QUESTION BANK
ENGINEERING MATHEMATICS-I

Module - I

Polar curves: angle between the radius vector and tangent, angle of intersection of polar curves. Pedal equation for polar curves. Curvature and radius of curvature -Cartesian (with proof) parametric. (without proof) , polar(without proof) and pedal forms. (with proof)

POLAR CURVES

I)Book work: With usual notations prove that $\tan\phi = r\frac{d\theta}{dr}$.

II) Book work: With usual notations prove that $\frac{1}{p^2} = \frac{1}{r^2}\left(1+\left(\frac{1}{r}\frac{dr}{d\theta}\right)^2\right)$

III) Show that the following pairs of polar curves intersect orthogonally:

(1) $r = a(1+\cos\theta),\ r = b(1-\cos\theta)$
(2) $r^n = a^n \cos n\theta,\ r^n = b^n \sin n\theta,$
(3) r = a(1+sin θ), r = b (1-sin θ)
(4) $r = a\theta, r = \frac{a}{\theta}$
(5) $r^2 \sin 2\theta = a^2,\ r^2 \cos 2\theta = b^2$
(6) $r = ae^{\theta},\ re^{\theta} = b.$

IV)Find the angle of intersection for each of the following pairs of polar curves:
(1) r = 2sin θ, r = sin θ+ cos θ
(2) r = a(1+cos θ), r=2 a cos θ
(3) r = a log θ, r= a/log θ
(4) $r = \frac{a\theta}{1+\theta}, r = \frac{a}{1+\theta^2}$
(5) $r = \frac{3\theta}{1+\theta}, r = \frac{10}{1+\theta^2}$
(6) $r = a\sin 2\theta, r = a\cos 2\theta$
(7) $r^2 \sin 2\theta = 4, r^2 = 16\sin 2\theta$

V) Show that the tangents to the polar curve r = a(1+cos θ) **at the points** $\theta = \pi/3$ *and* $\theta = 2\pi/3$ **are respectively parallel and perpendicular to the initial line.**

VI. Find the pedal equations of the following polar curves:
(1) $r = a\theta$
(2) $r = a(1-\cos\theta)$
(3) $r = a(1+\cos\theta)$
(4) $\frac{2a}{r} = 1-\cos\theta$

(5) $r^n = a^n \cos n\theta$
(6) $r^m \cos m\theta = a^m$
(7) $r^n = a^n(\cos n\theta + \sin n\theta)$
(8) $r^n = a^n \sin n\theta + b^n \cos n\theta$
(9) $\frac{l}{r} = 1 + e\cos\theta$
(10) $r = ae^{\theta \cot\alpha}$
(11) $r^2 = a^2 \sec 2\theta$
(12) $r = a\,\mathrm{cosec}^2\left(\frac{\theta}{2}\right)$
(13) $r = a\sin^3(\theta/3)$
(14) $r\theta = a$
(15) $r^2 = a^2 \sin 2\theta$

V. Show that for the curve $r\cos\left(\frac{1}{a}\sqrt{a^2-b^2}\right)\theta = \sqrt{a^2-b^2}$, the pedal equation is

$$p^2 = \frac{a^2r^2}{r^2+b^2}.$$

VI Radius of Curvature

1) The radius of curvature in the Cartesian form. (with proof)
2) The radius of curvature in the Parametric form.(without proof)
3) The radius of curvature in the polar form. (without proof) .
4) The radius of curvature in the Pedal form. . (with proof)
5) Find the radius of curvature of the curve $y = ax^2 + bx + c$ at the point for which $x = \frac{1}{2a}\left[\sqrt{a^2-1} - b\right]$
6) Show that the radius of curvature of the curve $y = 4\sin x - \sin 2x$ at $x = \frac{\pi}{2}$ $is 5\sqrt{5}$
7) Find the radius of curvature of the curve $x^3 + y^3 = 3axy$ at the point $\left(\frac{3a}{2}, \frac{3a}{2}\right)$
8) Find the radius curvature of the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ at the point where it cuts the line y= x
9) For the curve $y = \frac{ax}{a+x}$, where 'a' is a constant, prove that $\left(\frac{2\rho}{a}\right)^{\frac{2}{3}} = \left(\frac{y}{x}\right)^2 + \left(\frac{x}{y}\right)^2$

10) Find the radius of curvature for the curve $y^2 = \frac{a^2(a-x)}{x}$ at $(a,0)$.

11) Find the radius of curvature for the curve $y^2 = \frac{4a^2(2a-x)}{x}$ where the curve meets x-axis.

12) Find the radius of curvature of the curve $x^2y = a(x^2 + y^2)$ at the point (-2a, 2a)

13) Find the radius of curvature of the curve $a^2y = x^3 - a^3$ at the point where it cuts the x - axis

14) Prove that for the rectangular hyperbola $xy = c^2$, the radius of curvature at any point $p(x,y)$ is given by $\rho = \frac{r^3}{2c^2}$, where r is distance of the point p from the origin.

15) Prove that for the parabola $y^2 = 4ax$, the square of the radius of curvature at any point varies as the cube of the focal distance of the point.

16) Find the radius of curvature for the curve

$x = a(\theta + \sin\theta)$ $y = a(1 - \cos\theta)$

17) Find the radius of curvature for the curve

$x = a\cos^3\theta$ $y = a\sin^3\theta$

18) Find the radius of curvature for the curve

$x = a(\cos t + t\sin t)$ $y = a(\sin t - t\cos t)$

19) Show that radius of curvature for the curve $r = a(1 + Cos\theta)$ is $\rho = \frac{2}{3}\sqrt{2ar}$

20) For the curve $r = a(1 + \cos\theta)$, prove that $\frac{\rho^2}{r}$ = constant

21) Find the radius of curvature at any point of the Cardioid $r = a(1 - \cos\theta)$

22) Find the radius of curvature for the curve $r^n = a^n \cos n\theta$

23) Find the radius of curvature for the curve $\frac{2a}{r} = 1 + \cos\theta$