Unit -3 Ordinary Differential Equations (ODEs) of first order

Linear and Bernoulli's differential equations. Exact and reducible to exact differential equation. Integrating factors on $\frac{1}{N}\left(\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}\right)$ and $\frac{1}{M}\left(\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}\right)$.
Applications of ODE's - Orthogonal trajectories, Newton's law of cooling.
Nonlinear differential equations: Introduction to general and singular solutions, Solvable for p only, Clairaut's equations, reducible to Clairaut's equations. Problems.

Linear differential equations: Solve

1. $\frac{dy}{dx}+y\cot x=\cos x$
2. $\frac{dy}{dx}-\frac{2y}{x+1}=(x+1)^3$
3. $\frac{dy}{dx}+y\cot x=4x\operatorname{cosec} x$
4. $2y'\cos x+4y\sin x=\sin 2x$, given y=0 when x=π/3
5. $(x+1)\frac{dy}{dx}-y=e^{3x}(x+1)^2$
6. $\frac{dy}{dx}+3x^2y=x^5e^{x^3}$
7. $(1+y^2)dx=(\tan^{-1}y-x)dy$
8. $\sqrt{1-y^2}\,dx=(\sin^{-1}y-x)dy$
9. $y\log y\,dx+(x-\log y)dy=0$
10. $(1+y^2)dx+(x-e^{-\tan^{-1}y})dy=0$

Bernoulli's equation: Solve

1. $\frac{dy}{dx}+x\sin 2y=x^3\cos^2 y$
2. $\frac{dy}{dx}-\frac{\tan y}{1+x}=(1+x)e^x\sec y$
3. $x\frac{dy}{dx}+y=x^3y^6$
4. $\frac{dy}{dx}+y\tan x=y^3\sec x$
5. $2xy'=10x^3y^5+y$
6. $e^y\left(\frac{dy}{dx}+1\right)=e^x$
7. $\frac{dz}{dx}+\left(\frac{z}{x}\right)\log z=\left(\frac{z}{x}\right)(\log z)^2$ OR $\frac{dy}{dx}+\frac{y\log y}{x}=\frac{y}{x}(\log y)^2$
8. $(y\log x-2)y\,dx=x\,dy$
9. $x(x-y)dy+y^2dx=0$
10. $xy(1+xy^2)\frac{dy}{dx}=1$
11. $x^3\frac{dy}{dx}-x^2y=-y^4\cos x$

…erential equations: *Solve*

1. $(5x^4+3x^2y^2-2xy^3)dx+(2x^3y-3x^2y^2-5y^4)dy=0$
2. $\left[y\left(1+\frac{1}{x}\right)+\cos y\right]dx+\left[x+\log x-x\sin y\right]dy=0$
3. $\left(1+e^{\frac{x}{y}}\right)dx+e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)dy=0$
4. $y\sin 2x\,dx-(1+y^2+\cos^2 x)dy=0$
5. $\left[\cos x\tan y+\cos(x+y)\right]dx+\left[\sin x\sec^2 y+\cos(x+y)\right]dy=0$
6. $\frac{dy}{dx}+\frac{y\cos x+\sin y+y}{\sin x+x\cos y+x}=0$
7. $ye^{xy}dx+(xe^{xy}+2y)dy=0$
8. $(2xy+y-\tan y)dx+(x^2-x\tan^2 y+\sec^2 y)dy=0$
9. $\left(y^2e^{xy^2}+4x^3\right)dx+(2xye^{xy^2}-3y^2)dy=0$
10. $\frac{2x}{y^3}dx+\frac{y^2-3x^2}{y^4}dy=0$

Equations Reducible to Exact Differential equations

1. $y(2x-y+1)dx+x(3x-4y+3)dy=0$
2. $y(2xy+1)dx-xdy=0$
3. $(x^2+y^2+x)dx+xydy=0$
4. $y(x+y)dx+(x+2y-1)dy=0$
5. $(8xy-9y^2)dx+2(x^2-3xy)dy=0$

Orthogonal Trajectory

1) Find the orthogonal trajectories of the family of curves $y=ax^2$ where a is the parameter
2) Find the orthogonal trajectories of the family of curves $\frac{x^2}{a^2}+\frac{y^2}{a^2+\lambda}=1$, where λ is the parameter.
3) Find the orthogonal trajectories of the family of curves $\frac{x^2}{a^2}+\frac{y^2}{b^2+\lambda}=1$ where λ is the parameter.

4)Show that the family of curves $y^2=4a(x+a)$ is self -orthogonal. where a is a parameter

5)Show that the family of curves $x^2=4a(y+a)$ is self -orthogonal, where a is a parameter.

6) Show that the family of curves $\frac{x^2}{a^2+\lambda}+\frac{y^2}{b^2+\lambda}=1$ is self -orthogonal, where λ is a parameter

7) Find the orthogonal trajectories of the family of curves $x^2+y^2+2\lambda x+c=0$ where λ is the parameter

Civil and Mechanical engineering

1) A body originally at 80°C cools down to 60° in 20 minutes, the temperature of air being 40°C. What will be the temperature of the body after 40 minutes from the original?

EC and CSE Branches

1) Show that the differential equation for the current I in an electrical circuit containing an inductance L and a resistance R in series and acted on by an electromotive force E $\sin\omega t$ satisfies the equation $L dI/dt + RI = E\sin\omega t$. Find the value of the current at any time t, if initially there is no current in the circuit.

Solvable for p

Solve

1. $p^2 - 5p + 6 = 0$
2. $p^2 + p(x+y) + xy = 0$
3. $x^2p^2 + 3xyp + 2y^2 = 0$
4. $xyp^2 + p(3x^2 - 2y^2) - 6xy = 0$
5. $p - \frac{1}{p} = \frac{x}{y} - \frac{y}{x}$
6. $y\left(\frac{dy}{dx}\right)^2 + (x-y)\frac{dy}{dx} - x = 0$
7. $p(p+y) = x(x+y)$
8. $p^2 + 2py\cot x = y^2$

Clairaut's equations

Solve

1) $y = px + \frac{a}{p}$
2) $xp^3 - yp^2 + 1 = 0$
3) $xp^2 - py + kp + a = 0$
4) $xp^2 + px - py + 1 - y = 0$
5) Solve the equation $(px - y)(py + x) = 2p$ by reducing into Clairaut's form

   taking the substitution $X = x^2$, $Y = y^2$