Lab 4

Analog (continuous-time) filters: design and analysis systems

EECS3451

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1. Introduction:

Using MATLAB to design analog filters such as Butterworth, Chebyshev (Type I and II), and Elliptic filters and to learn how to apply filter to real-world problem. This report is mainly answering the provided questions using MATLAB. Results will demonstrate the use of MATLAB properly in analysing and designing Analog filters.

2. Equipment: MATLAB

3. Results and discussion:

The answers to provided question are as follows,

Laboratory Exercise 1

Using Matlab to design following filters. In your report, please include: (1) Transfer function of each filter; (2) Plot of the magnitude response and Bode plot for each filter; (3) Verify that the designed filter meets the given specifications; (4) Comments on the difference of these filters.

Q1

Q1. Design a Butterworth filter and a Type II Chebyshev filter to satisfy the following specifications:

Passband corner frequency: $0 \leq |\omega| \leq 10$ radians/s
Stopband corner frequency: $|\omega| > 20$ radians/s
Passband ripple: $0.9 \leq |H(\omega)| \leq 1$
Stopband ripple: $|H(\omega)| \leq 0.15$

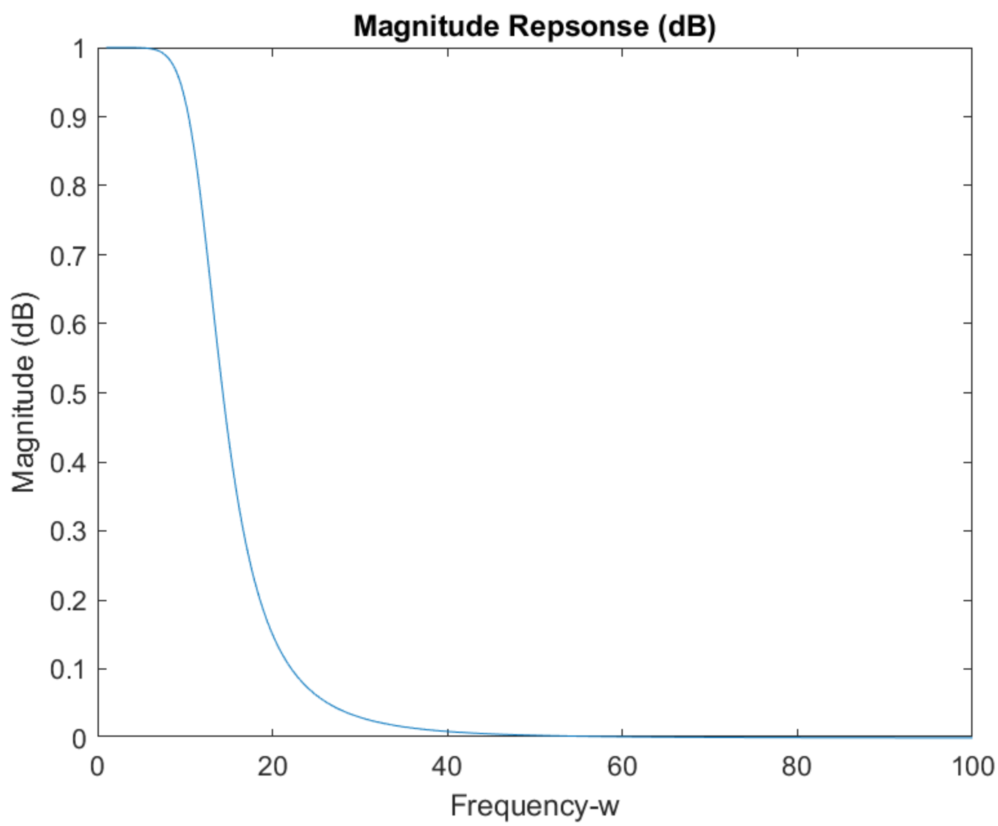
Butterworth filter

```
Wp =10;
Ws =20;
Pb = 0.9; %Passband
Sb = 0.15; %Stopband
Rp = -20*log10(Pb);
Rs = -20*log10(Sb);

[N,Wc]= buttord(Wp,Ws,Rp,Rs, 's')
[num,den]=butter(N,Wc,'s')
Ht=tf(num,den)
[H,w]=freqs(num,den);

% plot magnitude response
plot(w, abs(H));
xline(Wp,'--k',sprintf('Wp=%0.2f', Wp))
xline(Ws,'--k',sprintf('Ws=%0.2f', Ws))
yline(Pb,'--k',sprintf('delta_p=%0.2f', Pb))
yline(Sb,'--k',sprintf('delta_s=%0.2f', Sb))
xlabel('Frequency-w');
```

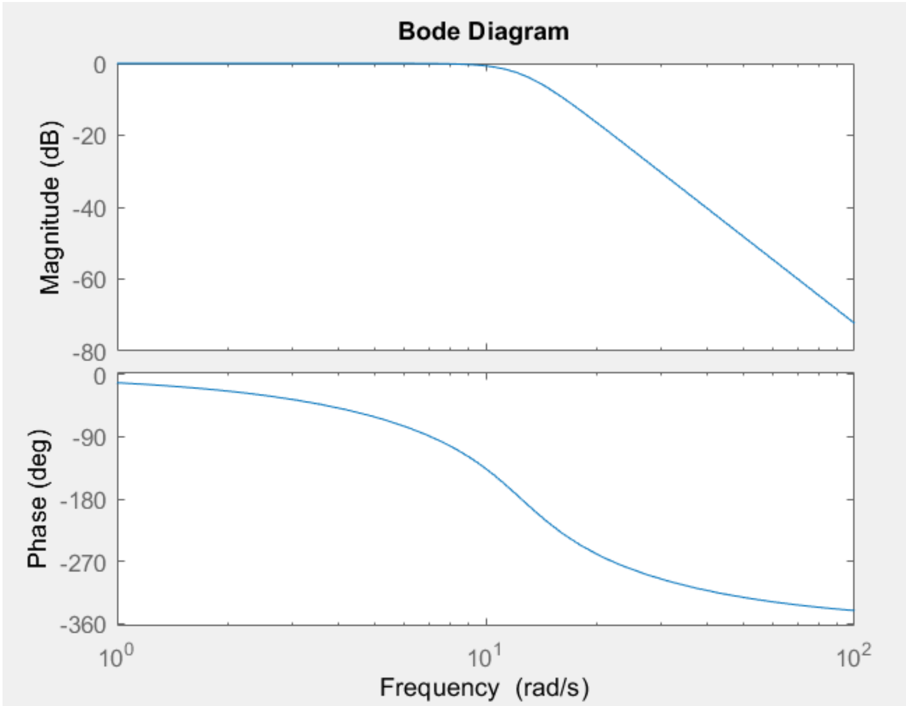
```
ylabel('Magnitude (dB)');
title('Magnitude Repsonse (dB)');
```

1) Transfer function = $\frac{2.428e04}{s^4 + 32.62\,s^3 + 532\,s^2 + 5082\,s + 2.428e04}$

