PS11

● Graded

Student

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Total Points

72 / 100 pts

Question 1

(no title) **10** / 10 pts

- ✓ **+ 10 pts** Correct

Question 2

(no title) **24** / 25 pts

- ✓ **+ 5 pts** A
- ✓ **+ 10 pts** B
- ✓ **+ 5 pts** C
- ✓ **+ 5 pts** D
- ✓ **– 1 pt** Missing proportionality constant leading to incorrect answer for (d)

Question 3

(no title) **0** / 10 pts

- ✓ **+ 0 pts** Missing/incorrect

Question 4

(no title) **8** / 10 pts

- ✓ **+ 10 pts** Correct
- ✓ **– 2 pts** Missing drawing in support vector space

Question 5

(no title) **12** / 20 pts

- ✓ **+ 20 pts** Correct
- ✓ **− 2 pts** Math error (should converge on [-1,2,-2,-1]
- ✓ **− 3 pts** Missing plane in drawing/Incorrect
- ✓ **− 3 pts** Stopping too early (not converging)

Question 6

(no title) **18** / 25 pts

- ✓ **+ 25 pts** Correct
- ✓ **− 2 pts** Incorrect converged values
- ✓ **− 5 pts** Output [-1,2,-2,-1]

Problem Set

1. $P(+b) = \frac{94}{100} = 0,94$

$P(-b) = \frac{6}{100} = 0.06$

$P(+l) = \frac{68}{100} = 0.68$

$P(-l) = \frac{32}{100} = 0.32$

$P(+g|-b) = \frac{0}{6} = 0$

$P(-g|-b) = \frac{6}{6} = 1$

$P(+g|+b) = \frac{89}{94} = 0.95$

$P(-g|+b) = \frac{5}{94} = 0.05$

$P(+m|+b,-l) = \frac{1}{30} = 0.03$

$P(+m|+b,+l) = \frac{57}{64} = 0.89$

$P(+m|-b,-l) = \frac{0}{2} = 0$

$P(+m|-b,+l) = \frac{0}{4} = 0$

$P(-m|+b,-l) = \frac{29}{30} = 0.97$

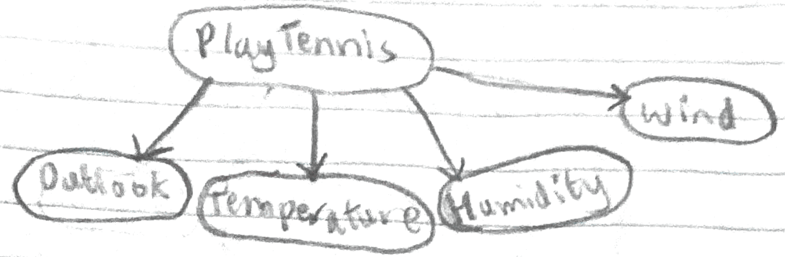
$P(-m|+b,+l) = \frac{7}{64} = 0.11$

$P(-m|-b,-l) = \frac{2}{2} = 1$

$P(-m|-b,+l) = \frac{4}{4} = 1$

Question assigned to the following page: 2

2a.



Class Name - PlayTennis.
Attributes - Outlook, Temperature, Humidity and Wind.

b. $P(Yes) = \frac{9}{14} = 0.64$
$P(No) = \frac{5}{14} = 0.36$
Play Tennis

$P(Sunny|Yes) = \frac{2}{9} = 0.22$
$P(Sunny|No) = \frac{3}{5} = 0.60$
$P(Overcast|Yes) = \frac{4}{9} = 0.44$
$P(Overcast|No) = \frac{0}{5} = 0$
$P(Rainy|Yes) = \frac{3}{9} = 0.33$
$P(Rainy|No) = \frac{2}{5} = 0.40$
Outlook

$P(Hot|Yes) = \frac{2}{9} = 0.22$
$P(Hot|No) = \frac{2}{5} = 0.40$
$P(Mild|Yes) = \frac{4}{9} = 0.44$
$P(Mild|No) = \frac{2}{5} = 0.40$
$P(Cool|Yes) = \frac{3}{9} = 0.33$
$P(Cool|No) = \frac{1}{5} = 0.20$
Temperature

$P(High|Yes) = \frac{3}{9} = 0.33$
$P(High|No) = \frac{4}{5} = 0.80$
$P(Normal|Yes) = \frac{6}{9} = 0.67$
$P(Normal|No) = \frac{1}{5} = 0.20$
Humidity

