Patrick Augustin
MGMTMFE-431-1 Credit Markets
Fall 2023

Structural Models of Credit Risk: Problems

1. (Merton Model) Use the following parameters to answer the questions, where V_0 denotes the value of firm assets as of time $t = 0$, μ denotes the expected return on firm assets, σ_V defines asset volatility, T is the maturity of the face value of debt, dt defines the time interval of the model and N denotes the nominal face value of debt.

$$
\begin{gathered}
V_0 = 100, \\
\mu = 9\%, \\
\sigma_V = 40\%, \\
T = 5 \text{ years}, \\
dt = 1/12 \text{ years}, \\
N = 75.
\end{gathered}
$$

   (a) Simulate seven scenarios for the asset value process (i.e., seven paths). Discuss which path results in default according to the definition of default provided in the Merton (1974) model.

   (b) Plot the term structure of credit spreads for a horizon up to 5 years, assuming a flat term structure of interest rates equal to 3% per annum.

   (c) Repeat question (b) for a lower leverage ratio implied by a face value of debt $N = 50$, as well as a lower asset volatility of $\sigma_V = 10\%$ (ceteris paribus). Discuss the shape of the credit spread term structure as a firm improves in credit quality.

   (d) In the Merton's model and previous questions, we have assumed that default is only possible at maturity. Now we add the possibility of bankruptcy before maturity.[1] For simplicity, we assume that bankruptcy happens when the asset value falls below ρN ($\rho = 0.9$), a constant times the face value of debt, before maturity. In this case, bondholders receive payoff equal to the asset value and the firm ceases to exist. We will use Monte Carlo simulation to calculate the price of the bond. The initial zero-coupon bond price can be written as

$$
P_0 = \mathbb{E}_0^{\mathbb{Q}}\left[e^{-r\tau} V_\tau \mathbb{I}_{\{\tau \leq T\}} + e^{-rT} \min\{V_T, N\} \mathbb{I}_{\{\tau > T\}}\right]
$$

   where $\tau = \inf\{t > 0 : V_t = \rho N\}$ is the default (bankruptcy) time. We can estimate the expected value by simulating a lot of sample paths of the underlying asset value before (including) maturity. For each path, we check if the underlying asset value falls below ρN before maturity. If so, calculate the present value of bankruptcy payoff; if not, calculate the present value of payoff at maturity by comparing the asset value at maturity with the face value of debt. As a result, we obtain the present value of payoff for each sample path. Finally, we take the average of these discounted payoffs, which is our estimation of the bond price. Repeat (b) by plotting in one graph the term structure of credit spreads calculated from this extended model using simulations of 1000 paths, together with that from Merton's

[1] This is a simplified version of Black and Cox (1976 JF).

model. What do you observe? Can you explain your observation? Are you satisfied with the simulated credit spread?

(e) (optional) If you are not satisfied with the simulated credit spread in (d), try a larger number of sample paths and smaller time step, e.g. 5000 paths and $dT = 1/120$. Can you re-use the code in (d)? (Hint: how long does it take to run the code?) How can you improve the code to make its computational time acceptable?[2] Then, repeat (d) using 5000 paths and $dT = 1/60$. What do you observe?

2. (Calibrating Asset Value and Asset Volatility) Assume that the continuously compounded default-free riskless interest rate is constant at 0.40% per month. Using the following data of SIFCO Industrial Inc. provided in Table 1, calibrate the firm's asset value and asset volatility:

Table 1: Balance Sheet Information

Year	Short-term debt ($mil.)	Long-term debt ($mil.)
2021	16.334	18.294

(a) Using the monthly stock returns provided in *ps3_q2_data.csv*, compute the monthly stock return volatility.

(b) Using the iterative approach developed by KMV (see also Vassalou and Xing, 2004), estimate the monthly asset value and asset volatility. Assume $T = 12$ months.

(c) Using the estimated monthly asset values, compute the average monthly asset return during the year 2021. Using this average value as the expected asset return, compute the distance to default of Tupperware Brands Corp. at the end of of year 2021. What is the physical probability of the company being in default over the next 12 months (i.e., 1 year horizon)?

3. (Capital Structure Arbitrage) Using the data provided for Boeing (Boeing.csv), answer the following questions given the information on the default intensity λ, the recovery rate R, the time horizon T and the leverage ration $\bar{L}$:

$$\lambda = 0.15,$$
$$R = 0.5,$$
$$T = 5 \text{ years},$$
$$\bar{L} = 0.6.$$

(a) Compute the 5-year CDS spread implied by the CreditGrades (CG) model. Plot the joint time series of the CG-implied CDS spread and the market spread, respectively. Can you identify capital structure arbitrage opportunities during this time period? If so, discuss which prices you think are undervalued or overvalued when considered CDS and equity prices.

[2] Compared with compiled languages, python is notoriously slow. For computationally intensive tasks, there are two common choices to improve the performance: numba or cython. The former is much easier to learn and implement. Check out this tutorial (`https://python-programming.quantecon.org/numba.html`) by Thomas J. Sargent & John Stachurski and this excellent video (`https://www.youtube.com/watch?v=x58W9A2lnQc`).

(b) Using the CG model, determine the hedge ratio of equity (i.e., how many shares to trade) for \$1-notional CDS contract when implementing the capital structure arbitrage strategy.

(c) Suppose that you enter into the capital structure arbitrage when you observe a divergence (i.e., difference) between the 5-year at-market and CG-implied CDS spreads of more than 200% for the past 1-year average spread difference. You hedge your CDS position using equities. Assume that you hold the convergence trading position for three month (63 trading days) once you open it. Determine the profit of this strategy.