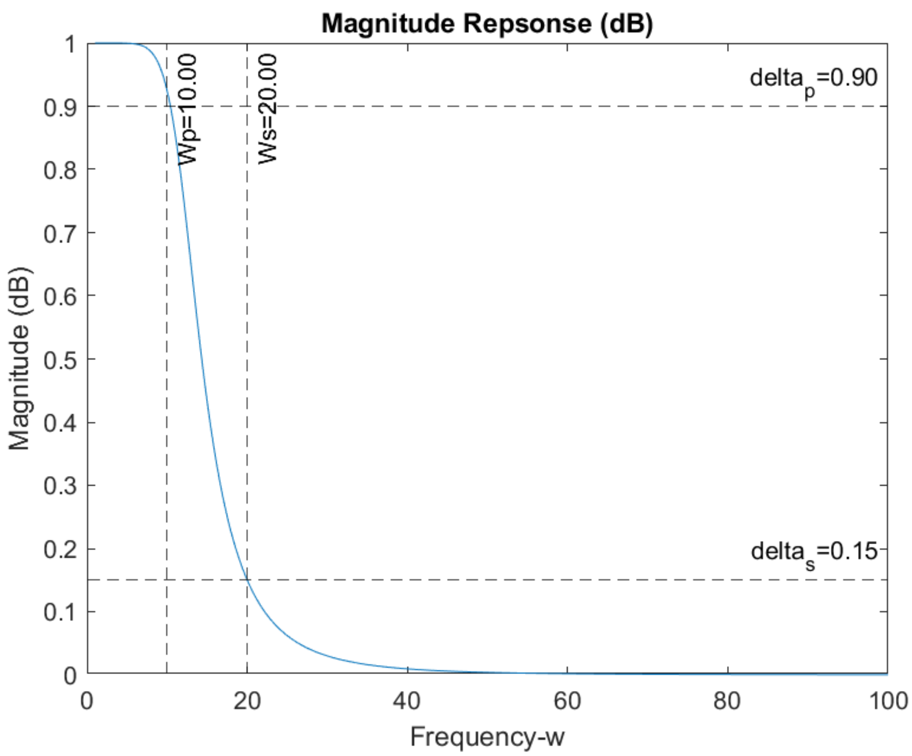
2) Magnitude response plot



Bode plot



3) Verify



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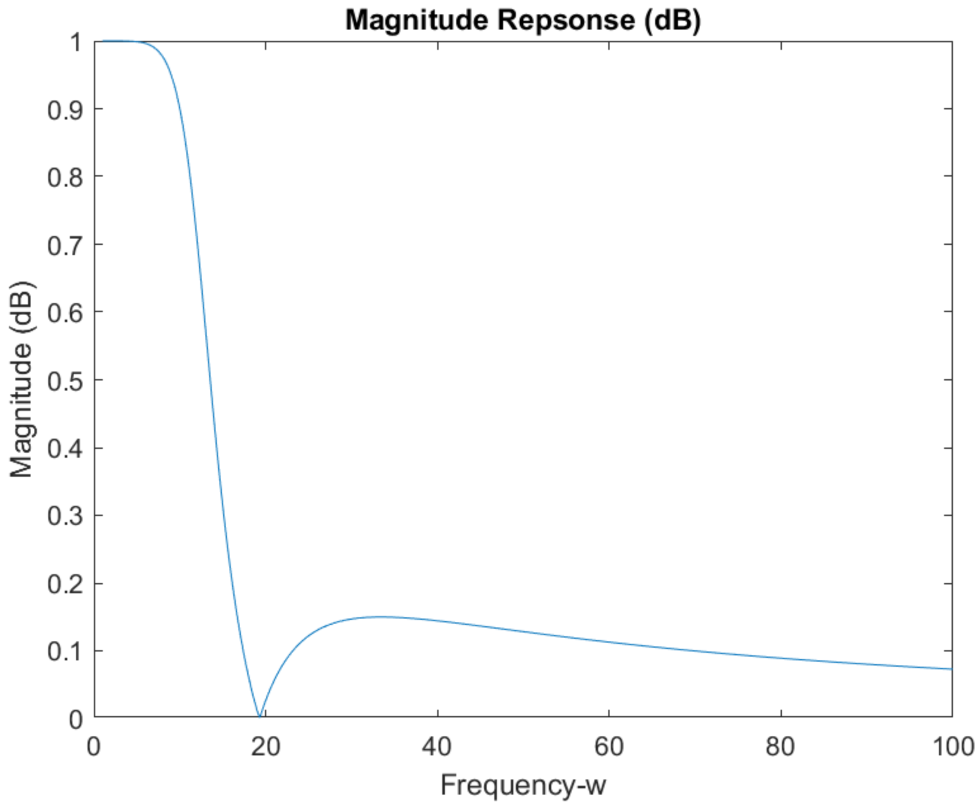
Type II Chebyshev Filter

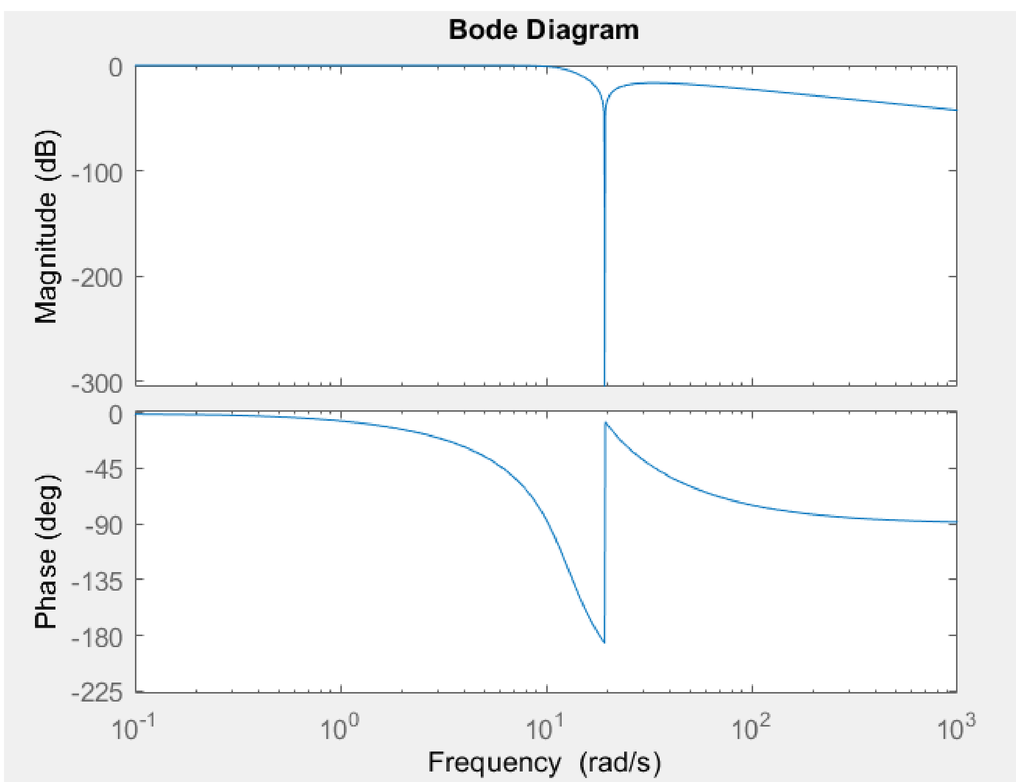
```
Wp =10;
Ws =20;
Pb = 0.9; %Passband
Sb = 0.15; %Stopband
Rp = -20*log10(Pb);
Rs = -20*log10(Sb);

[N,Wn]= cheb2ord(Wp,Ws,Rp,Rs, 's')
[num,den]=cheby2(N,Rs,Wn,'s')
Ht=tf(num,den)
[H,w]=freqs(num,den);

% plot magnitude response
plot(w, abs(H));
xline(Wp,'--k',sprintf('Wp=%0.2f', Wp))
xline(Ws,'--k',sprintf('Ws=%0.2f', Ws))
yline(Pb,'--k',sprintf('delta_p=%0.2f', Pb))
yline(Sb,'--k',sprintf('delta_s=%0.2f', Sb))
xlabel('Frequency-w');
ylabel('Magnitude (dB)');
title('Magnitude Repsonse (dB)');

% Bode plot
bode(Ht);
```

1) Transfer function = $\frac{7.599\,s^2 + 2825}{s^3 + 26.75\,s^2 + 328.9\,s + 2825}$

2) Magnitude response plot


Magnitude Repsonse (dB)
Magnitude (dB)
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
20
40
60
80
100
Frequency-w

Bode plot


Bode Diagram
Magnitude (dB)
0
-100
-200
-300
Phase (deg)
0
-45
-90
-135
-180
-225
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
Frequency (rad/s)

