J. Medhi ↓

Stochastic Process

Seq^n of Random Variable

✓ X_n = fortune of gambler ~~a~~ after n^{th} game

$X_n = X_{n-1} + Z_n$ $\qquad Z_n = \begin{cases} +1 & \text{w.p. } p \ ✓ \\ -1 & \text{w.p } 1-p \end{cases}$

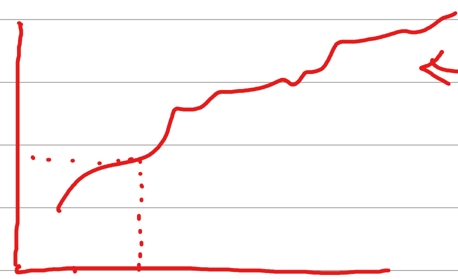
100 → 99, 101

$X_0 \quad X_1 \quad X_2 \ \ldots \quad X_n \ldots$

Discrete seq^n of Random ✓ $\{X_i\}_{i=1}^{\infty}$

9.00 → 9.12 $\quad$ 9.12 — 9.15 $\quad$ 9-30 —— 3:30

SENSEX

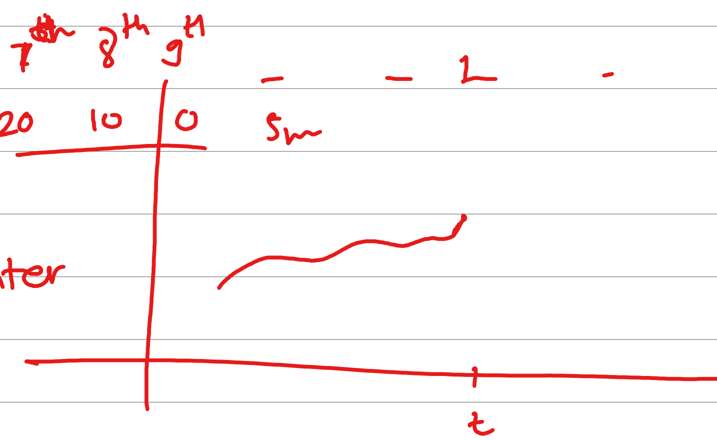


continous $\cancel{X_n}$ $\quad \{X_t\}_{t \in I}$

I = Time index set

Rain

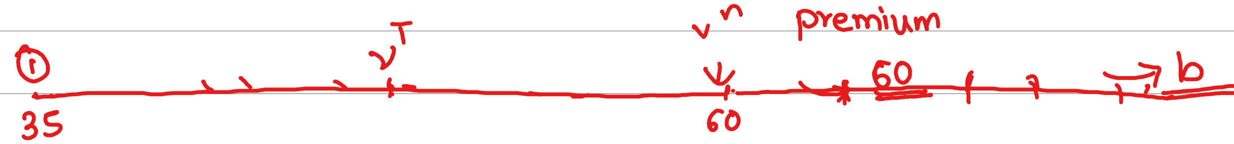
Cash counter



	Statespace	Time domain	Example
1	Discrete	Discrete	* No. of covid patients on n^{th} day * No. of customers on a shop · -"-
2	Discrete	Conti	* Pop^n size → conti time discrete pop^n * No. of accidents up to time t
3	Continuous	Discrete	* Milk by cow on n^{th} day * Min^m/Max^m temp on n^{th} day
4	Continuous	Contin.	* Fever - body temp on t time * Blood conc up to time t
5	Mixture	Discrete Cont	* Speed of internet * Rainfall up to time t

