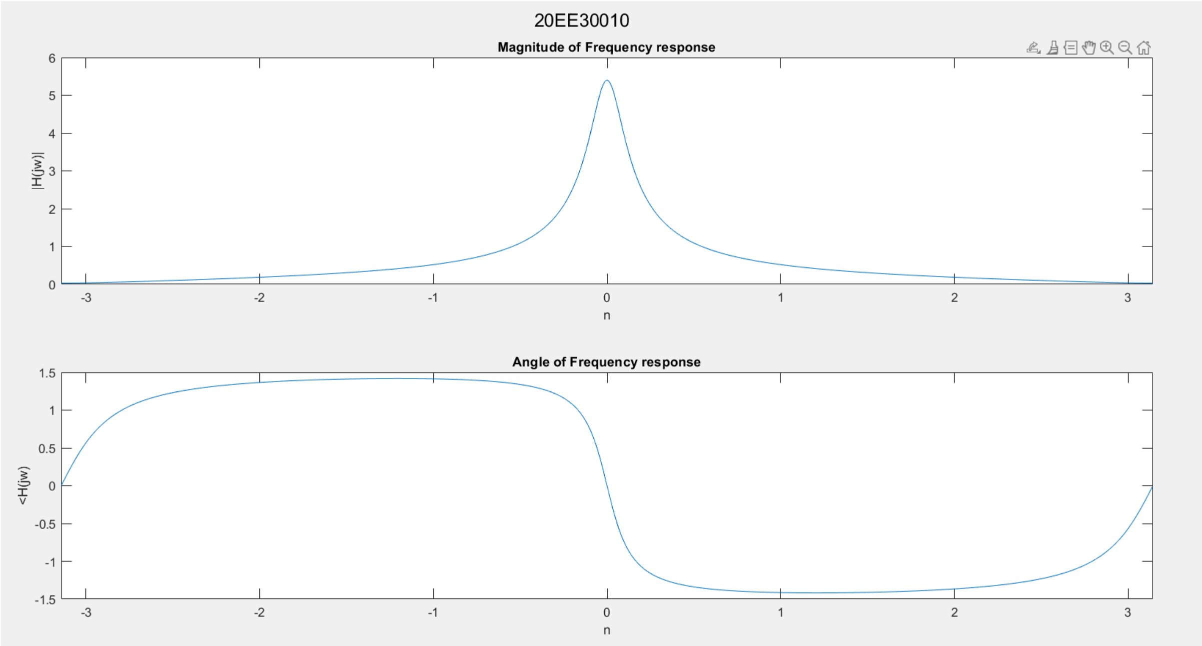
```
n = -2:1:60;


x = @(n) n==0;
% system causal so y(i) = 0 for i < 0
y = zeros(1,length(n));


for i = 2:length(n)
y(i) = 0.9.*y(i-1) + 0.3.*x(n(i)) + 0.24.*x(n(i-1));
end


subplot(1,1,1);
plot(n,y,”r-o"LineWidth",2);
title("Impulse response of y[n] = 0.9y[n-1] + 0.3x[n] + 0.24x[n-1]");
```




```
n = -2:1:60;
% for impulse response we have x[n] = delta[n]
x = @(n) n==0;
y = zeros(1,length(n));


for i = 2:length(n)
y(i) = 0.9.*y(i-1) + 0.3.*x(n(i)) + 0.24.*x(n(i-1));
end
y = y(1,2:end);


dw = 0.01;
w = -pi:dw:pi;
y1 = DTFT(y,1,dw);

% plotting the Frequency response
subplot(1,1,1);
plot(w,y1);
title("Frequency response of y[n] ");

xlim([-pi pi]);



function X = DTFT(x, no, dw)
w=-pi:dw:pi;
[~,s] = size(w);
X = zeros(1,s);
n = 0:length(x);
j = sqrt(-1);

for k=1:length(x)
X=X+x(k).*exp(-j.*w.*(n(k)+no-1));
end
end
```




```
w = -pi:0.01:pi;
n = length(w);
j = sqrt(-1);
num = 0.3 + 0.24.*exp(-j.*w);
den = 1 - 0.9.*exp(-j.*w);
y = num./den;

% plotting the magnitude
subplot(2,1,1);
plot(w,abs(y),"LineWidth",2);
title("Magnitude of Frequency response");
xlim([-pi pi]);

% plotting the angle
subplot(2,1,2);
plot(w,angle(y));
title("Angle of Frequency response");
xlim([-pi pi]);
```



$$y[n] = 0.9\,y[n-1] + 0.3\,x[n] + 0.24\,x[n-1]$$

for impulse response,

$$x[n] = \delta[n]$$

$$h[n] = 0 \quad \forall \quad n<0$$

$$h[0] = (0.9)(0) + 0.3 + (0.24)(0) = 0.3$$

$$h[1] = (0.9)(0.3) + 0.24 = 0.3 \times 1.7$$

$$h[2] = (0.9)(0.3)(1.7)$$

$$h[3] = (0.9)^2(0.3)(1.7)$$

and

$$h[n] = \begin{cases} 0.3 & , n=0 \\ 0.5 & , n=1 \\ (0.9)^{n-1}(0.51) & n \geq 2 \end{cases}$$

given,

$$y[n] = 0.9\,y[n-1] + 0.3x[n] + 0.24\,x[n-1]$$

Taking DTFT,

$$Y(\omega) = 0.9\,Y(\omega)e^{-j\omega} + 0.3\,X(\omega) + 0.24\,X(\omega).e^{-j\omega}$$

$$\Rightarrow Y(\omega)\{1 - 0.9e^{-j\omega}\} = X(\omega)\{0.3 + 0.24e^{-j\omega}\}$$