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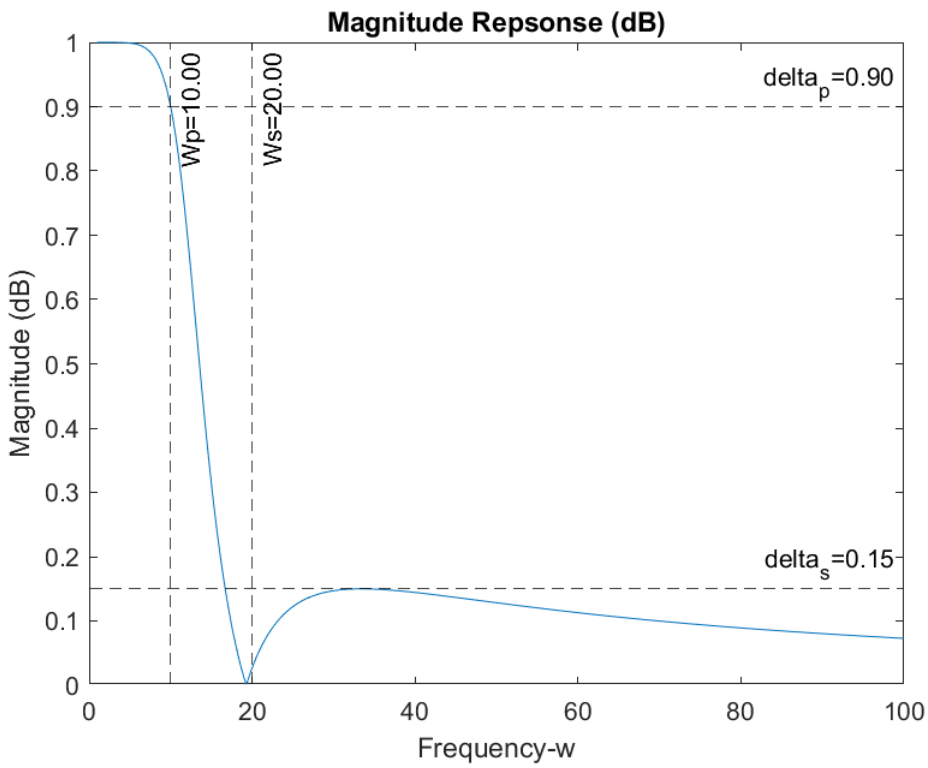
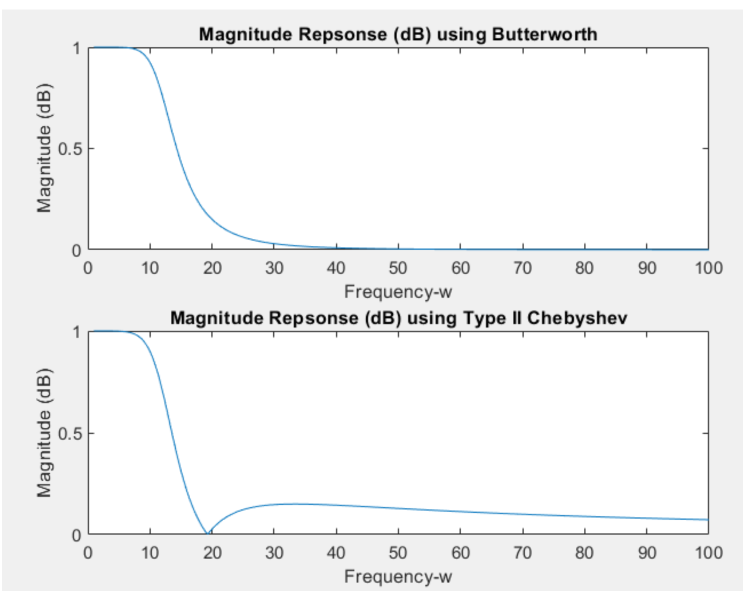
3) Verify



Difference between Butterworth and Type II Chebyshev Filters



Butterworth filter is flat during pass band and stop band, while in Type II Chebyshev it is flat during pass band but has ripples during stop band.

Transition band of Butterworth filter is larger than Type II Chebyshev.

Q2

Q2. Design a Type-I Chebyshev filter and an Elliptic filter to satisfy the following specifications:

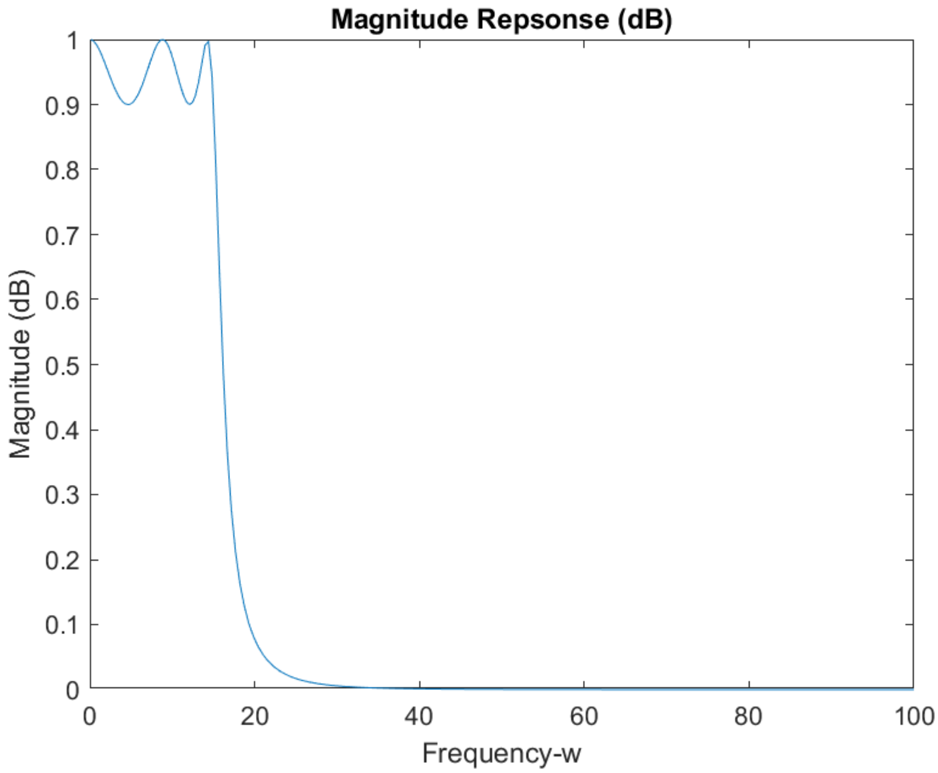
Passband corner frequency: $0 \leq |\omega| \leq 15$ radians/s
Stopband corner frequency: $|\omega| > 20$ radians/s
Passband ripple: $0.9 \leq |H(\omega)| \leq 1$
Stopband ripple: $|H(\omega)| \leq 0.15$

Type I Chebyshev filter

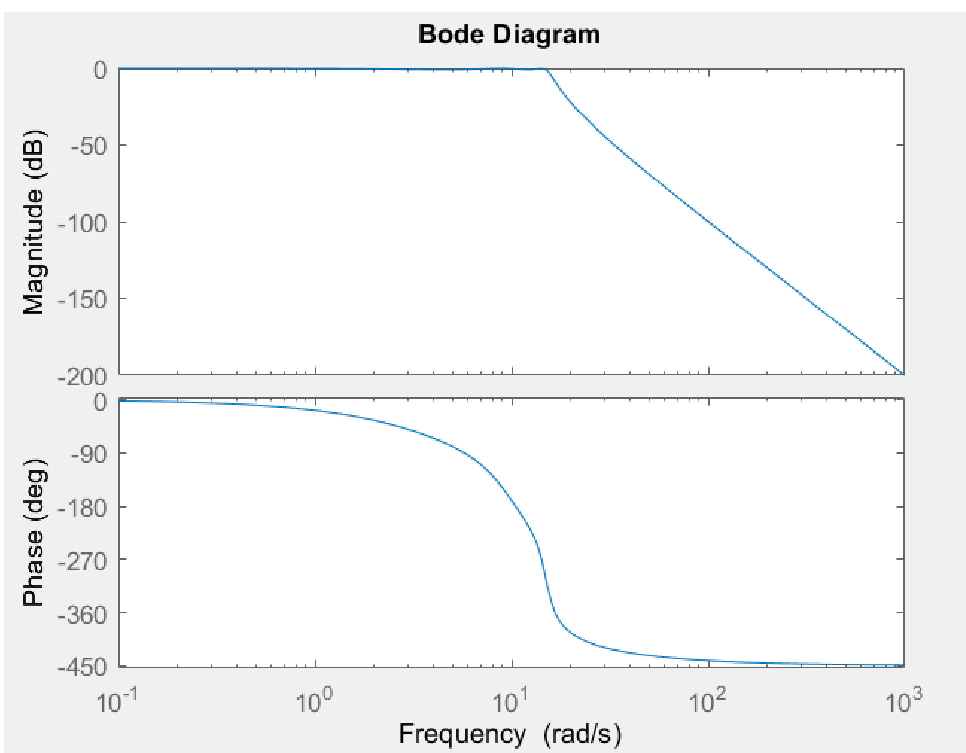
```
Wp =15;
Ws =20;
Pb = 0.9; %Passband
Sb = 0.15; %Stopband
Rp = -20*log10(Pb);
Rs = -20*log10(Sb);

[N,Wn]= cheb1ord(Wp,Ws,Rp,Rs, 's');
[num,den]=cheby1(N,Rp,Wn,'s');
Ht=tf(num,den)
[H,w]=freqs(num,den);

% plot magnitude response
plot(w, abs(H));
xlabel('Frequency-w');
ylabel('Magnitude (dB)');
title('Magnitude Repsonse (dB)');
xline(Wp,'--k',sprintf('Wp=%0.2f', Wp))
xline(Ws,'--k',sprintf('Ws=%0.2f', Ws))
yline(Pb,'--k',sprintf('delta_p=%0.2f', Pb))
yline(Sb,'--k',sprintf('delta_s=%0.2f', Sb))

% Bode plot
bode(Ht);
```

4) Transfer function = $\dfrac{9.799e04}{s^5 + 14.5\, s^4 + 386.4\, s^3 + 3422\, s^2 + 3.035e04\, s + 9.799e04}$

