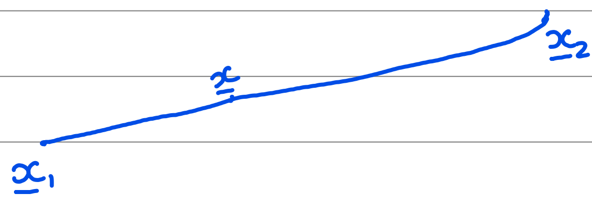
* Line segment : $\underline{x} \in \mathbb{R}^n$



scaler

$\underline{x} = \alpha \underline{x}_1 + (1-\alpha)\underline{x}_2$ $\qquad \alpha \in (0,1)$

line segment joining $\underline{x}_1, \underline{x}_2 \in \mathbb{R}^n$ $\{\underline{x} / \underline{x} = \alpha \underline{x}_1 + (1-\alpha)\underline{x}_2,\ \alpha \in (0,1)\}$

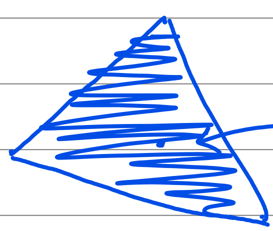
* Line passing $\underline{x}_1, \underline{x}_2 \in \mathbb{R}^n$ $\qquad \alpha + 1 - \alpha$

line segment ← ✓ convex combⁿ $\underline{x} = \alpha \underline{x}_1 + (1-\alpha)\underline{x}_2$ $\quad \alpha \in (0,1)$

line, linear combⁿ $\underline{x} = \alpha \underline{x}_1 + (1-\alpha)\underline{x}_2$ $\quad \alpha \in \mathbb{R}$

$\therefore$

vector space → $\boxed{\underline{x} = \sum \alpha_i \underline{x}_i}$ $\quad$ Linear Combⁿ $\quad \alpha_i \in \mathbb{R}$



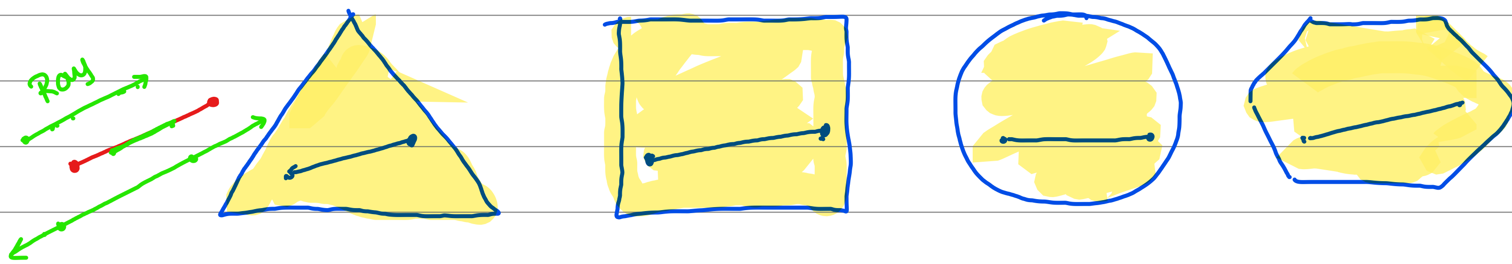
$\underline{x} = \sum \alpha_i \underline{x}_i \qquad \sum \alpha_i = 1, \qquad \alpha_i \geq 0$

↳ Convex Combination

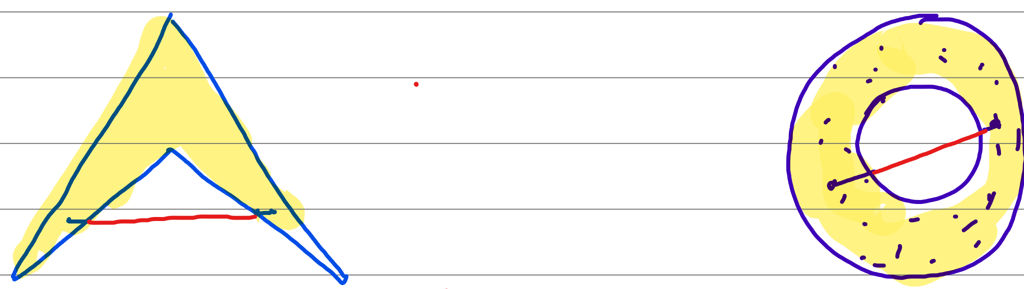
$$\left\{ \underline{x} \,/\, \underline{x} = \sum_{i=1}^{n} \alpha_i x_i,\ \alpha_i \geq 0,\ \sum \alpha_i = 1 \right\}$$

Convex set if $\forall \underline{x}_1$ & $\underline{x}_2 \in A$, $\underline{x} = \alpha \underline{x}_1 + (1-\alpha)\underline{x}_2 \quad \forall \alpha \in (0,1)$

then if $\underline{x} \in S \ \forall \alpha$ A is convex set



Convex sets



Non convex sets

* Ray is convex set

$$\underline{x} = \underline{x}_0 + \underline{d}\alpha \qquad \alpha \geq 0$$

$\underline{x}_1$ & $\underline{x}_2 \in A$

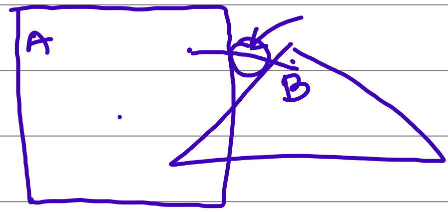
$$\lambda \qquad \underline{x}_1 = \underline{x}_0 + \underline{d}\alpha_1$$

$$1-\lambda \qquad \underline{x}_2 = \underline{x}_0 + \underline{d}\alpha_2$$

$$\lambda \underline{x}_1 + (1-\lambda)\underline{x}_2 = (\lambda + 1 - \lambda)\underline{x}_0 + \underline{d}\left(\lambda\alpha_1 + (1-\lambda)\alpha_2\right)$$