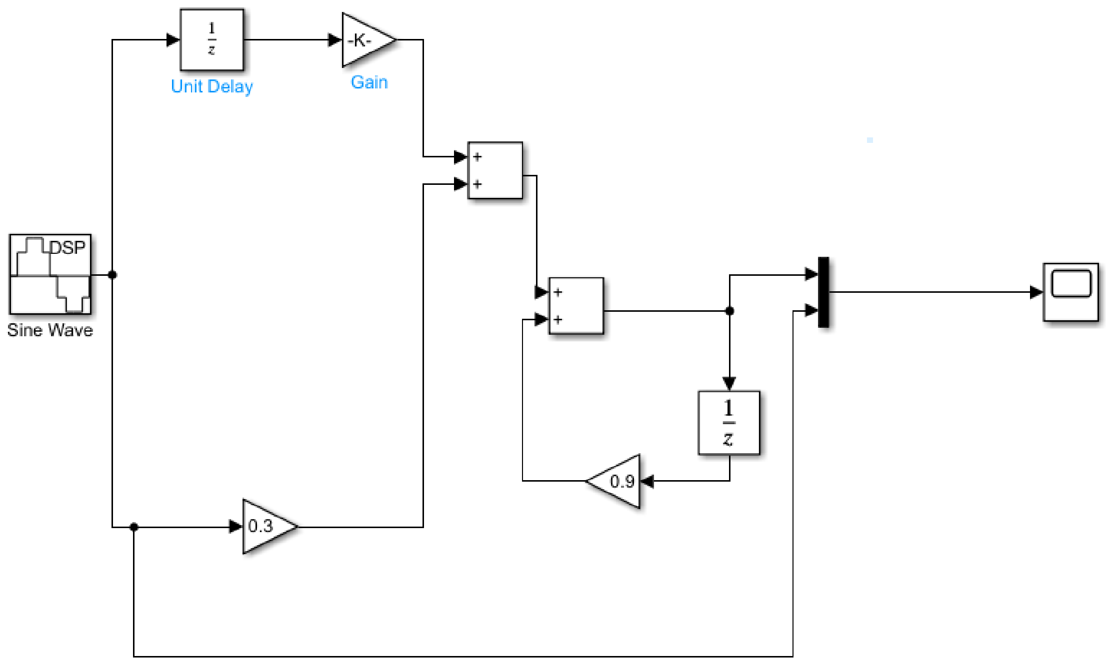
$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{0.3 + 0.24e^{-j\omega}}{-0.9\,e^{-j\omega} + 1}$$

2.3 System Analysis

Hand in the following:

- Figure of your completed block diagram
- Table of both the amplitude measurements you made and their theoretical values.
- Figure with the impulse response, and the magnitude and phase of the frequency response.

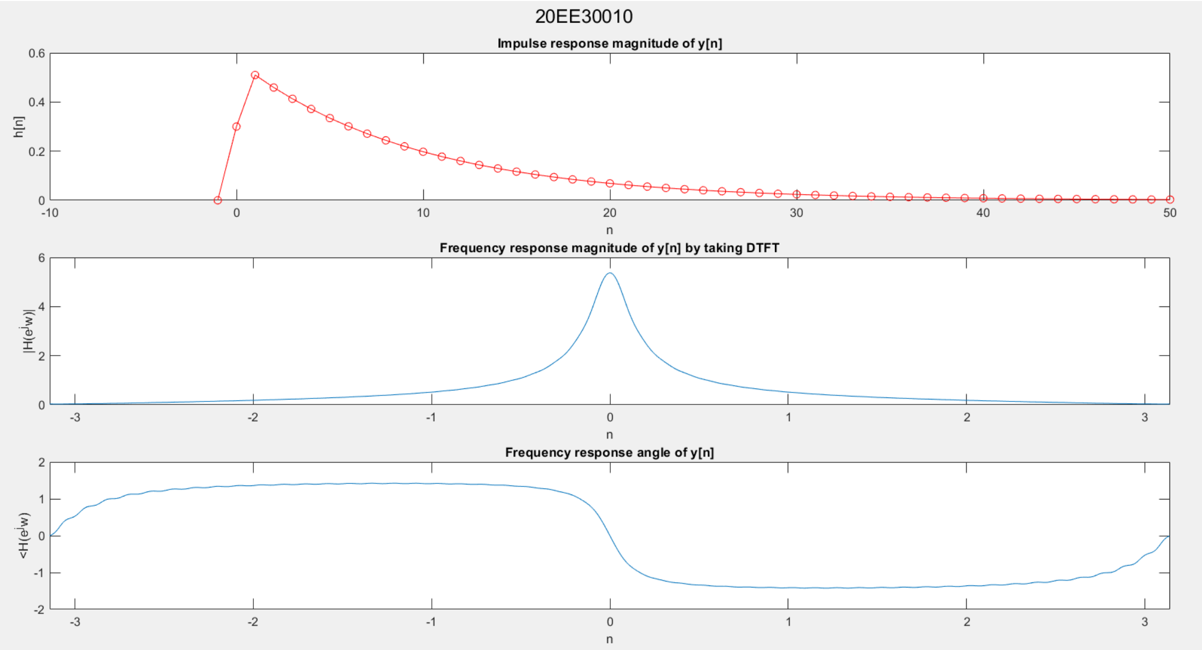
Model:



Sn no.	frequency	Observed amplitude	Calculated amplitude
1	$\pi/16$	2.52	2.545
2	$\pi/8$	1.36	1.381
3	$\pi/4$	0.678	0.6814

