Module-5 Linear Algebra

Introduction of liner algebra:

Elementary row transformation of a matrix, Rank of a matrix. Consistency and solution of a system of linear equations - Gauss-elimination method, Gauss-Jordan method and approximate solution by Gauss-Seidel method. Eigenvalues and Eigenvectors, Rayleigh's power method to find the dominant Eigenvalue and Eigenvector problems.

1. Determine the rank of the following matrices:

1) $A = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$ 2) $A = \begin{bmatrix} 0 & 2 & 3 & 4 \\ 2 & 3 & 5 & 4 \\ 4 & 8 & 13 & 12 \end{bmatrix}$ 3) $A = \begin{bmatrix} 2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$

4) $A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 0 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$ 5) $A = \begin{bmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$ 6) $A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$

7) $A = \begin{bmatrix} 0 & 1 & 2 & -2 \\ 4 & 0 & 2 & 6 \\ 2 & 1 & 3 & 1 \end{bmatrix}$

2. Test for consistency and solve.

1) $x + y + z = 6 \quad x - y + 2z = 5 \quad 3x + y + z = 8$

2) $x + 2y + 3z = 14 \quad 4x + 5y + 7z = 35 \quad 3x + 3y + 4z = 21$

3) $x - 4y + 7z = 14 \quad 3x + 8y - 2z = 13 \quad 7x - 8y + 26z = 5$

4) $4x - 5y + z = -3 \quad 2x + 3y - z = 3 \quad 3x - y + 2z = 5 \quad x + 2y - 5z = -9$

5) $5x_1 + x_2 + 3x_3 = 20 \quad 2x_1 + 5x_2 + 2x_3 = 18 \quad 3x_1 + 2x_2 + x_3 = 14$

6) $5x + 3y + 7z = 5 \quad 3x + 26y + 2z = 9 \quad 7x + 2y + 10z = 5$

7) $3x + 3y + 2z = 1 \quad x + 2y = 4 \quad 2x - 3y - z = 5$

8) Investigate the values of λ *and* μ such that the system is consistent
$x + y + z = 6 \quad x + 2y + 3z = 10 \quad x + 2y + \lambda z = \mu$ may have
i) no solution ii) a unique solution and iii) an infinite number of solutions.

9) Find all the values of μ and λ for which the system has i) no solution ii) a unique solution iii) Infinitely many solution. $x+y+z=2, \quad x+2y+z=3, \quad x+2y+(\mu^2-5)z=\lambda.$

3\. Apply Gauss elimination method to solve the following equations.

1) $x + y + z = 6 \quad x - y + 2z = 5 \quad 3x + y + z = 8$

2) $x + y + z = 9 \quad x - 2y + 3z = 8 \quad 2x + y - z = 3$

3) $2x_1 + x_2 + 4x_3 = 12 \quad 4x_1 + 11x_2 - x_3 = 33 \quad 8x_1 - 3x_2 + 2x_3 = 20$

4) $5x_1 + x_2 + x_3 + x_4 = 4 \quad x_1 + 7x_2 + x_3 + x_4 = 12 \quad x_1 + x_2 + 6x_3 + x_4 = -5$
$x_1 + x_2 + x_3 + 4x_4 = -6$

5) $x + 2y + 3z = 14 \quad 4x + 5y + 7z = 35 \quad 3x + 3y + 4z = 21$

6) $x - 4y + 7z = 14 \quad 3x + 8y - 2z = 13 \quad 7x - 8y + 26z = 5$

7) $2x+y+z=10$: $3x+2y+3z=18$: $x+4y+9z=16$

8) $2x+2y+z=12$: $3x+2y+2z=8$: $5x+10y-8z=10$

9) $2x-y+3z=9$: $x+y+z=6$: $x-y+z=2$

10) $2x+2y+z=3$: $3x+2y+2z=-2$: $x-y+z=6$

4\. Apply Gauss Jordan method to solve the following equations.

1) $x + y + z = 9 \quad x - y + 2z = 5 \quad 3x + y + z = 8$

2) $x + y + z = 9 \quad x - 2y + 3z = 8 \quad 2x + y - z = 3$

3) $2x + 5y + 7z = 52 \quad 2x + y - z = 0 \quad x + y + z = 9$

i) $2x+y+z=10$: $3x+2y+3z=18$: $x+4y+9z=16$

ii) $2x-3y+z==1$: $x+4y+5z=25$: $3x-4y+z=2$

i) $x+3y+3z=16$: $x+4y+3z=18$: $x+3y+4z=19$

11) $2x+y+5z+w=5$: $x+y-3z+4w=-1$: $3x+6y-2z+w=8$: $2x+2y+2z-3w=2$

5\. Apply Gauss – Seidal iterative method to solve the following equations.

1) $10x + y + z = 12 \quad x + 10y + z = 12 \quad x + y + 10z = 12$

2) $x + y + 54z = 110 \quad 27x + 6y - z = 85 \quad 6x + 15y + 2z = 72$

3) $20x + y - 2z = 17 \quad 3x + 20y - z = 18 \quad 2x - 3y + 20z = 25$

6\. Find the eigen values and eigen vectors of the following:

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} \quad A = \begin{bmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 5 \end{bmatrix} \quad A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix} \quad A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} \quad A = \begin{bmatrix} 6 & -2 & 0 \\ -2 & 9 & 0 \\ 5 & 8 & 3 \end{bmatrix} \quad A = \begin{bmatrix} 2 & 4 & 3 \\ -4 & -6 & -3 \\ 3 & 3 & 1 \end{bmatrix} \quad A = \begin{bmatrix} 4 & 2 & 3 \\ -1 & 1 & -3 \\ 2 & 4 & 9 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 3 & 3 \\ -3 & -5 & -3 \\ 3 & 3 & 1 \end{bmatrix} \quad A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & -3 & 0 \\ 4 & -13 & 1 \end{bmatrix} \quad A = \begin{bmatrix} 4 & -1 & 6 \\ 2 & 1 & 6 \\ 2 & -1 & 8 \end{bmatrix} \quad A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

$$\lambda^3 - (\textstyle\sum d)\lambda^2 + (\textstyle\sum md)\lambda - |A| = 0$$

7.

1) Find the largest (Dominant) eigen value and the corresponding eigen vector of the matrix by power method $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ taking the initial eigen vector $[1 \quad 0 \quad 0]'$

2) Find the largest (Dominant) eigen value and the corresponding eigen vector of the matrix $A = \begin{bmatrix} 6 & 2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ by power method taking the initial eigen vector $[1 \quad 1 \quad 1]'$

3) Find the largest (Dominant) eigen value and the corresponding eigen vector of the matrix by power method $A = \begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix}$ taking the initial eigen vector $[1 \quad 0 \quad 0]'$

4) Find the largest (Dominant) eigen value and the corresponding eigen vector of the matrix by power method $A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ taking the initial eigen vector $[1 \quad 0 \quad 0]'$

5) Find the largest (Dominant) eigen value and the corresponding eigen vector of the matrix by power method $A = \begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix}$ taking the initial eigen vector $[1 \quad 0 \quad 1]'$

6) Find the largest (Dominant) eigen value and the corresponding eigen vector of the matrix by power method $A = \begin{bmatrix} 10 & 2 & 1 \\ 2 & 10 & 1 \\ 2 & 1 & 10 \end{bmatrix}$ taking the initial eigen vector $[1 \quad 1 \quad 1]'$