$P(Weak|Yes) = \frac{6}{9} = 0.67$
$P(Weak|No) = \frac{2}{5} = 0.40$
$P(Strong|Yes) = \frac{3}{9} = 0.33$
$P(Strong|No) = \frac{3}{5} = 0.60$
Wind

c and d. $P(Yes \mid Sunny, Cool, High, Strong)$

$= \alpha P(Yes) P(Sunny \mid Yes) P(Cool, Yes) P(High, Yes) P(Strong \mid Yes)$

$= \alpha \left(\frac{9}{14}\right)\left(\frac{2}{9}\right)\left(\frac{3}{9}\right)\left(\frac{3}{9}\right)\left(\frac{3}{9}\right)$

$= \alpha\, 0.0053$

$P(No \mid Sunny, Cool, High, Strong)$

$= \alpha P(No) P(Sunny \mid No) P(Cool, No) P(High, No) P(Strong, No)$

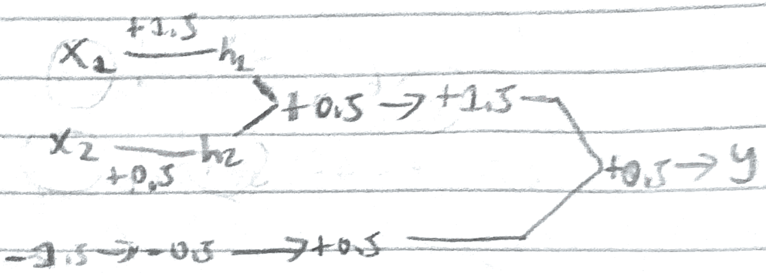
$= \alpha \left(\frac{5}{14}\right)\left(\frac{3}{5}\right)\left(\frac{1}{5}\right)\left(\frac{4}{5}\right)\left(\frac{3}{5}\right)$

$= \alpha\, 0.0206$

$P(No \mid Sunny, Cool, High, Strong) > P(Yes \mid Sunny, Cool, High, Strong)$, so the predicted target attribute of the example is No.

3.

x_1	x_2	$x_1 \oplus x_2 = y$
0	0	0
0	1	1
1	0	1
1	1	0



Question assigned to the following page: 4

4.

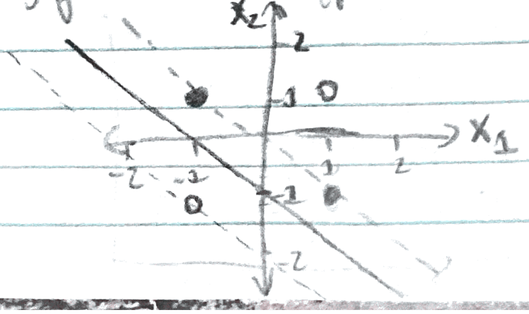
x_1	x_2	$x_1 \oplus x_2$		x_1, x_1x_2
−1	−1	−1	NExample −([−1,−1],−1) →	[−1,+1]
−1	+1	+1	P Example −([−1,+1],+1) →	[−1,−1]
+1	−1	+1	PExample −([+1,−1],+1) →	[+1,−1]
+1	+1	−1	NExample −([+1,+1],−1) →	[+1,+1]

Positive mappinge = $[x_1 x_2] = -1$.

Negative mapping = $[x_1 x_2] = +1$.

Maximum margin Separator = $x_1x_2 = -1$ when margin = 2.

The seperator has x_1 and x_2 in the original space or the limit of hyperbolic Separator. So, the separating line is −1.



5a. Epoch 1 -

$(x_1, x_2, x_3, y) = (1, 0, 0, 1)$

Weighted sum $= 0 + 1(0) + 0(0) + 0(0) = 0$.

Threshold $= 1$, and actual output $= 1 = y$. Don't update the weight.

Epoch 2 -

$(0, 1, 1, 0)$

Weighted sum $= 0 + 0(0) + 1(0) + 1(0) = 0$.

Threshold $= 1$, and actual output $= 1 \neq y$. Update the weight.

$w_0 = 0 + \alpha(y - Y)(1) = 0 + 1(0-1)(1) = -1$.

$w_1 = 0 + \alpha(y - Y)(0) = 0 + 1(0-1)(0) = 0$.

$w_2 = 0 + \alpha(y - Y)(1) = 0 + 1(0-1)(1) = -1$.

$w_3 = 0 + \alpha(y - Y)(1) = 0 + 1(0-1)(1) = -1$.

