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1. mean → θ_1, var → θ_2

$$L(\theta_1, \theta_2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\theta_2}} \left(e\right)^{-\frac{(x_i-\theta_1)^2}{2\theta_2}}$$

taking log

$$\log(L(\theta_1, \theta_2)) = -n/_2 \log(2\pi\theta_2) - {}^1/_{2\theta_2} \sum_{i=1}^{n} (x_i - \theta_1)^2$$

for θ_1 → differentiate $\log(L(\theta_1, \theta_2))$ w.r.t. θ_1 set it to zero.

$$\frac{\partial \log(L)}{\partial \theta_1} = \frac{1}{\theta_2} \sum_{i=1}^{n} (x_i - \theta_1) = 0$$

$$\theta_1 = {}^1/_n \sum_{i=1} x_i$$

M.L.E of θ_1 is simple mean.

for θ_2 → diff. w.r.t θ_2 set it zero

$$\boxed{\theta_2 = {}^1/_n \sum_{i=1}^{n} (x_i - \theta_1)^2}$$

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(2) $B(m, \theta)$ Binomial dist.

$m \to$ no. of trials

$\theta = (0,1)$ prob. of success.

$$L\theta = \prod_{i=1}^{n} (f(x_i, n, \theta))$$

p.m.f.

$$f(x, n, \theta) = {}^{n}C_{x}\, \theta^{n} (1-\theta)^{m-x}$$

$$L(\theta) = \prod_{i=1}^{n} {}^{n}C_{x_i}\, \theta^{x_i} (1-\theta)^{m-x_i}$$

taking log and diff. w.r.t θ

$$\frac{\partial \log(L)}{\partial \theta} = \frac{1}{\theta} \sum_{i=1}^{n} x_i - \frac{1}{1-\theta} \sum_{i=1}^{n} (m - x_i)$$

and set to zero.

$$\frac{1}{\theta}\left(\sum_{i=1}^{n} x_i\right) = \frac{1}{1-\theta}\left(\sum_{i=1}^{n} (m - x_i)\right)$$

$$1-\theta\left(\sum_{i=1}^{n} x_i\right) = \theta\left(\sum_{i=1}^{n} (m - x_i)\right)$$

$$\boxed{\theta = \sum_{i=1}^{n} x_i / mn}$$ m.l.e of θ

MLE for $B(n, \theta)$ is $\bar{x}$ where

$$\bar{x} = \sum_{i=1}^{n} x_i / n$$