Unit - III

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3D- Transformation

3D - Transformation ⇒ It is very similar to our 2D transformation. Rather in 3D we are extending 2D transformation by considering one additional parameter i.e z - co-ordinate

Like 2D transformation, we are having scaling, translation, rotation transformation for 3D also.

In 3D, to explain all transformation, we are making use of homogeneous co-ordinate system.

1 **Scaling** ⇒ The formula for 2D and 3D scaling are the same with addition of z co-ordinate.

The normal scaling matrix in 3D will be :-

$$\begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & S_z \end{bmatrix}$$

$S_x, S_y, S_z \Rightarrow$ Represent the scaling factor for the three dimension

3D scaling matrix with homogeneous co-ordinate will be

$$\begin{vmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

$$\begin{vmatrix} x' \\ y' \\ z' \\ 1 \end{vmatrix} = \begin{vmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} * \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$x' = S_x \cdot x$ $\quad z' = S_z \cdot z$

$y' = S_y \cdot y$

2. Translation ~~Scaling~~ $\Rightarrow$ The ~~translation~~ normal matrix will be

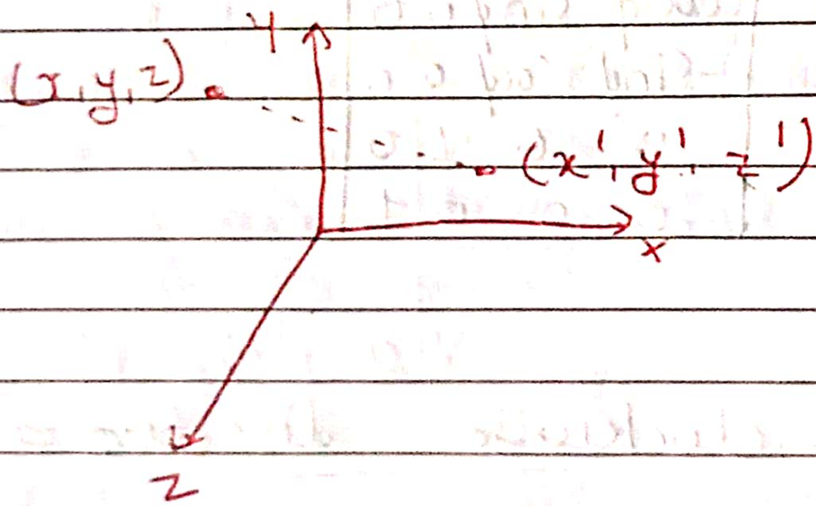
$$\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ t_x & t_y & 1 \end{vmatrix}$$

t_x and t_y ⇒ Translation factors in x and y direction

3D translation matrix are.

$$\begin{vmatrix} 1 & 0 & 0 & \cancel{0}\ t_x \\ 0 & 1 & 0 & \cancel{0}\ t_y \\ 0 & 0 & 1 & \cancel{0}\ t_z \\ \cancel{t_x}\ 0 & \cancel{t_y}\ 0 & \cancel{t_z}\ 0 & 1 \end{vmatrix}$$

t_z ⇒ Translating factor for z-direction



$$x' = x + T_x$$
$$y' = y + T_y$$
$$z' = z + T_z$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \cancel{0}\ t_x \\ 0 & 1 & 0 & \cancel{0}\ t_y \\ 0 & 0 & 1 & \cancel{0}\ t_z \\ \cancel{t_x}\ 0 & \cancel{t_y}\ 0 & \cancel{t_z}\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$x' = x + T_x$$
$$y' = y + T_y$$
$$z' = z + T_z$$

3. Rotation $\Rightarrow$

For anticlockwise direction $\Rightarrow$

$$\Rightarrow \begin{vmatrix} \cos\phi & \sin\phi & 0 & 0 \\ -\sin\phi & \cos\phi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

For clockwise direction $\Rightarrow$

$$\begin{vmatrix} \cos\phi & -\sin\phi & 0 & 0 \\ \sin\phi & \cos\phi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

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Projection ⇒ Projection is nothing but a shadow of the object (Transform 3D object at 2D plane using Projection)

Ideally an object is projected by projecting each of its points. As there are infinitely many points in an object, we are not producing projections of all those points. Instead of that we will take projection of only corner points of an object on 2D plane & then we will join these projected points by straight line in 2D plane.

- Projection
  - Parallel
    - Oblique
      - Cavalier
      - Cabinet
    - Orthographic
      - (Axonometric O.P)
  - Perspective

Proofs.

