Terms

Literals	l	$::= true \mid false \mid Bool \mid \mathbb{R} \mid Float \mid () \mid Unit \mid Type \mid \square$
Variables	v	$::= [a-z\|A-Z][a-z\|A-Z\|1-9]*$
Expressions	e, t, k	$::= \mid l$
		$\mid v$
		$\mid \lambda v : t.e$
		$\mid \forall v : t.e$
		$\mid e_f\ e_a$
		$\mid if\ e_p\ then\ e_c\ else\ e_a$
		$\mid let\ v\ =\ e_v\ in\ e_b$
		$\mid letrec\ v\ :\ t\ = e_v\ in\ e_b$
		$\mid (e_l, e_r)$
		$\mid e.L$
		$\mid e.R$
		$\mid e.R$
		$\mid (t_l \wedge t_r)$
		$\mid (e : t)$
		$\mid (e)$

Typing

Literals

$$\frac{}{\vdash true : Bool}\ \text{BLit} \qquad \frac{}{\vdash false : Bool}\ \text{BLit} \qquad \frac{}{\vdash Bool : Type}\ \text{BTLit}$$

$$\frac{}{\vdash \mathbb{R} : Float}\ \text{FLit} \qquad \frac{}{\vdash Float : Type}\ \text{FTLit}$$

$$\frac{}{\vdash () : Unit}\ \text{ULit} \qquad \frac{}{\vdash Unit : Type}\ \text{UTLit}$$

$$\frac{}{\vdash Type : \square}\ \text{TLit}$$

Variables

$$\frac{}{\Gamma, v : t \vdash v : t}\ \text{Var}$$

Compounds

$$\frac{\Gamma \vdash t : \mathit{Type}}{\Gamma \vdash t :^t \mathit{Type}}\ \text{IsType} \qquad \frac{\Gamma \vdash t : \Box}{\Gamma \vdash t :^t \Box}\ \text{IsKind}$$

$$\frac{\Gamma \vdash t_{uv} :^t k_{uv} \qquad t_{uv} \Downarrow t_v \qquad \Gamma, t_v : k \vdash e : t_b}{\Gamma \vdash (\lambda v : t_{uv}.e) : (\forall v : t_v.t_b)}\ \lambda\text{I}$$

$$\frac{\Gamma \vdash e_f : (\forall v : t_a.t_b) \qquad \Gamma \vdash e_a : t_a \qquad (\lambda v : t_a.t_b)e_a \Downarrow t_o}{\Gamma \vdash e_f\ e_a : t_o}\ \lambda\text{E}$$

$$\frac{\Gamma \vdash t_{uv} :^t k_v \qquad t_{uv} \Downarrow t_v \qquad \Gamma, v : t_v \vdash t_{ub} :^t k_b}{\Gamma \vdash (\forall v : t_{uv}.t_{ub}) : k_b}\ \forall\text{I}$$

$$\frac{\Gamma \vdash e_p : \mathit{Bool} \qquad \Gamma \vdash e_c : t \qquad \Gamma \vdash e_a : t}{\Gamma \vdash (\mathit{if}\ e_p\ \mathit{then}\ e_c\ \mathit{else}\ e_a) : t}\ \text{If}$$

$$\frac{\Gamma \vdash e_v : t_v \qquad \Gamma, v : t_v \vdash e_b : t_b}{\Gamma \vdash (\mathit{let}\ v\ =\ e_v\ \mathit{in}\ e_b) : t_b}\ \text{Let}$$

$$\frac{\Gamma \vdash t_{uv} : k \qquad t_{uv} \Downarrow t_v \qquad \Gamma, v : t_v \vdash e_v : t_v \qquad \Gamma, v : t_v \vdash e_b : t_b}{\Gamma \vdash (\mathit{letrec}\ v\ :\ t_{uv}\ =\ e_v\ \mathit{in}\ e_b) : t_b}\ \text{Letrec}$$

$$\frac{\Gamma \vdash e_l : t_l \qquad \Gamma \vdash e_r : t_r}{\Gamma \vdash (e_l, e_r) : (t_l \wedge t_r)}\ \text{Pair} \qquad \frac{\Gamma \vdash t_l :^t k \qquad \Gamma \vdash t_r :^t k}{\Gamma \vdash (t_l \wedge t_r) : k}\ \text{PairT}$$

$$\frac{\Gamma \vdash e : (t_l \wedge t_r)}{\Gamma \vdash e.L : t_l}\ \text{PairL} \qquad \frac{\Gamma \vdash e : (t_l \wedge t_r)}{\Gamma \vdash e.R : t_r}\ \text{PairR}$$

$$\frac{\Gamma \vdash t_u :^t k \qquad t_u \Downarrow t \qquad \Gamma \vdash e : t}{\Gamma \vdash (e : t_u) : t}\ \text{Asc}$$

Normalization

$$\frac{}{l \Downarrow l}\ \text{Lit} \qquad \frac{}{v \Downarrow v}\ \text{Var}$$

$$\frac{t \Downarrow t' \qquad e \Downarrow e'}{\lambda v : t.e \Downarrow \lambda v : t'.e'}\ \lambda \qquad \frac{t_v \Downarrow t'_v \qquad t_b \Downarrow t'_b}{\forall v : t_v.t_b \Downarrow \forall v : t'_v.t'_b}\ \forall$$

$$\frac{e_f \Downarrow e'_f \qquad e_a \Downarrow e'_a}{e_f\ e_a \Downarrow e'_f\ e'_a}\ \text{NeuApp}$$

$$\frac{e_f \Downarrow \lambda v : t.e_b \qquad e_b[e_a/v] \Downarrow e'_b}{e_f\ e_a \Downarrow e'_b}\ \beta$$

$$\frac{e_p \Downarrow e'_p \qquad e_c \Downarrow e'_c \qquad e_a \Downarrow e'_a}{if\ e_p\ then\ e_c\ else\ e_a \Downarrow if\ e'_p\ then\ e'_c\ else\ e'_a}\ \text{NeuIf}$$

$$\frac{e_p \Downarrow true \qquad e_c \Downarrow e'_c}{if\ e_p\ then\ e_c\ else\ e_a \Downarrow e'_c}\ \text{IfT} \qquad \frac{e_p \Downarrow false \qquad e_a \Downarrow e'_a}{if\ e_p\ then\ e_c\ else\ e_a \Downarrow e'_a}\ \text{IfF}$$

$$\frac{e_b[e_v/v] \Downarrow e'_b}{let\ v\ =\ e_v\ in\ e_b \Downarrow e'_b}\ \text{Let}$$

$$\frac{e_l \Downarrow e'_l \qquad e_r \Downarrow e'_r}{(e_l, e_r) \Downarrow (e'_l, e'_r)}\ \text{Pair} \qquad \frac{t_l \Downarrow t'_l \qquad t_r \Downarrow t'_r}{(t_l \wedge t_r) \Downarrow (t'_l \wedge t'_r)}\ \text{PairT}$$

$$\frac{e \Downarrow e'}{e.L \Downarrow e'.L}\ \text{NeuPairL} \qquad \frac{e \Downarrow (e_l, e_r)}{e.L \Downarrow e_l}\ \text{PairL}$$

$$\frac{e \Downarrow e'}{e.R \Downarrow e'.R}\ \text{NeuPairR} \qquad \frac{e \Downarrow (e_l, e_r)}{e.R \Downarrow e_r}\ \text{PairR}$$

$$\frac{e \Downarrow e'}{(e : t) \Downarrow e'}\ \text{Asc}$$