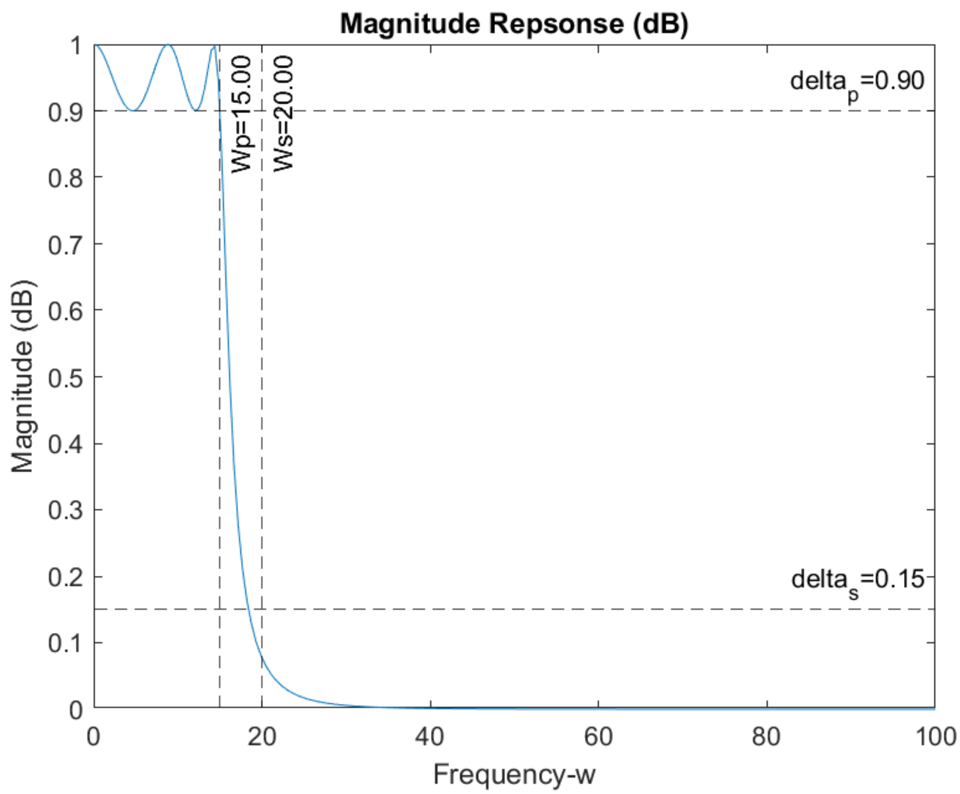
5) Magnitude response plot



Bode plot



6) Verify



Elliptic Filter

```
Wp =10;
Ws =20;
Pb = 0.9; %Passband
Sb = 0.15; %Stopband
Rp = -20*log10(Pb);
Rs = -20*log10(Sb);

[N,Wn]= cheb2ord(Wp,Ws,Rp,Rs, 's')
[num,den]=cheby2(N,Rs,Wn,'s')
Ht=tf(num,den)
[H,w]=freqs(num,den);

% plot magnitude response
plot(w, abs(H));
xline(Wp,'--k',sprintf('Wp=%0.2f', Wp))
xline(Ws,'--k',sprintf('Ws=%0.2f', Ws))
yline(Pb,'--k',sprintf('delta_p=%0.2f', Pb))
yline(Sb,'--k',sprintf('delta_s=%0.2f', Sb))
xlabel('Frequency-w');
ylabel('Magnitude (dB)');
title('Magnitude Repsonse (dB)');
```

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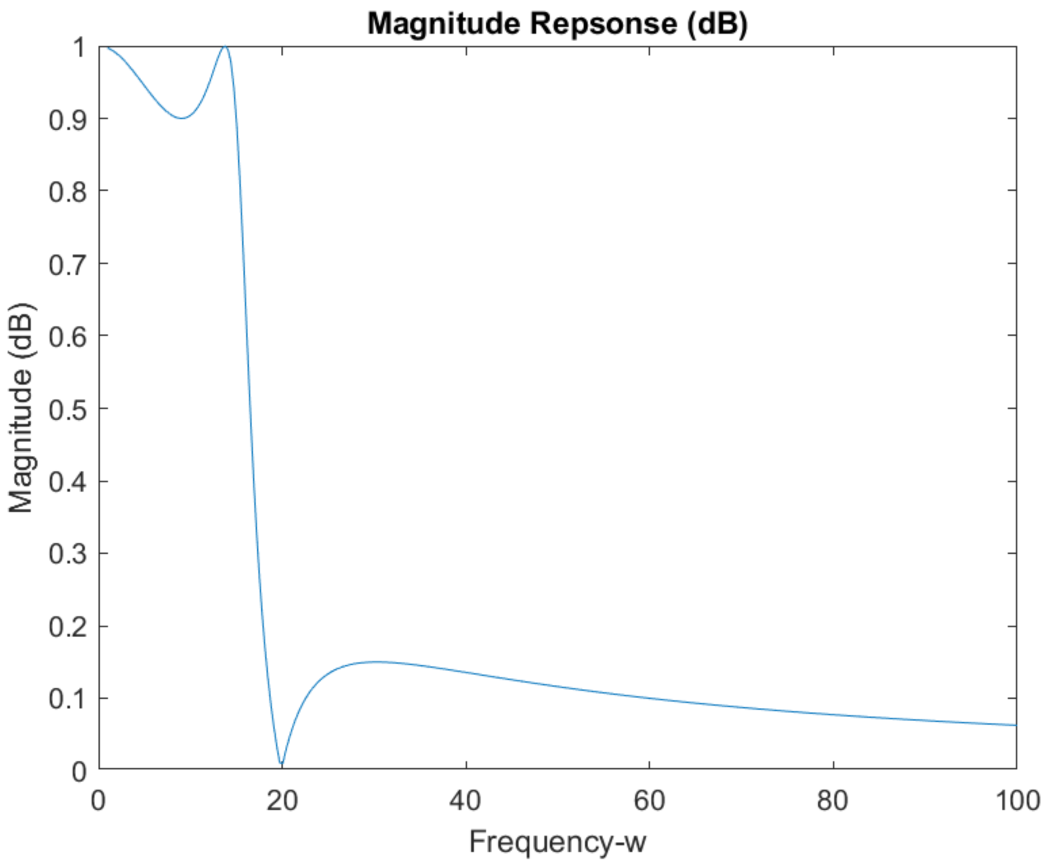
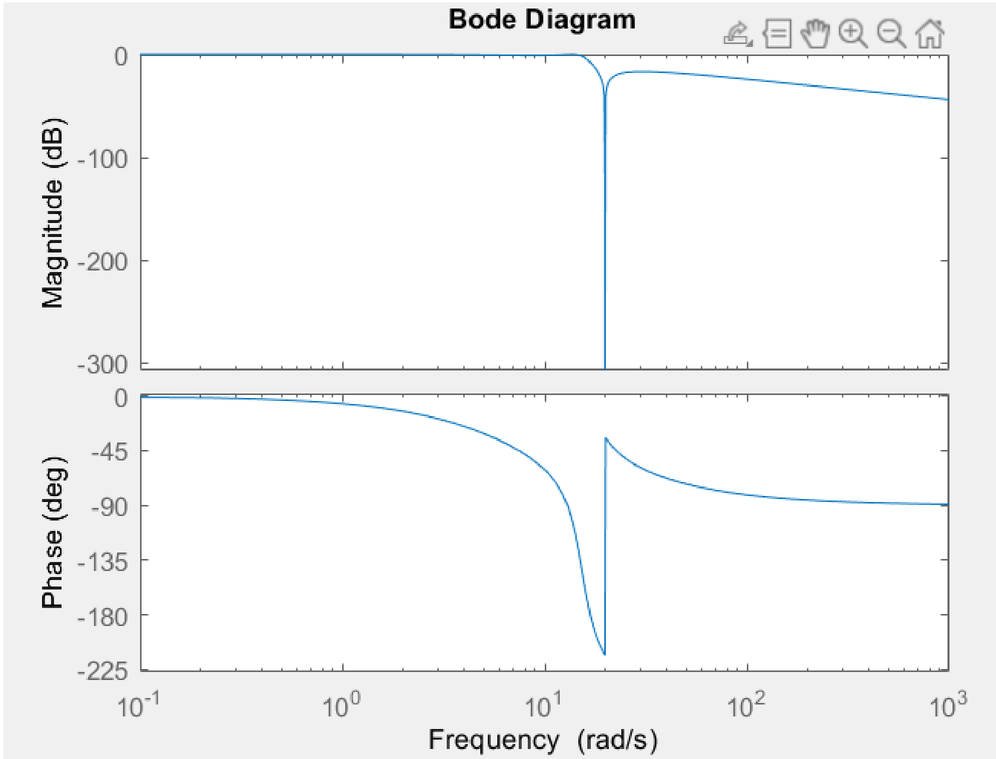
```
% Bode plot
bode(Ht);
```

4) Transfer function = $\frac{6.412\, s^2 + 2538}{s^3 + 15.08\, s^2 + 282.8\, s + 2538}$