Code:

```matlab
n = -1:1:50;
% for impulse response we have x[n] = delta[n]
x = @(n) n==0;
y = zeros(1,length(n));


for i = 2:length(n)
y(i) = 0.9.*y(i-1) + 0.3.*x(n(i)) + 0.24.*x(n(i-1));
end

% plotting impulse response
subplot(3,1,1);
plot(n,y,"r-o");
title("Impulse response magnitude of y[n]");
xlabel("n");
ylabel("h[n]");

y = y(2:end);
% finding DTFT of the impulse response
dw = 0.01;
w = -pi:dw:pi;
y1 = DTFT(y,1,dw);

% plotting the Frequency response magnitude and angle
subplot(3,1,2);
plot(w,abs(y1));
xlim([-pi pi]);
subplot(3,1,3);
plot(w,angle(y1));
title("Frequency response angle of y[n]");

% function to calculate the DTFT of the signals
function X = DTFT(x, no, dw)
w=-pi:dw:pi;
[~,s] = size(w);
X = zeros(1,s);
n = 0:length(x);
j = sqrt(-1);

for k=1:length(x)
 X=X+x(k).*exp(-j.*w.*(n(k)+no-1));
 end
end
```



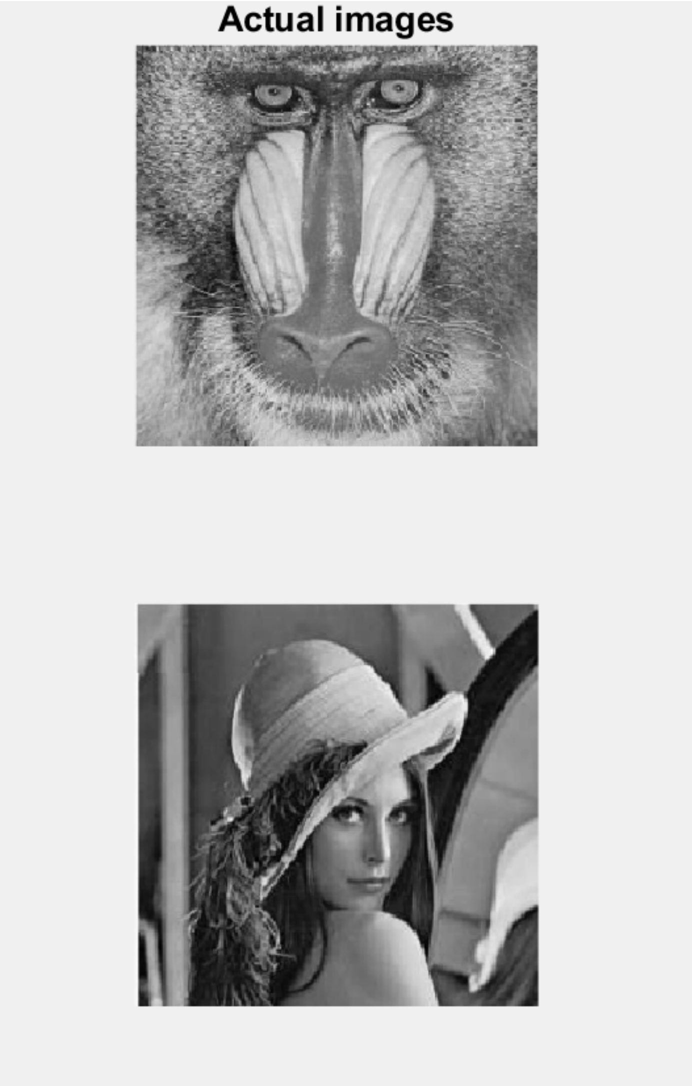
Part 3

Additional Tasks

3.1 Pick two appropriate pictures (256x256 resolution).From their frequency response, extrapolate the magnitude and phase.To create two new images, switch the phase components of the two frequency responses.

Code:

```matlab
% Read the images
PIC1=double(imread('p1.png'))/255;
PIC2=double(imread('p2.png'))/255;

% converting RGB to gray
Image2 = rgb2gray(PIC2);

figure(1);
subplot(2,1,1);
imshow(PIC1, []);
title("Actual images", "FontSize",16);
subplot(2,1,2);
imshow(Image2, []);


% Find dimensions and extent of the FFT
[rows1, cols1] = size(PIC1);
[rows2, cols2] = size(Image2);
rows = max(rows1, rows2);
cols = max(cols1, cols2);

% Take the FFT
PIC1_FFT=fft2(PIC1, rows, cols);
Image2_FFT=fft2(Image2, rows, cols);

% Find the magnitudes and phase responses
mag1 = abs(PIC1_FFT);
mag2 = abs(Image2_FFT);
phase1 = angle(PIC1_FFT);
phase2 = angle(Image2_FFT);

% Recompute frequency responses by swapping the phases
out1 = mag1 .* exp(j*phase2);
out2 = mag2 .* exp(j*phase1);

% Find the inverse images
out1 = real(ifft2(out1));
out2 = real(ifft2(out2));

% Show the images
figure(2);
subplot(2,1,1);
imshow(out1, []);
title("images following phase switching and reconstruction");
subplot(2,1,2);
imshow(out2, []);
sgtitle("20EE30010");
```



