Hybrid SABR-Local Volatility Engine
Theoretical Framework and Industrial Applications

Quantitative Development Repository

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1 Introduction

Stochastic volatility models like SABR are standard for modeling the "smile" but often fail to re-price every market quote exactly. Conversely, Local Volatility (LV) models fit all quotes but lack realistic forward-volatility dynamics. The **Hybrid SABR-LV** approach reconciles these by using SABR as a "prior" and a leverage function for exact calibration.

2 The Mathematical Foundation

2.1 Shifted SABR Dynamics

To accommodate negative interest rates, we define the forward rate F_t with a constant shift s:

$$d(F_t + s) = \alpha_t (F_t + s)^\beta dW_{1,t} \tag{1}$$

$$d\alpha_t = \nu \alpha_t dW_{2,t} \tag{2}$$

$$\langle dW_{1,t}, dW_{2,t} \rangle = \rho dt \tag{3}$$

Where:

- α: Initial volatility (ATM level).

- β: Elasticity (fixed at 0.5 for FX/Rates stability).

- ρ: Correlation (controls skew/asymmetry).

- ν: Vol-of-vol (controls curvature).

2.2 Dupire Local Volatility Mapping

The local volatility $\sigma_{loc}(K, T)$ is extracted from the SABR-implied density using the Dupire equation:

$$\sigma_{loc}^2(K, T) = \frac{\frac{\partial C}{\partial T} + (r - q)K\frac{\partial C}{\partial K} + qC}{\frac{1}{2}K^2 \frac{\partial^2 C}{\partial K^2}} \tag{4}$$

3 Industrial Use Cases

3.1 Pricing Continuous Barrier Options

Barrier options (e.g., Down-and-Out Calls) are highly sensitive to the local shape of the volatility surface. While SABR provides a good global fit, the Hybrid engine ensures the model re-prices the specific vanillas near the barrier level, reducing "model risk" during knock-out events.

3.2 Market Making in FX Skew

In FX markets, risk reversals (ρ) and butterfly (ν) quotes move rapidly. This engine allows a market-making desk to update their 3D surface in real-time, providing consistent prices for client-driven exotics like One-Touch or No-Touch options.

3.3 Structured Products & TARNs

Target Redemption Notes (TARNs) require accurate forward-starting volatility. The SABR component provides the realistic stochastic "movement" of the smile, while the LV component ensures today's risk is perfectly hedged against vanilla instruments.