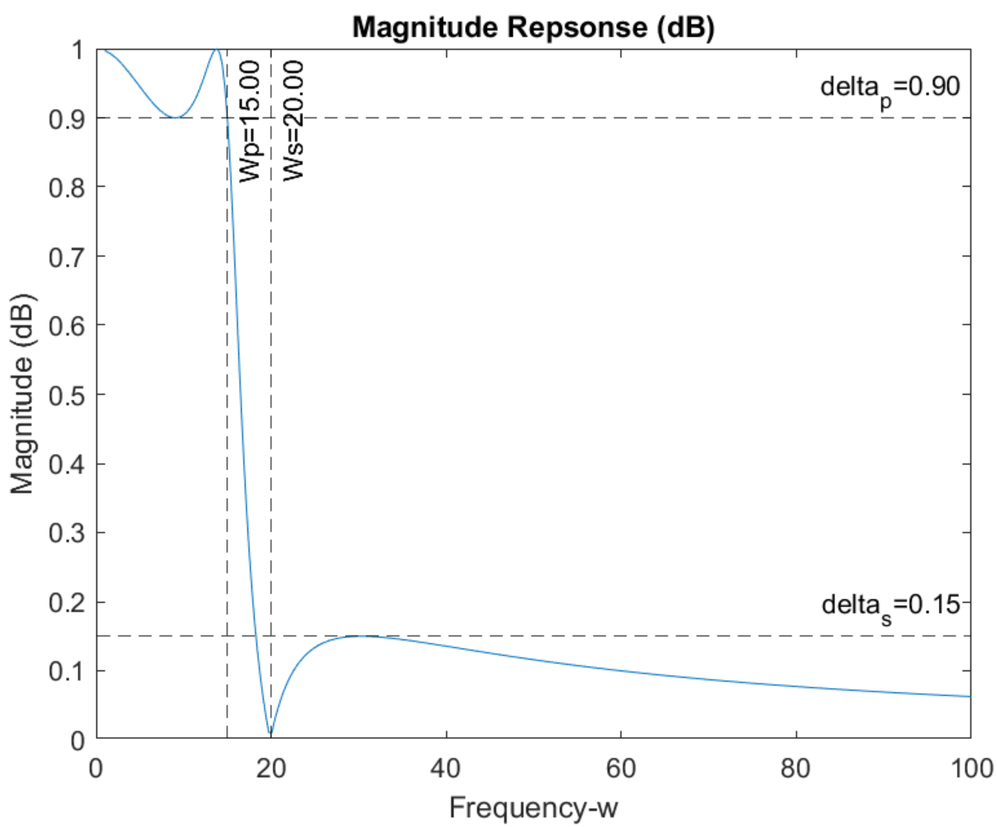
5) Magnitude response plot



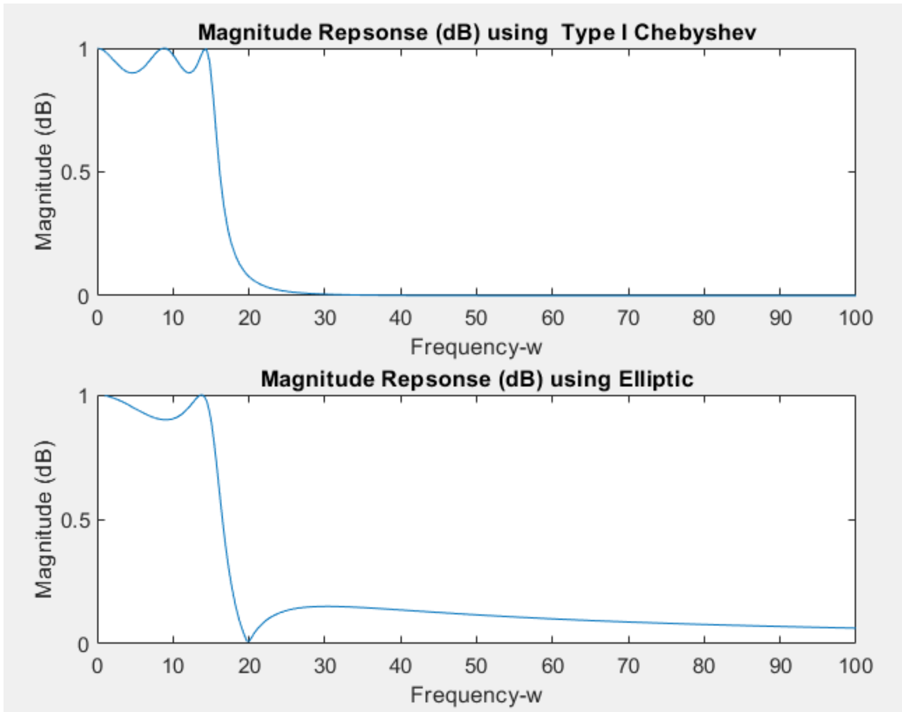
Bode plot


Bode Diagram
Magnitude (dB)
0
-100
-200
-300
Phase (deg)
0
-45
-90
-135
-180
-225
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
Frequency (rad/s)

6) Verify


Magnitude Repsonse (dB)
Magnitude (dB)
Wp=15.00
Ws=20.00
delta$_p$=0.90
delta$_s$=0.15
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
20
40
60
80
100
Frequency-w

Difference between Type I Chebyshev Filters and Elliptic Filter



Type I Chebyshev filter is flat during stop band and pass band ripples, while in Elliptic it has ripples in both pass band and stop band.

Transition band of Type I Chebyshev filter is larger than Elliptic.

Laboratory Exercise 2

The exercise will have you to design a filter for an application in the area of electro-cardiology. Electro-cardiology is the study of the electric signals produced by the heart. These signals maintain the heart's rhythmic beat, and they are measured by an instrument called an electrocardiograph (ECG). This instrument must be capable of detecting periodic signals whose frequency is about 1Hz. The instrument must also be able to operate in the presence of sinusoidal noise consisting of signals from the surrounding electrical environment, whose fundamental frequency is 60Hz (i.e. frequency at which electric power is supplied).

Your task is to design a filtering system such that it could be used in an ECG system *to filter out any noises below 1Hz and above 10Hz* and *pass at least 90% of the ECG signals between 1Hz to 10Hz*. To achieve this, the filter needs to meet the following requirements:

- The attenuation at 0.1Hz is at least 20dB,
- The ripple within passband from 1Hz to 10Hz is between 0.9 and 1,
- The attenuation at 60Hz is at least 40dB.

Based on the requirements above, design this filter system and verify its performance by plotting its magnitude response and Bode plot. In your report, please include : (1) filter specifications based on given requirements, (2) Transfer function of the filter, (3) plot of magnitude response and Bode plot, and (4) verify that the designed filter meet the given requirements.

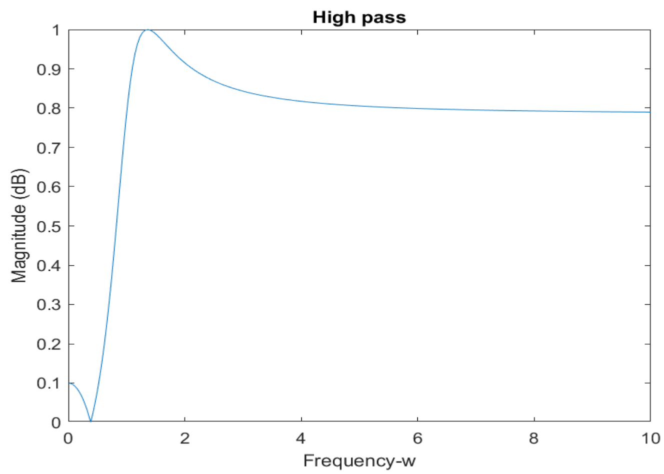
This Exercise is done using Elliptic and Butterworth filters

```
% 1) Using Elliptic filter

% high pass filter specifications
wp=1; ws=0.1; Rp=-20*log(0.9); Rs=20;
[N,Hwn] = ellipord(wp,ws,Rp,Rs,'s') ;
[num4,den4] = ellip(N,Rp,Rs,Hwn,'high','s') ;
H4 = tf(num4,den4);
```