3.2 **Analysis of Speech Acoustics and Spectrogram Understanding**

3.2.1 **Vowel Synthesis**

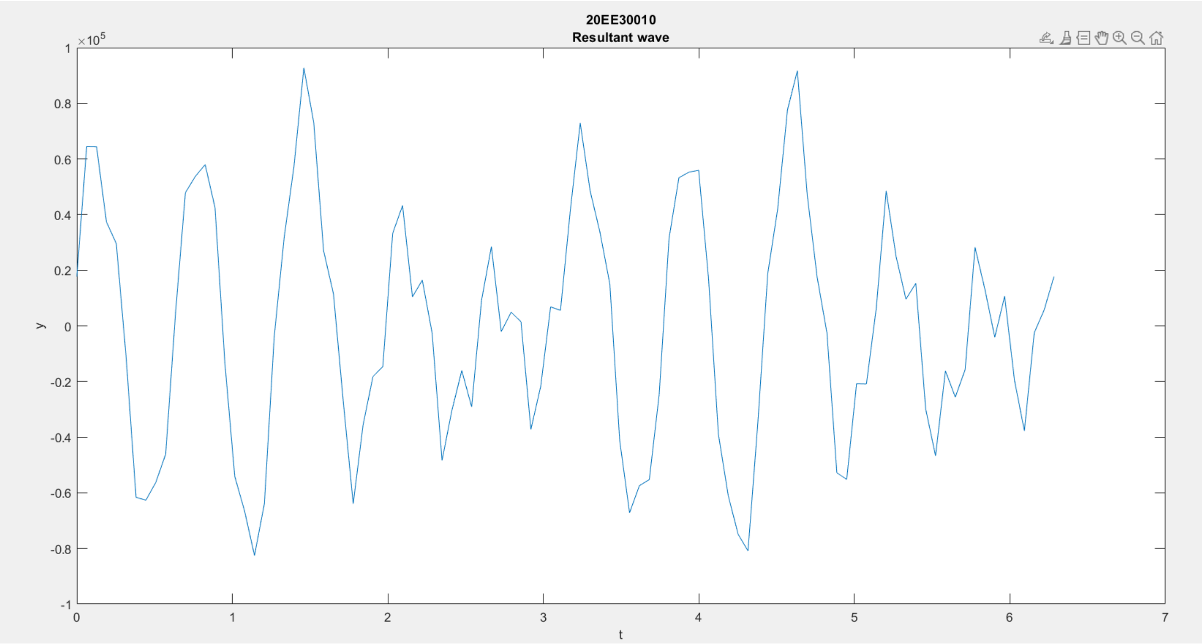
Create a synopsis of the corresponding signals using the provided magnitude and phase charts.

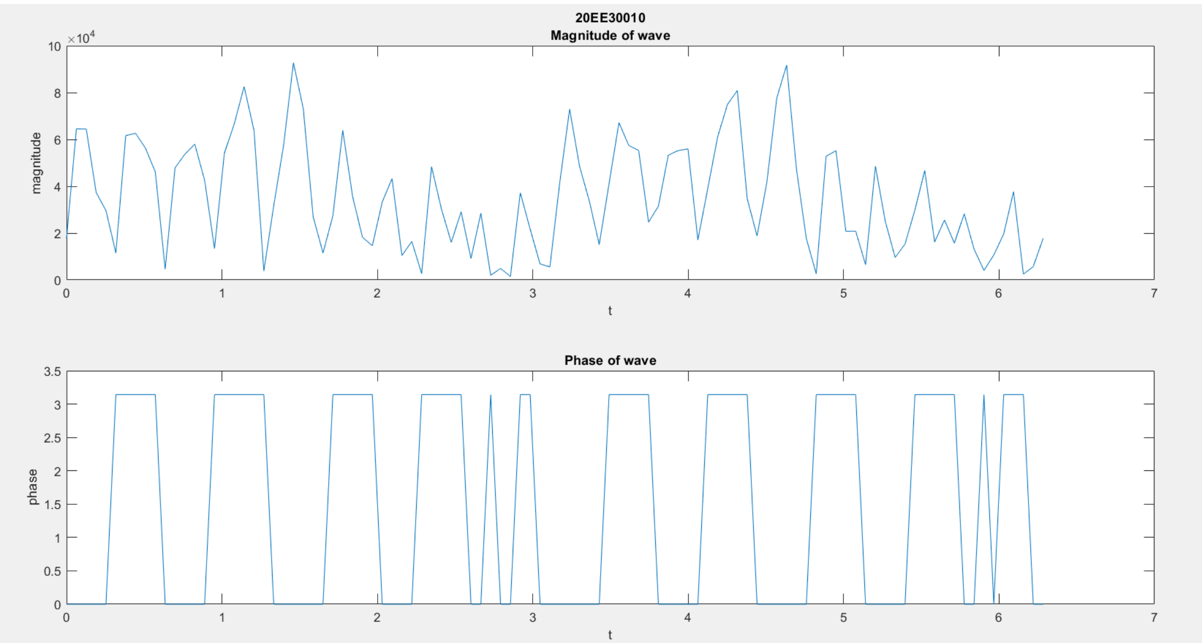
f_k (Hz)	X_k	Mag	Phase (rad)
200	$(771 + j12202)$	12,226	1.508
400	$(-8865 + j28048)$	29,416	1.876
500	$(48001 - j8995)$	48,836	-0.185
1600	$(1657 - j13520)$	13,621	-1.449
1700	$4723 + j0$	4723	0