Endowment → $x \to n$ 35 → 60 → b

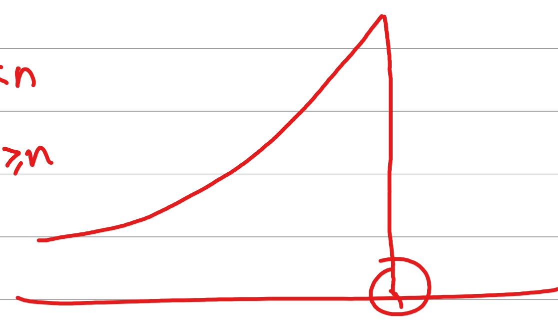
premium



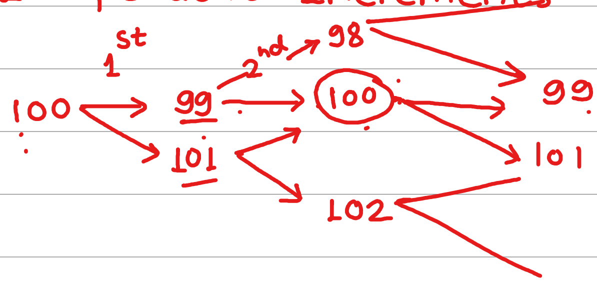
mixture

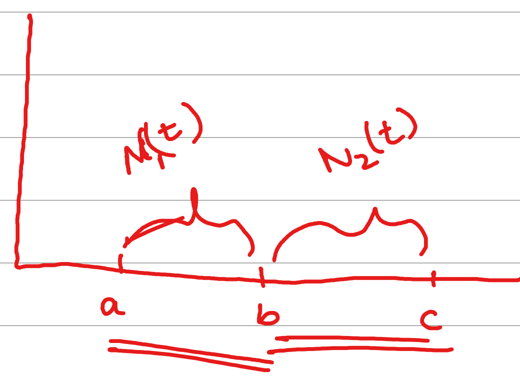
Present = v^T T < n

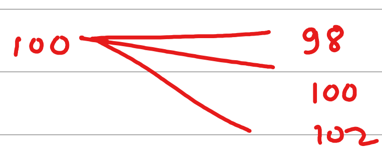
v^n T ≥ n



Independent Increments







disjoint time interval

$N_1(t)$ & $N_2(t)$

dist^n indep

independent increments

$t_0 < t_1 < t_2 < \dots < t_{n-1} < t_n$

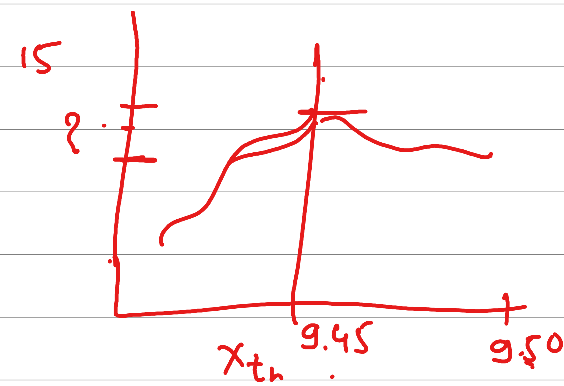
$\rightarrow X_{t_0}, X_{t_1}, X_{t_2}, \dots \dots < X_{t_{n-1}} < X_{t_n}$

$X_{t_1} - X_{t_0}, X_{t_2} - X_{t_1}, \dots, X_{t_n} - X_{t_{n-1}}$ indep. inc.

12 - 2

12 - 1

12.30 - 1.30



Joint same X_{t_0+u} X_{t_1+u} X_{t_2+u} X_{t_n}

t_0 t_1 t_2 t_{n-1} t_n

Joint $(X_{t_0}, X_{t_1}, X_{t_2}, \quad X_{t_{n-1}}, X_{t_n})$

$X_{t_1} - X_{t_0}$ $X_{t_2} - X_{t_1}$ $X_{t_n} - X_{t_{n-1}}$

independent Statinarity strictly

$s < t$ $u > 0$

increment $(s, t]$ $X(t) - X(s)$ 12-2 No. of students

✓interval length $(t-s)$

$(s+u, t+u]$ $X(t+u) - X(s+u)$ 7-9

✓interval length $(t-s)$

X_{t_0}

weak stationarily μ
$$\begin{cases} E(X(t)) = \mu & \text{constant } \forall t \in I \\ Cov(X(t), X(s)) = fun(t-s) \end{cases}$$

Gambler → Random walk

Renewal

Stationarity

Probalilistic Replica

Find the p.g.f. of the sum Sn = X1 + + Xn of n independent and identical zero-truncated Poisson variates. Find E (Sn) and Pr {Sn = m}, m = n, n + 1, n + 2..

$X_i \sim$ zero-truncated poisson

$$P[X=x] = \frac{e^{-\lambda}\lambda^x}{x!} \Big/ (1-e^{-\lambda}) \rightarrow E(X)$$

$$S_n = \sum_{i=1}^{n} X_i \Rightarrow E(S_n) = n \cdot E(X) = \frac{n \cdot \lambda}{1-e^{-\lambda}}$$

$$P_S(t) = E(t^S) = E\left(t^{\sum_{i=1}^{n} x_i}\right) = [P_X(t)]^n$$

$$P_X(t) = \frac{1}{1-e^{-\lambda}} \, e^{-\lambda} \sum_{x=1}^{\infty} \frac{(\lambda t)^x}{x!}$$

$$= \frac{e^{-\lambda}}{1-e^{-\lambda}}\left[e^{\lambda t} \sum_{x=0}^{\infty} \frac{e^{-\lambda t}(\lambda t)^x}{x!} - e^{-\lambda t}\right]$$

$$\sum_{x=0}^{\infty} \frac{e^{-\lambda t}(\lambda t)^x}{x!} \rightarrow 1$$

$$= \frac{e^{-\lambda}}{1-e^{-\lambda}}\left[e^{\lambda t} - e^{-\lambda t}\right]$$