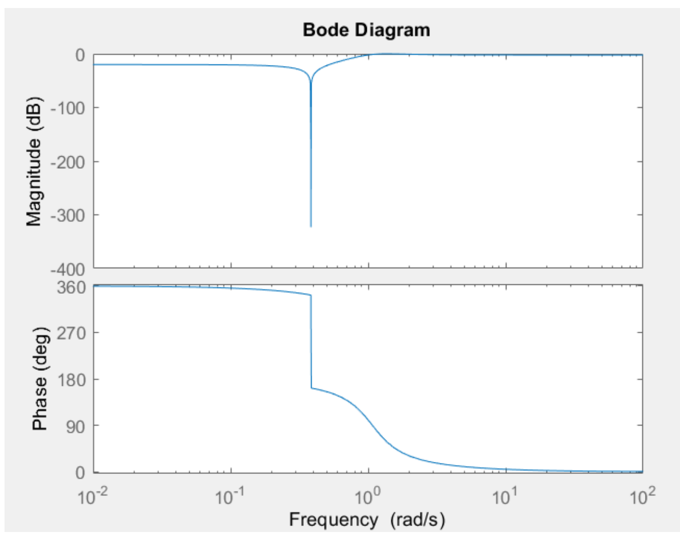
$$Transfer\ function = \frac{0.7846\,s^2 - 8.711e{-}17\,s + 0.1155}{s^2 + 0.8387\,s + 1.155}$$

```
[H,w]=freqs(num4,den4);
plot(w, abs(H));
xlabel('Frequency-w');
ylabel('Magnitude (dB)');
```

```
title('High pass');
```




```
bode(H4);
```

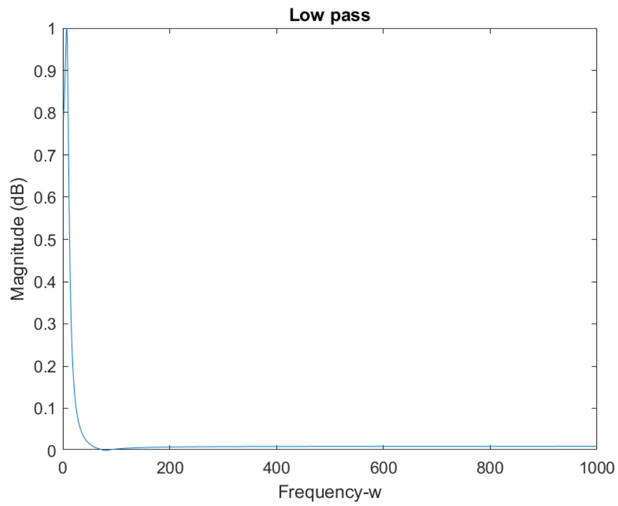



```
% low pass filter specifications
wp=10; ws=60; Rp=-20*log(0.9); Rs=40;
[N,Lwn] = ellipord(wp,ws,Rp,Rs,'s') ;
[num4,den4] = ellip(N,Rp,Rs,Lwn,'s') ;
H4 = tf(num4,den4);
```

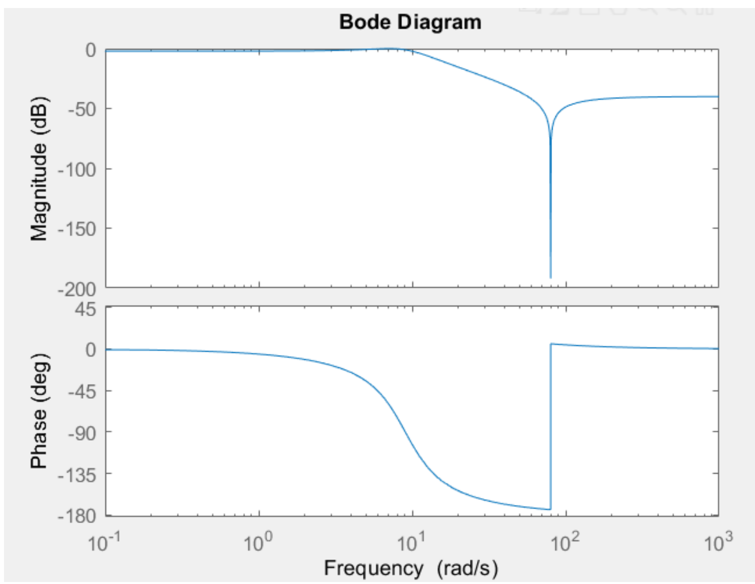
$$Transfer\ function = \frac{0.01\, s^2 + 63.77}{s^2 + 7.778\, s + 81.28}$$

```
[H,w]=freqs(num4,den4);
plot(w, abs(H));
xlabel('Frequency-w');
ylabel('Magnitude (dB)');
```

```
title('Low pass');
```



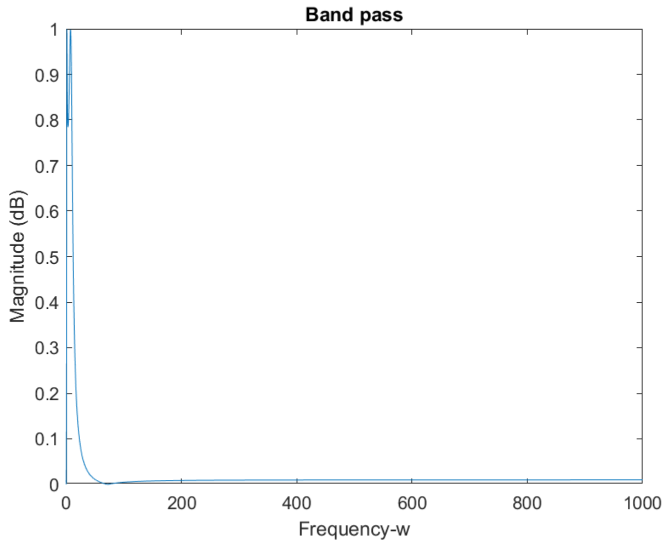

```
bode(H4)
```



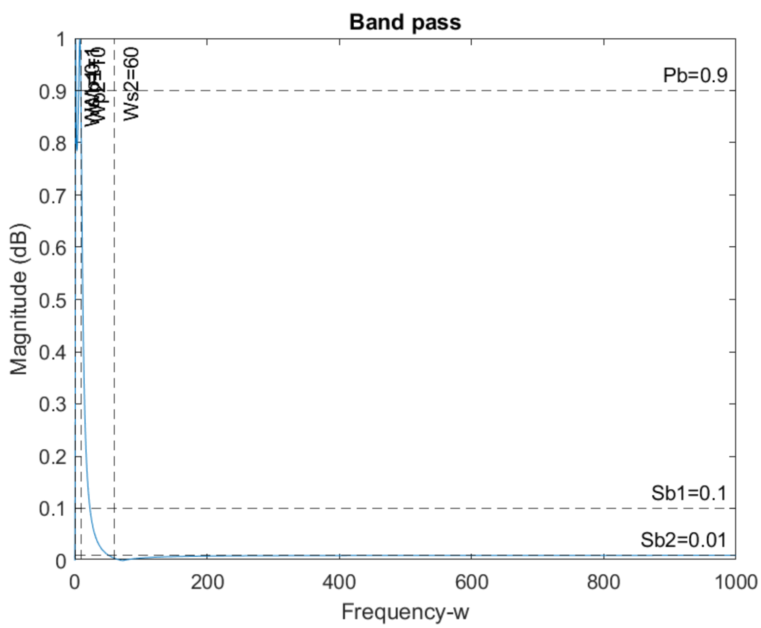

```
% Band pass filter specifications
[num,den] = ellip(N,Rp,Rs,[Hwn Lwn],'s') ;
H = tf(num,den);
```

$$Transfer\ function = \frac{0.01\,s^4 - 1.776e{-17}\,s^3 + 51.85\,s^2 - 1.776e{-16}\,s + 1}{s^4 + 7\,s^3 + 85.83\,s^2 + 70\,s + 100}$$

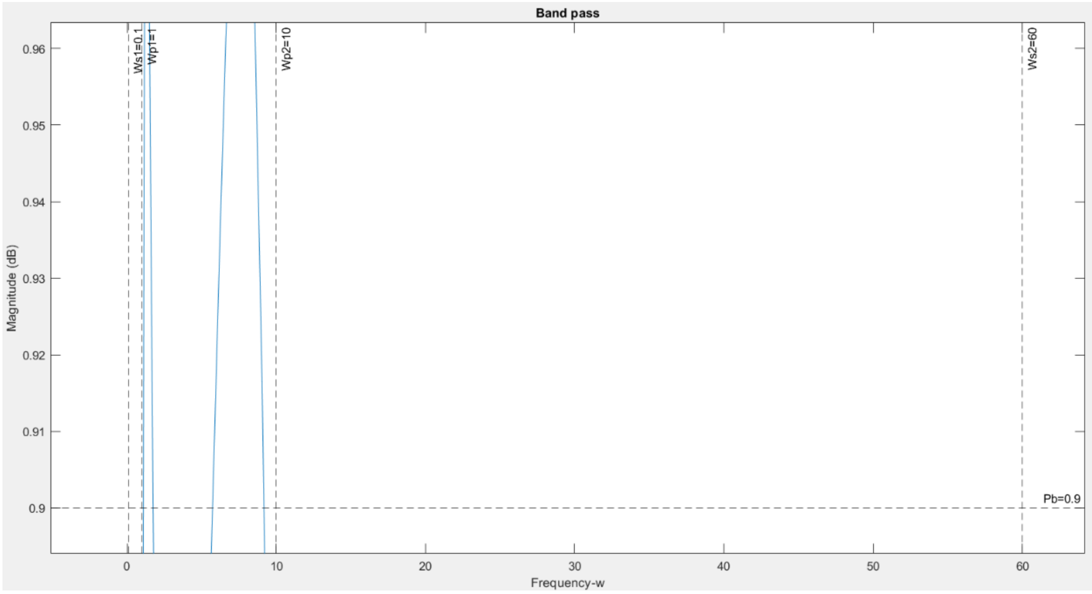
```
[PH,w]=freqs(num,den);
plot(w, abs(PH));
xlabel('Frequency-w');
ylabel('Magnitude (dB)');
title('Band pass');
```