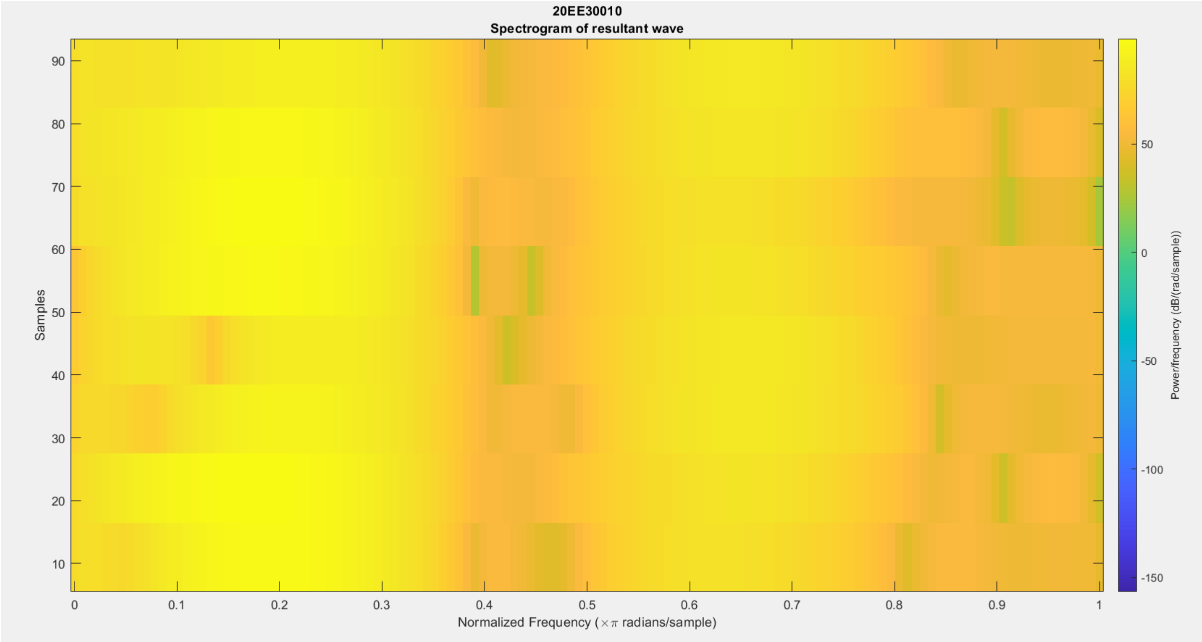
Draw a plot and a picture of the resulting waveform. Plot the vowel's magnitude and phase spectrum. Plot the spectrogram for the speech signal seen above and display

Code:

```

f1 = 200; phi1 = 1.508;
f2 = 400; phi2 = 1.876;
f3 = 500; phi3 = -0.185;
f4 = 1600; phi4 = -1.449;
f5 = 1700; phi5 = 0;

% individual freq components given
t = linspace(0,2.*pi,100);
y1 = 12226.*sin(2.*f1.*t+phi1);
y2 = 29416.*sin(2.*f2.*t+phi2);
y3 = 48836.*sin(2.*f3.*t+phi3);
y4 = 13621.*sin(2.*f4.*t+phi4);
y5 = 4273.*sin(2.*f5.*t+phi5);

y6 = y1+y2+y3+y4+y5;
figure(1);
subplot(1,1,1);
plot(t,y6);
xlabel("t");
ylabel("y");

figure(2);
subplot(2,1,1);
plot(t,abs(y6));

subplot(2,1,2);
plot(t,angle(y6));


figure(3);
spectrogram(y6);