Epoch 3 -

$(1, 1, 0, 1)$

Weighted sum $= -1 + 1(0) + 1(-1) + 0(-1) = -2$.

Threshold $=$ actual output $= 0 \neq y$. Update the weight.

$w_0 = -1 + 1(1-0)(1) = 0$.

$w_1 = 0 + 1(1-0)(1) = 1$.

$w_2 = -1 + 1(1-0)(1) = 0$.

$w_3 = -1 + 1(1-0)(0) = -1$.

Epoch 4 -

$(1, 1, 1, 0)$

Weighted sum $= 0 + 1(1) + 1(0) + 1(-1) = 0$.

Threshold $=$ actual output $= 1 \neq y$. Update the weight.

$w_0 = 0 + 1(0-1)(1) = -1$.

$w_1 = 1 + 1(0-1)(1) = 0$.

$w_2 = 0 + 1(0-1)(1) = -1$.

$w_3 = -1 + 1(0-1)(1) = -2$.

Epoch 5 -

$(0, 0, 1, 0)$

Weighted sum $= -1 + 0(0) + 0(-1) + 1(-2) = -3$.

Threshold $=$ actual output $= 0 = y$. Don't update the weight.

Epoch 5-

(1, 0, 1, 1)

Weighted sum= $-1+1(0)+0(-1)+1(-2)=-3$.

Threshold= actual output=0 ≠ y. Update the weights.

$w_0=-1+1(1-0)(1)=0$.

$w_1=0+1(1-0)(1)=1$.

$w_2=-1+1(1-0)(0)=-1$.

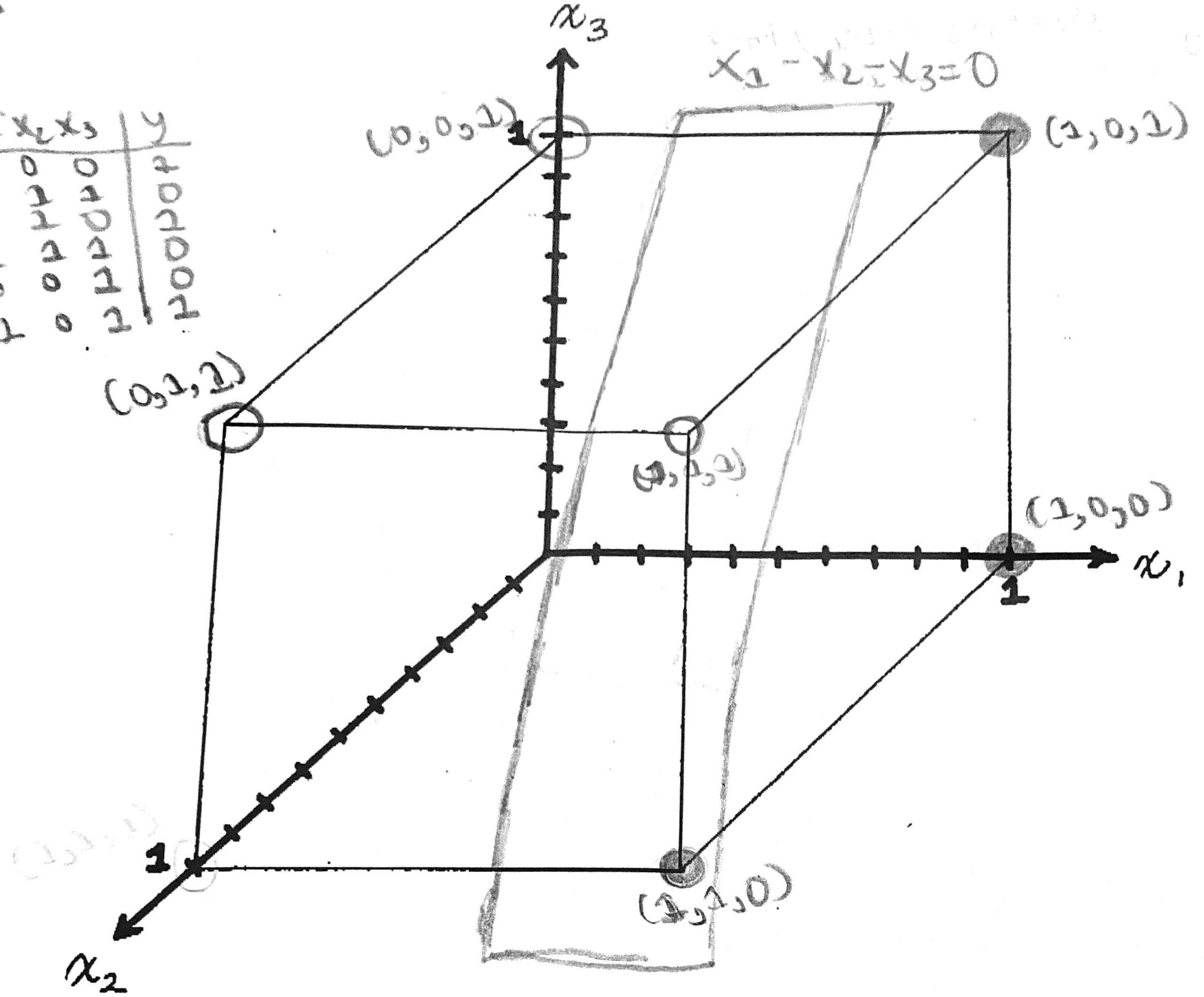
$w_3=-2+1(1-0)(1)=-1$.