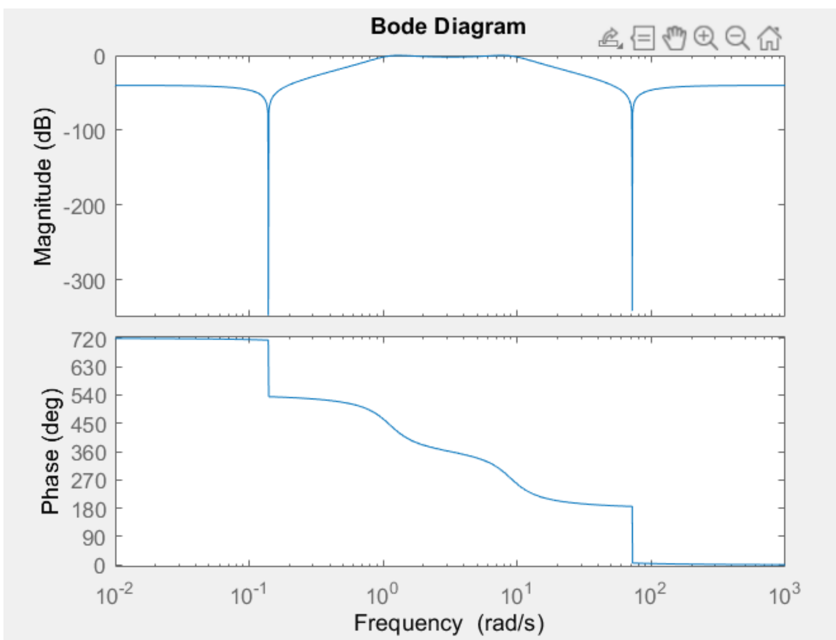

```
% Verification
xline(1,'--k','Wp1=1')
xline(10,'--k','Wp2=10')
xline(0.1,'--k','Ws1=0.1')
xline(60,'--k','Ws2=60')
yline(0.9,'--k','Pb=0.9')
yline(0.1,'--k','Sb1=0.1')

yline(0.01,'--k','Sb2=0.01')
```



closeup of a small range



bode(H)

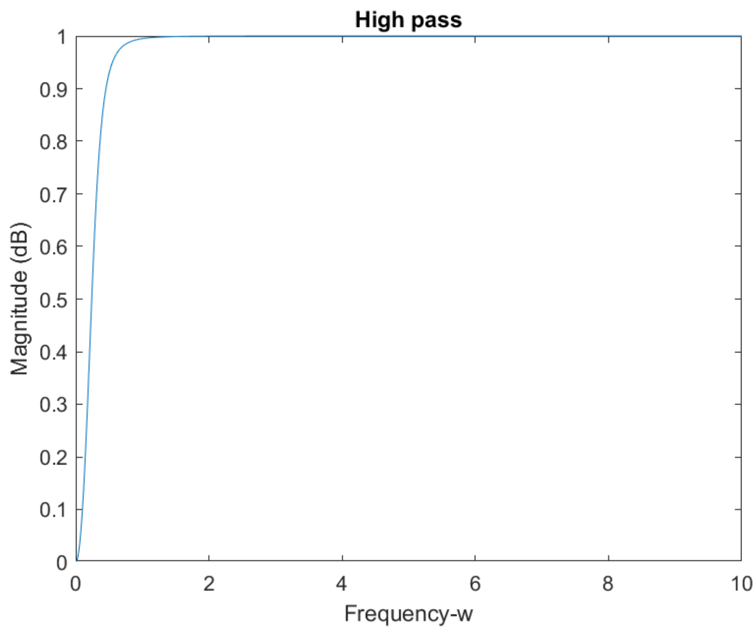



```
% 2) Using Butterworth filter

% high pass filter specifications
wp=1; ws=0.1; Rp=-20*log(0.9); Rs=20;
[N, Hwc] = buttord(wp,ws,Rp,Rs,'s');
```

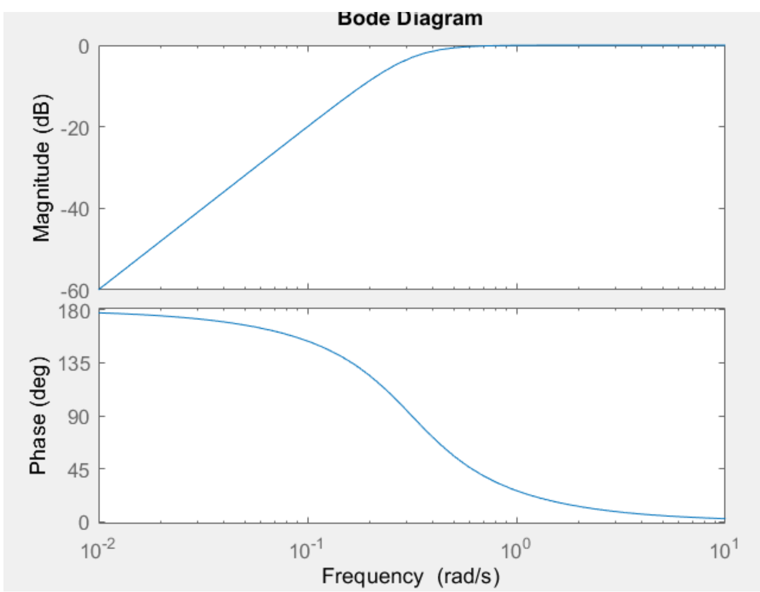
```
[num1,den1] = butter(N,Hwc,'high','s');
H4 = tf(num1,den1);
```

$$Transfer\ function = \frac{s^2}{s^2 + 0.4461\, s + 0.0995}$$

```
[H,w]=freqs(num1,den1);
plot(w, abs(H));
xlabel('Frequency-w');
ylabel('Magnitude (dB)');
title('High pass');
```




```
bode(H4)
```




```
% low pass filter specifications
```

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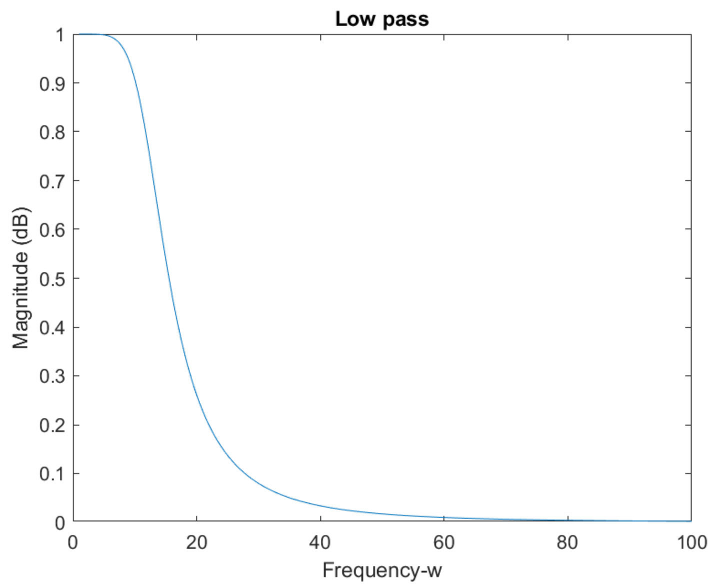
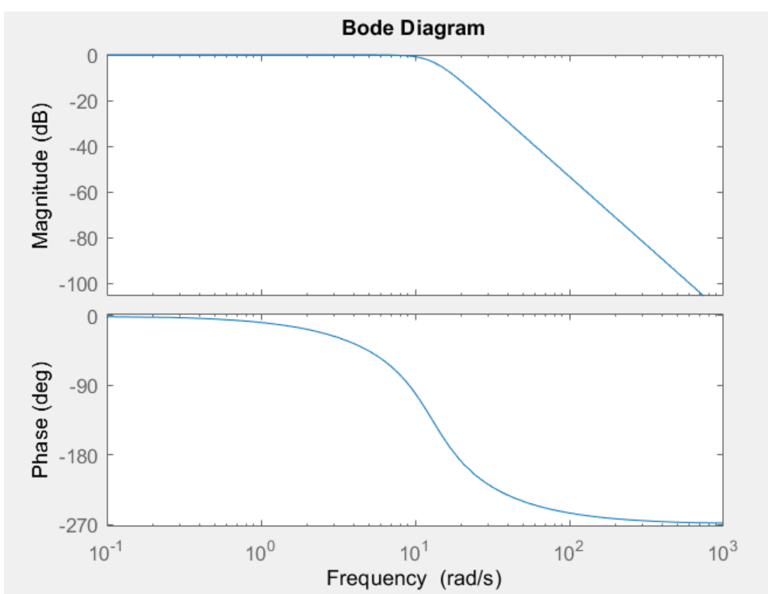
```
wp=10; ws=60; Rp=-20*log(0.9); Rs=40;
[N, Lwc] = buttord(wp,ws,Rp,Rs,'s');
[num1,den1] = butter(N,Lwc,'s');
H4 = tf(num1,den1);
```

$$Transfer\ function = \frac{2160}{s^3 + 25.85\, s^2 + 334.2\, s + 2160}$$

```
[H,w]=freqs(num1,den1);
plot(w, abs(H));
xlabel('Frequency-w');
ylabel('Magnitude (dB)');
title('Low pass');
```