```


20EE30010
Resultant wave
y
t


20EE30010
Magnitude of wave
magnitude
t
Phase of wave
phase
t

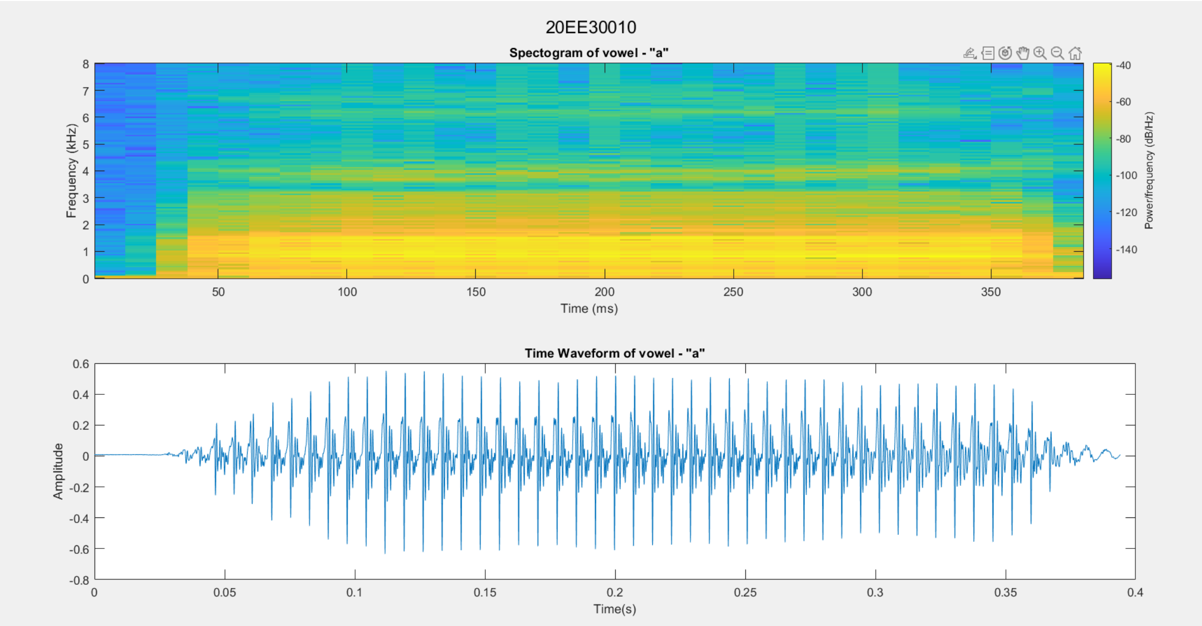


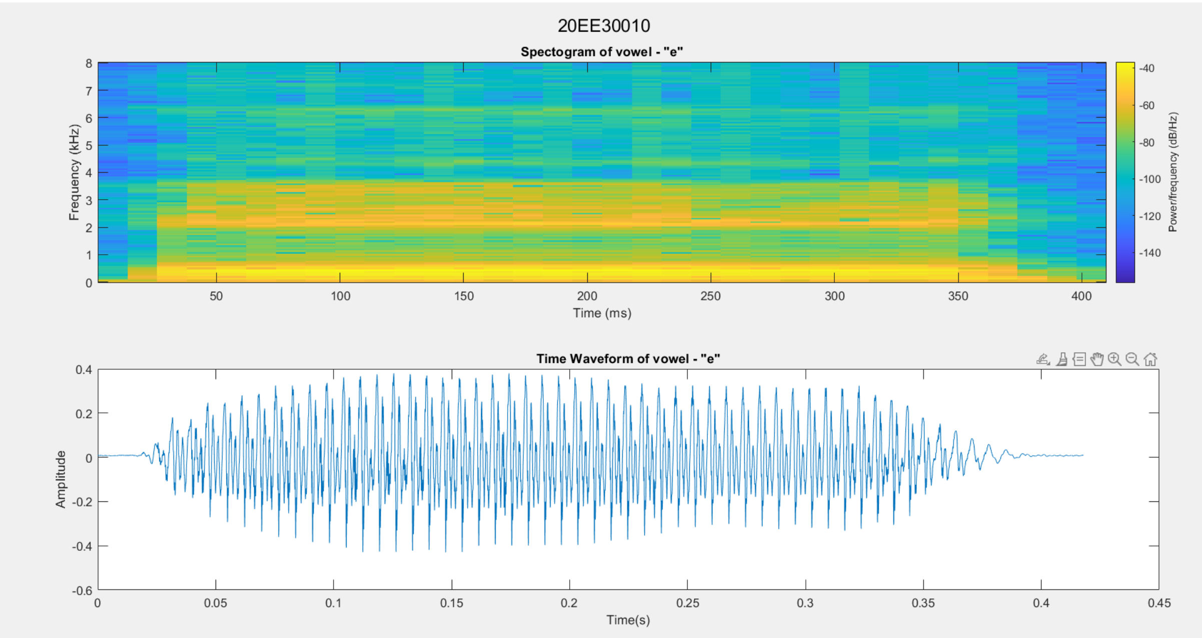
3.2.2

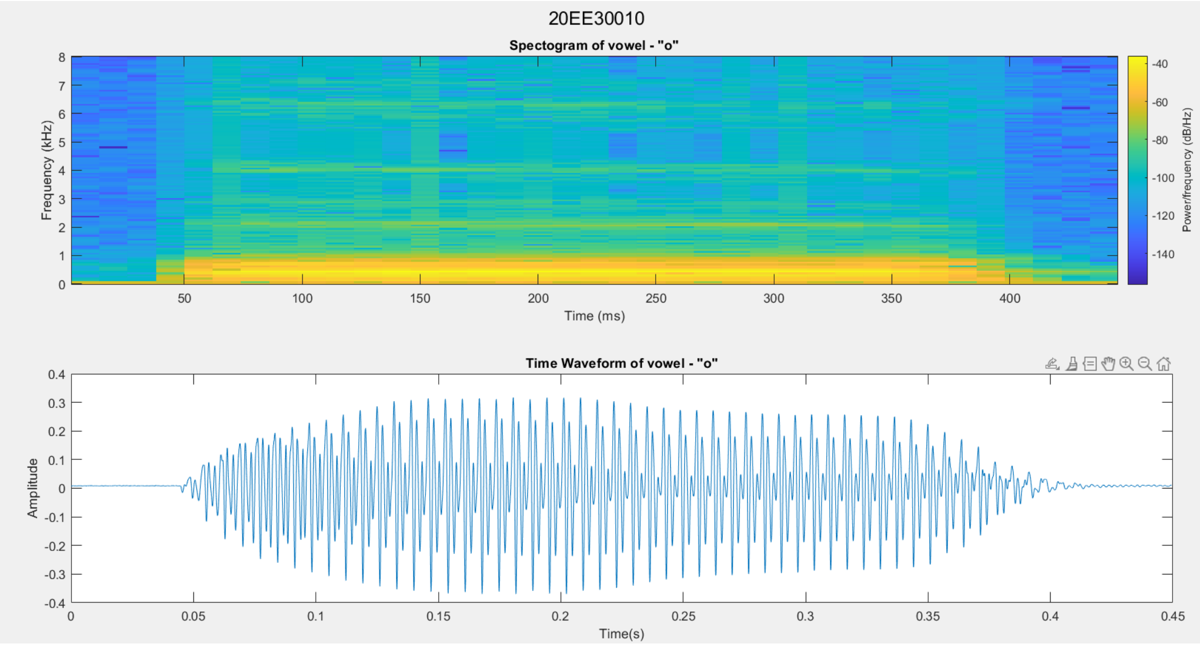
The voice of three vowels, a consonant, and a fricative sound should be recorded. You should then view the voice's time waveform and spectrogram and remark on your findings. Record your name, view its spectrogram, and provide comments on the many types of sounds that are present in the waveform.