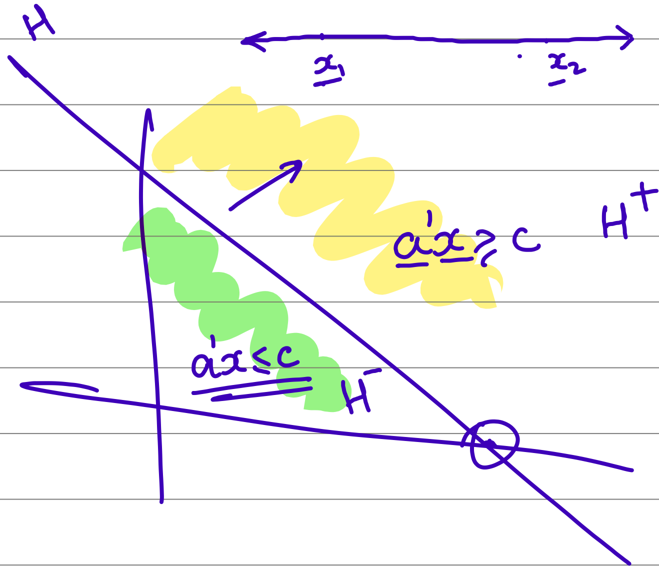
$$= x_0 + d(\qquad) \quad \geq 0$$

$\in A$



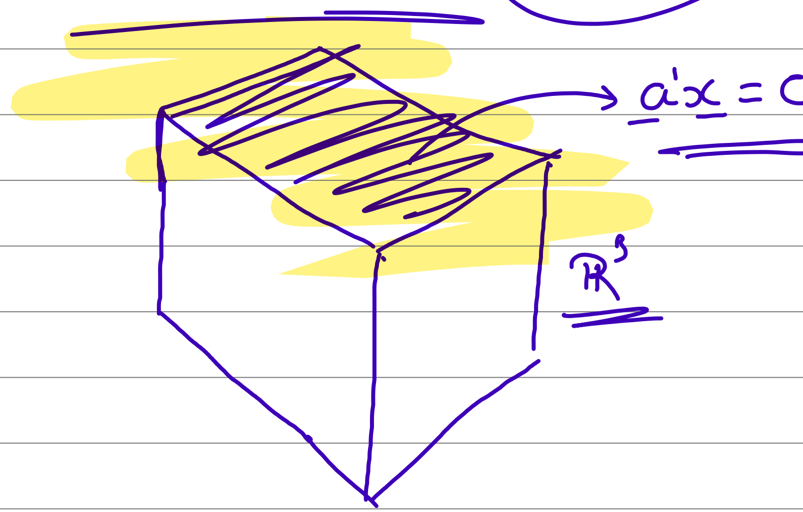
* Union of convex sets may or may not be convex.

* Intersection of convex sets is also convex.

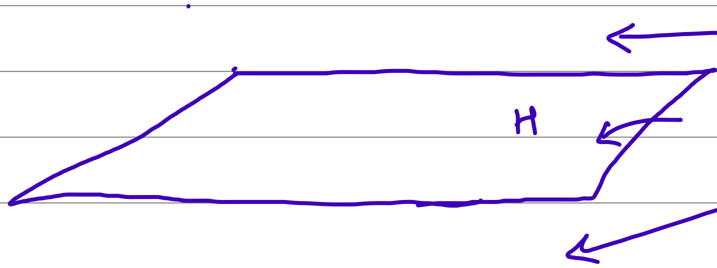


$$\sum_{i=1}^{2} \alpha_i x_i$$

$$\sum_{i=1}^{n} \alpha_i x_i = c \qquad \underline{\alpha}'\underline{x} = c$$



Hyperplane



$H_0^+ = \{\underline{x} / a'x > c\}$ +ve open half space

$H^+ = \{\underline{x} / \underline{a}'\underline{x} \geq c\}$ ← positive closed half space

$H = \{\underline{x} / \underline{a}'\underline{x} = c\}$ Hyperplane

$H^- = \{\underline{x} / a'x \leq c\}$ ← Negative closed half space

$H_0^- = \{\underline{x} / \underline{a}'\underline{x} < c\}$ -ve open half space

Hyperplane $H = \{\underline{x} / \underline{a}'\underline{x} = c\}$

let $\underline{x}_1, \underline{x}_2 \in H \Rightarrow \underline{a}'\underline{x}_1 = c$ & $\underline{a}'\underline{x}_2 = c$

$\alpha \underline{x}_1 + (1-\alpha)\underline{x}_2$

$$\left[\begin{aligned} \underline{a}'(\alpha \underline{x}_1 + (1-\alpha)\underline{x}_2) &= \alpha \underline{a}'\underline{x}_1 + (1-\alpha)\underline{a}'\underline{x}_2 \\ &= \alpha c + (1-\alpha)c \\ &= c \end{aligned}\right]$$

$\Rightarrow \alpha \underline{x}_1 + (1-\alpha)\underline{x}_2 \in H$

$\Rightarrow$ Hyperplane is convex set

H^+, H^-, H_0^+, H_0^- all are convex sets