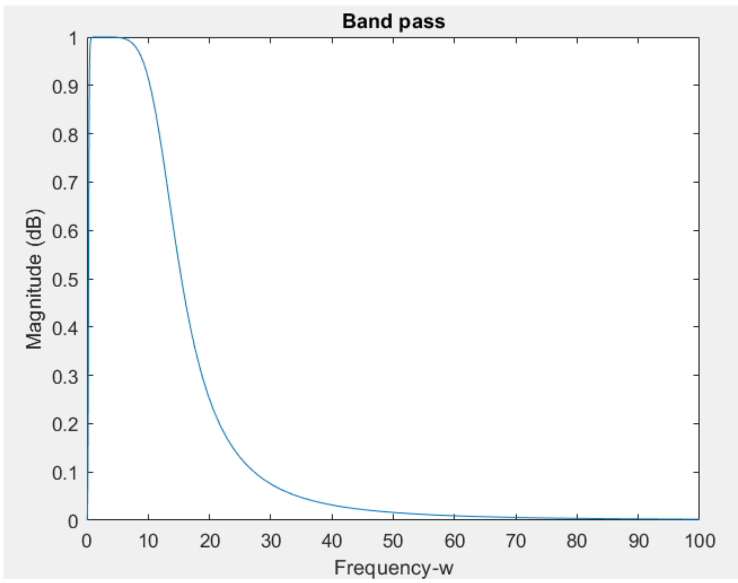

```
bode(H4)
```



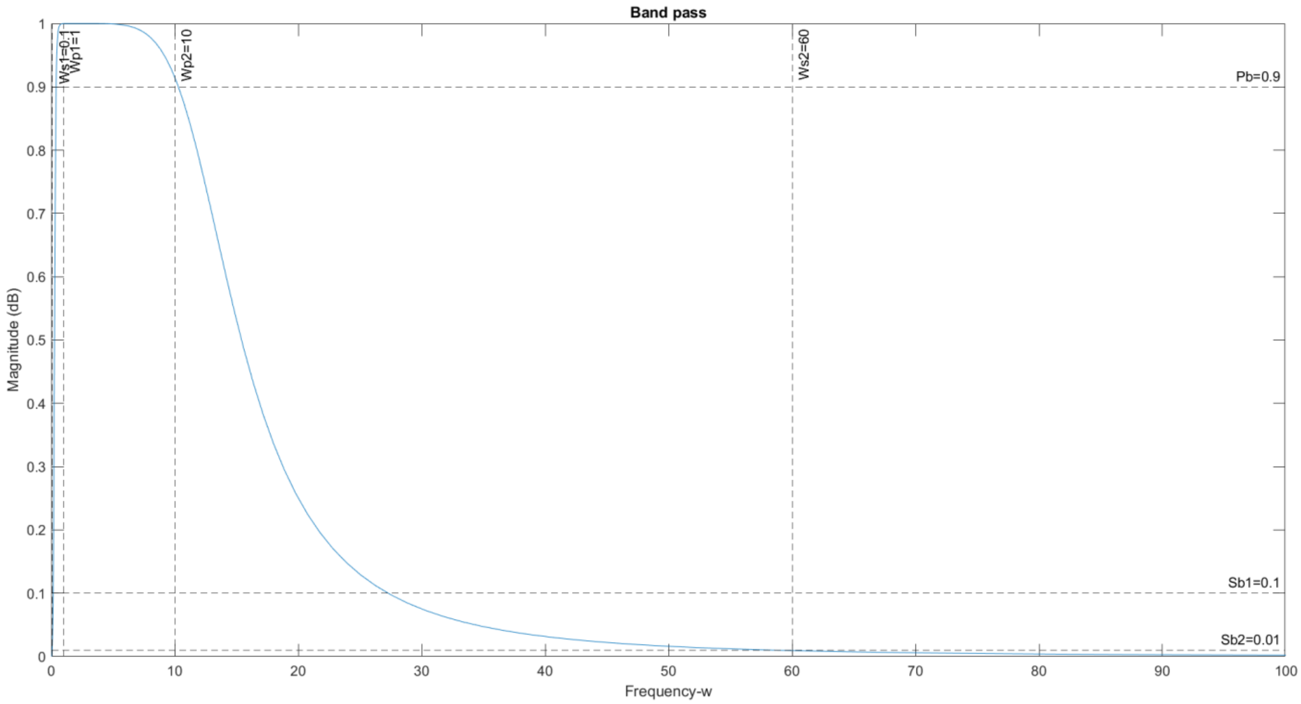
```
% Band pass filter specifications
[num,den] = butter(N,[Hwc Lwc],'s') ;
H = tf(num,den);
```

$$Transfer\ function = \frac{2006\,s^3}{s^6 + 25.22\,s^5 + 330.3\,s^4 + 2212\,s^3 + 1347\,s^2 + 419.4\,s + 67.8}$$

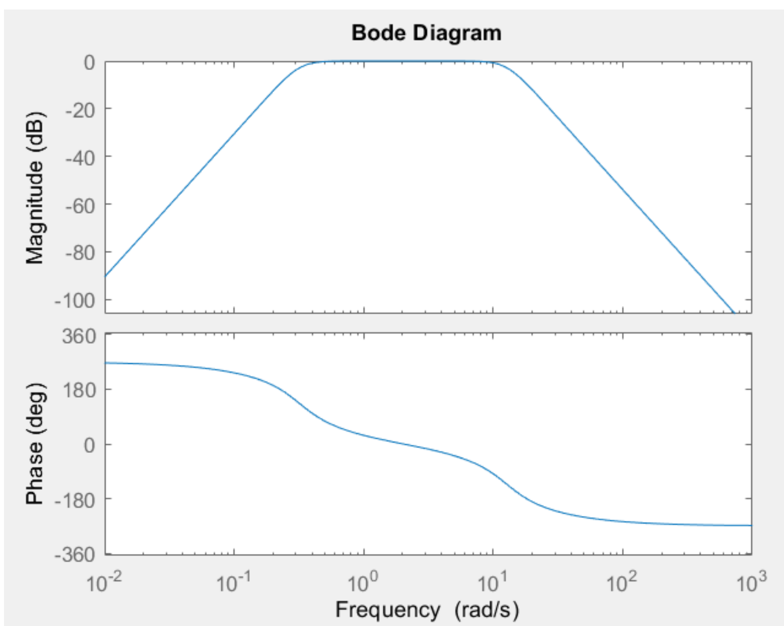
```
[PH,w]=freqs(num,den);
plot(w, abs(PH));
xlabel('Frequency-w');
ylabel('Magnitude (dB)');
title('Band pass');
```




```
% Verifications
xline(1,'--k','Wp1=1')
xline(10,'--k','Wp2=10')
xline(0.1,'--k','Ws1=0.1')
xline(60,'--k','Ws2=60')
yline(0.9,'--k','Pb=0.9')
yline(0.1,'--k','Sb1=0.1')
yline(0.01,'--k','Sb2=0.01')
```


Band pass
Magnitude (dB)
Frequency-w
Ws1=0.1
Wp1=1
Wp2=10
Ws2=60
Pb=0.9
Sb1=0.1
Sb2=0.01

```
bode(H)
```


Bode Diagram
Magnitude (dB)
Phase (deg)
Frequency (rad/s)

4. Conclusion: state what you learn from this lab, lab objectives you achieved, and any difficulties you met.

Learned how to use Butterworth, Chebyshev (Type I and II), and Elliptic filters in MATLAB to design analog filters and how to build a filter that could be used in an ECG system.

All the questions were answered using MATLAB and was able to get a better understanding of high pass, low pass and band pass filters.

Building a band pass filter for an EGC system was challenging, but turns out its not that hard to make one.