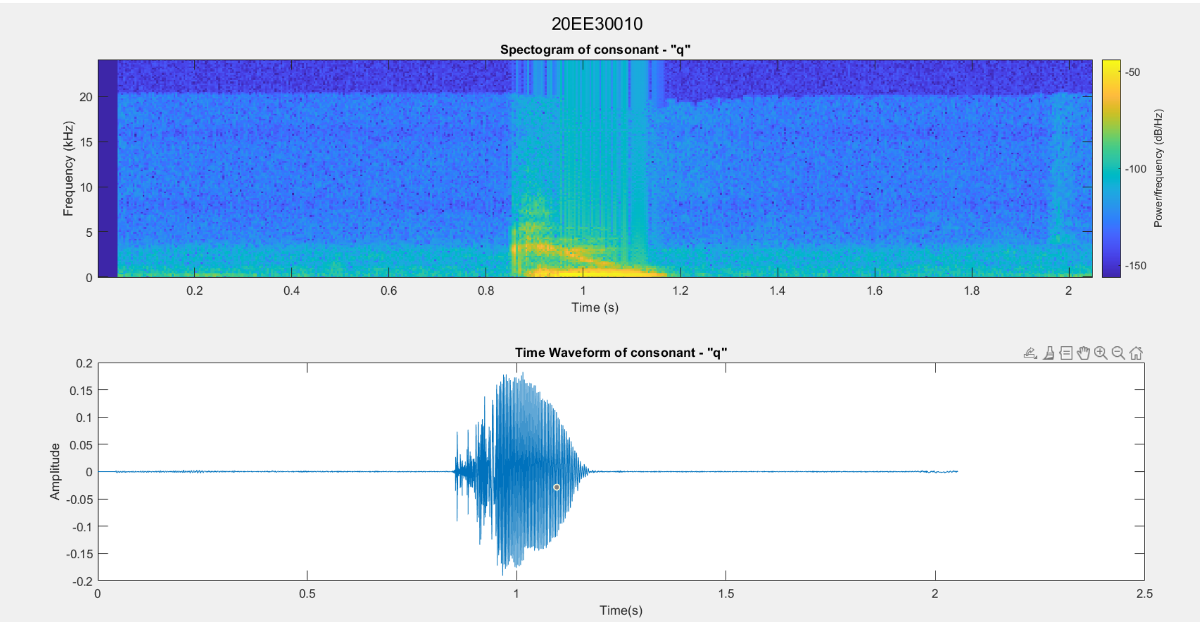
Code:

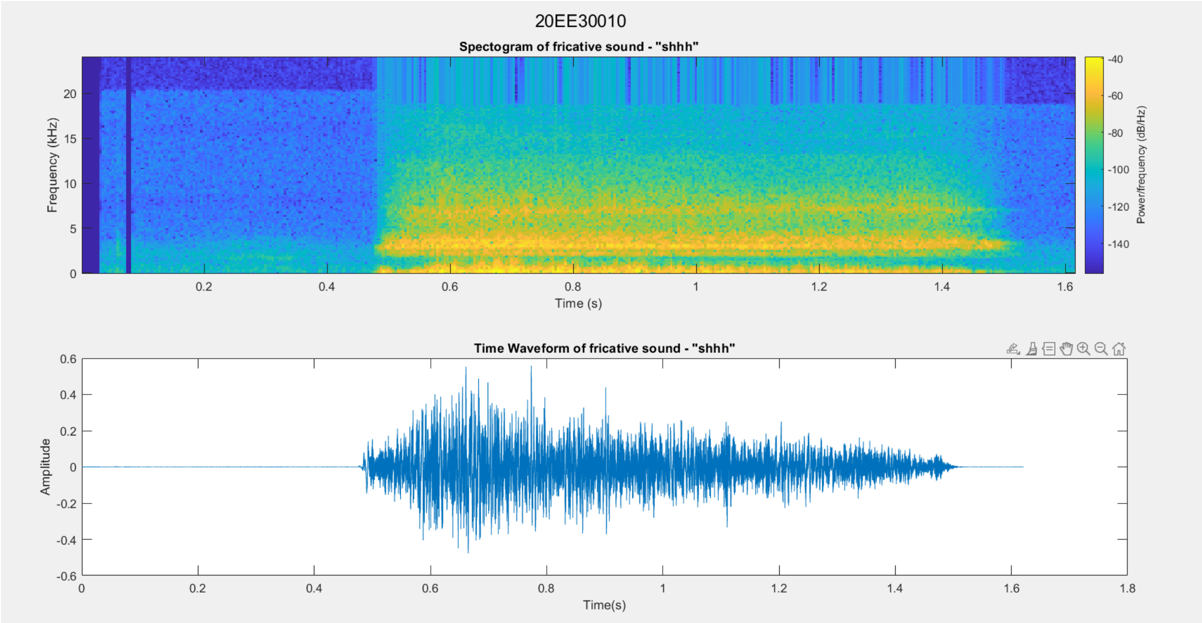
```
% reading vowels
[y1,Fs1]= audioread('a.wav');
[y2,Fs2]= audioread('e.wav');
[y3,Fs3]= audioread('o.wav');
windowsize=256;
% reading a consonant
[y4,Fs4]= audioread('q.mp3');
% reading a fricative sound
[y5,Fs5]= audioread('shhh.mp3');
% reading my name
[y6,Fs6]= audioread('name.mp3');
% plotting time waveforms and spectograms
figure(1);
subplot(2,1,1);
spectrogram(y1(:,1),windowsize,windowsize/4,[],Fs1,'yaxis');
% imagesc(T,F, db(S,40));
title('Spectogram of vowel - "a"');
subplot(2,1,2);
N = size(y1,1);
t = (0:N-1)/Fs1;
plot(t, y1);
xlabel('Time(s)');
ylabel('Amplitude');
title('Time Waveform of vowel - "a"');
sgtitle("20EE30010");
figure(2);
subplot(2,1,1);
spectrogram(y2(:,1),windowsize,windowsize/4,[],Fs2,'yaxis');
title('Spectogram of vowel - "e"');
subplot(2,1,2);
N = size(y2,1);
t = (0:N-1)/Fs2;
plot(t, y2);
xlabel('Time(s)');
ylabel('Amplitude');
title('Time Waveform of vowel - "e"');
sgtitle("20EE30010");
figure(3);
subplot(2,1,1);
spectrogram(y3(:,1),windowsize,windowsize/4,[],Fs3,'yaxis');
title('Spectogram of vowel - "o"');
subplot(2,1,2);
N = size(y3,1);
t = (0:N-1)/Fs3;
plot(t, y3);
xlabel('Time(s)');
ylabel('Amplitude');
title('Time Waveform of vowel - "o"');
sgtitle("20EE30010");
figure(4);
subplot(2,1,1);
spectrogram(y4(:,1),windowsize,windowsize/4,[],Fs4,'yaxis');
title('Spectogram of consonant - "q"');
subplot(2,1,2);
N = size(y4,1);
t = (0:N-1)/Fs4;
plot(t, y4);
xlabel('Time(s)');
ylabel('Amplitude');
title('Time Waveform of consonant - "q"');
sgtitle("20EE30010");
figure(5);
subplot(2,1,1);
spectrogram(y5(:,1),windowsize,windowsize/4,[],Fs5,'yaxis');
title('Spectogram of fricative sound - "shhh"');
subplot(2,1,2);
N = size(y5,1);
t = (0:N-1)/Fs5;
plot(t, y5);
xlabel('Time(s)');
ylabel('Amplitude');
title('Time Waveform of fricative sound - "shhh"');
sgtitle("20EE30010");
figure(6);
subplot(1,1,1);
spectrogram(y6(:,1),windowsize,windowsize/4,[],Fs6,'yaxis');
title('Spectogram of my name',"FontSize", 16);
sgtitle("20EE30010");
```