Final weights: $w_0=0$, $w_1=1$, $w_2=-1$, and $w_3=-1$.

Question assigned to the following page: 5

5b.

x_1	x_2	x_3	y
1	0	0	1
0	1	1	0
1	1	0	1
1	1	1	0
0	0	1	0
1	0	1	1



$w_0 + w_1 x_1 + w_2 x_2 + w_3 x_3 = 0$

$0 + (1)(x_1) + (-1)(x_2) + (-1)(x_3) = 0$

$x_1 - x_2 - x_3 = 0$

Question assigned to the following page: 6

6.

Sources- This code is used, borrowed, and modified from EECS 649: Lec 21's Learning from Examples 3's Perceptron Learning Rule Algorithm, R&N Textbook's Perceptron Learning Algorithm, Perceptron Learning Example, and Geeks for Geeks' What is Perceptron-The Simplest Artificial Neuron Network Article's Python code.

```
import numpy as np
import matplotlib.pyplot as plt

class PerceptronLearning:
    def __init__(self, input_size, alpha = 1):
        self.alpha = alpha
        self.weights = np.zeros(input_size + 1)

    def predict(self, inputs):
        activation = np.dot(inputs, self.weights[1:]) + self.weights[0]
        if (activation >= 0):
           return 1
        else:
           return 0
```

Question assigned to the following page: 6

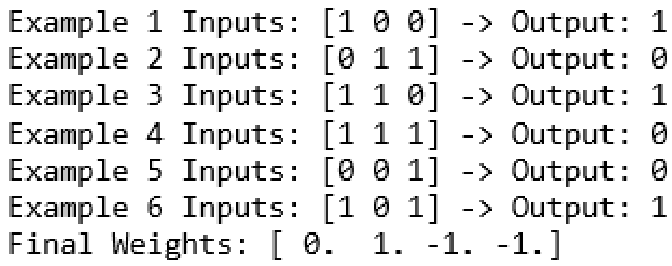
```
    def train(self, inputs, labels, epochs=100):
        for _ in range(epochs):
            weights_updated = False
            for x, y in zip(inputs, labels):
                y_pred = self.predict(x)
                error = y - y_pred
                if (error != 0):
                     self.weights[1:] += self.alpha * error * x
                     self.weights[0] += self.alpha * error
                     weights_updated = True
            if not weights_updated:
                break

X_train = np.array([[1, 0, 0], [0, 1, 1], [1, 1, 0], [1, 1, 1], [0, 0, 1],
[1, 0, 1]])
y_train = np.array([1, 0, 1, 0, 0, 1])
p = PerceptronLearning(input_size = 3, alpha = 1)
p.train(X_train, y_train)
for i, x in enumerate(X_train):
    print("Example", (i+1), "Inputs:", x, "-> Output:", p.predict(x))
print("Final Weights:", p.weights)

plt.plot(p.weights)
plt.title("Weight Vector")
plt.xlabel("Index")
plt.ylabel("Weight Value")
plt.show()
```