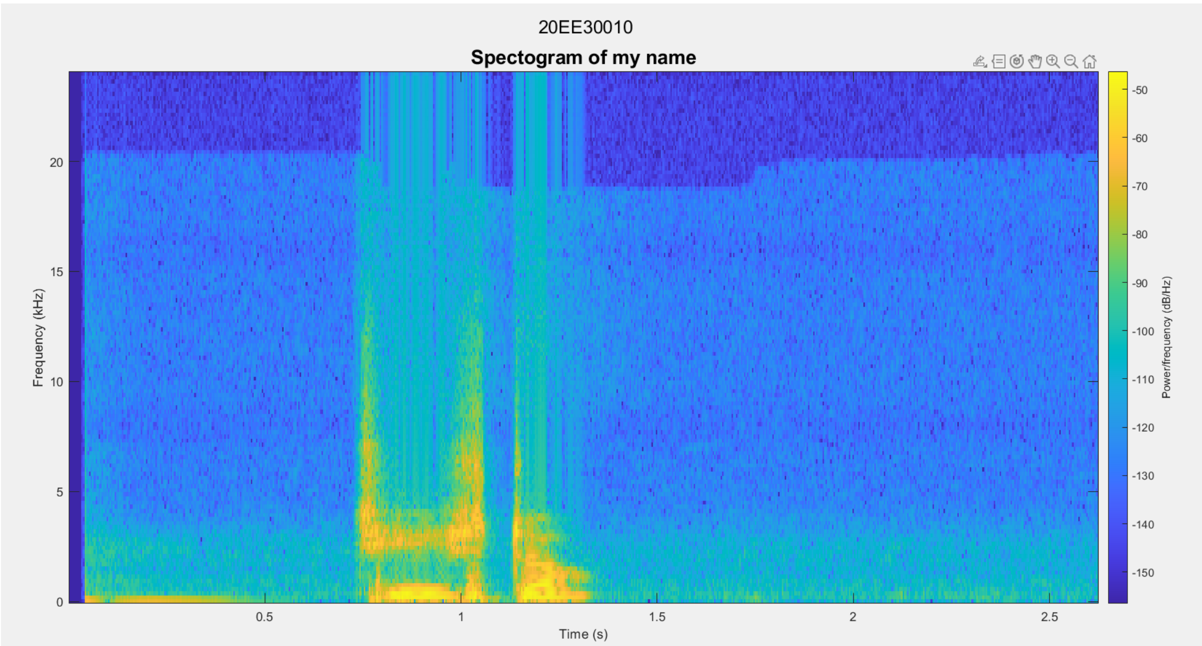

20EE30010
Spectogram of vowel - "a"
Frequency (kHz)
Time (ms)
Power/frequency (dB/Hz)
Time Waveform of vowel - "a"
Amplitude
Time(s)


20EE30010
Spectogram of vowel - "e"
Frequency (kHz)
Time (ms)
Power/frequency (dB/Hz)
Time Waveform of vowel - "e"
Amplitude
Time(s)


20EE30010
Spectogram of vowel - "o"
Frequency (kHz)
Time (ms)
Power/frequency (dB/Hz)
Time Waveform of vowel - "o"
Amplitude
Time(s)


20EE30010
Spectogram of consonant - "q"
Frequency (kHz)
Time (s)
Power/frequency (dB/Hz)
Time Waveform of consonant - "q"
Amplitude
Time(s)





Comments:
The turbulent airstream of fricatives produces a chaotic mix of random frequencies, each lasting for a very limited period of time, in contrast to the often very well defined formant bars of vowels. The end result has a sound that is remarkably similar to static noise, and when viewed on a spectrogram, it resembles the static noise that we may hear on a television.
As the tongue body passes through the mouth during diphthongs, we may observe the formants changing in frequency. Complete silence occurs during the mediaeval period of a voiceless plosive. This shows up as a white blank on a spectrogram.
Nasals typically resemble very little, low-amplitude vowels at the higher frequencies. We can easily see that my name has a variety of vowels and consonants in the spectrogram it has generated. For instance, it has formant bars that show vowel sounds are present in the waveform.