Question assigned to the following page: 6

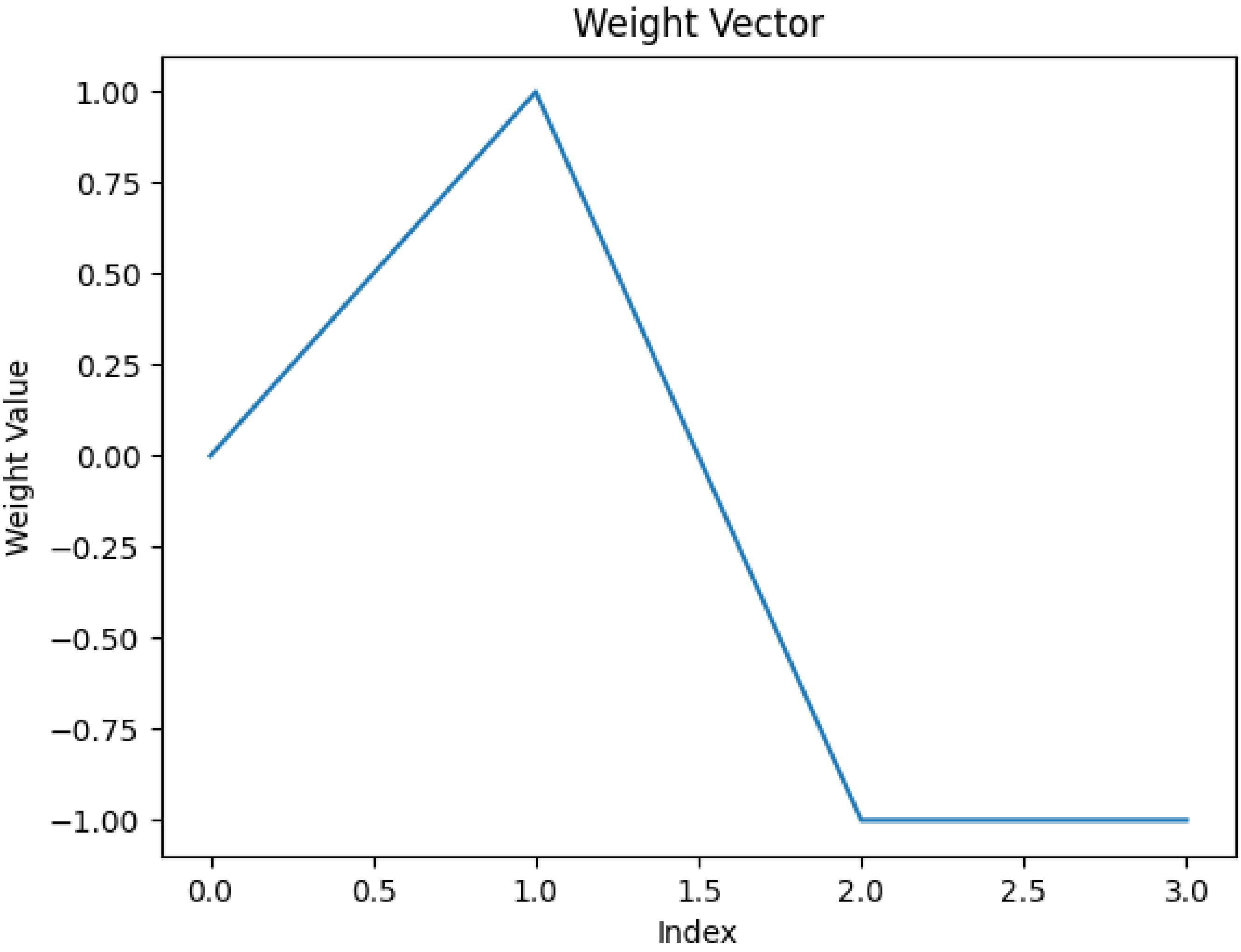
Result-



```
Example 1 Inputs: [1 0 0] -> Output: 1
Example 2 Inputs: [0 1 1] -> Output: 0
Example 3 Inputs: [1 1 0] -> Output: 1
Example 4 Inputs: [1 1 1] -> Output: 0
Example 5 Inputs: [0 0 1] -> Output: 0
Example 6 Inputs: [1 0 1] -> Output: 1
Final Weights: [ 0.  1. -1. -1.]
```

I have made the program that uses Perceptron Learning Algorithm to calculate the final weights of the 6 examples that use alpha=1 and plot the graph of the weight vector. Then, I have included the examples and their respective inputs and outputs of the fifth question to make sure that the answers of these questions are same. I have compiled the program, which is showing the examples' corresponding inputs and outputs and the final weights. Also, I have generated a graph to make sure that the weight vector is displayed correctly. The graph shows the plot of the transpose of the weight vector or the row vector to show the final results. The final weights of this question and the final weights of the fifth question are equal and same, so the results are displayed successfully.

Question assigned to the following page: 6


Weight Vector
Weight Value
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
0.0
0.5
1.0
1.5
2.0
2.5
3.0
Index