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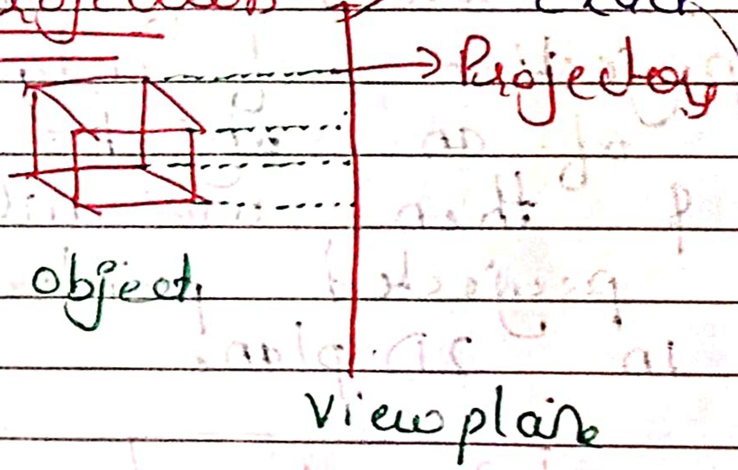
Parallel Projection =>

[The plane on which are taking projection is called View plane]

Projection plane => When geometric objects are formed by the intersection of lines with a plane the plane is called view plane

Projectors => The lines are called projectors

Parallel Projection => Exact measurement



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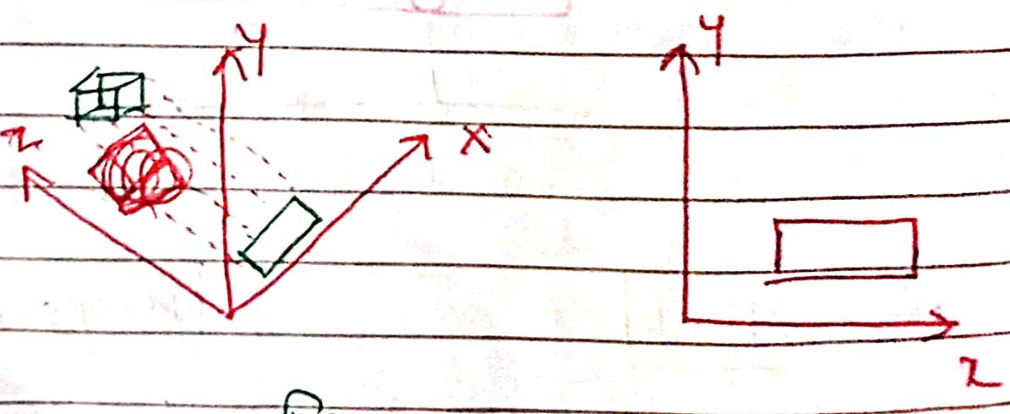


Fig - 5.1

A parallel projection is formed by extending parallel lines from each vertex of the object until they intersect the plane of the screen.

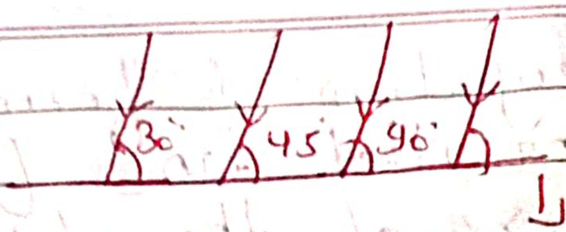
→ The point of intersection is the projection of the vertex

→ Then we connect the projected vertices by line segments which correspond to connections on the original object (see fig 5.1).

→ If we want to represent a 3D object on 2D plane, the simple way to discard z- coordinate

→ Parallel projections meet a infinity

(i) Oblique projection ⇒ It is a type of parallel projection where the lines of sight (Projectors lines) are not perpendicular to the projection plane.



Projection plane
/ View plane

→ exact दिखा रखते है

2) Orthographic projection ⇒ When the projection lines are normal (⊥) to the projection plane, the projection is called Orthographic projection. It is used in engineering drawing to produce front, side & top view of an object (multi view pro...)

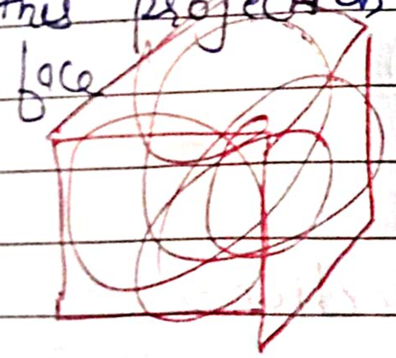
Axnometric Orthographic projection ⇒

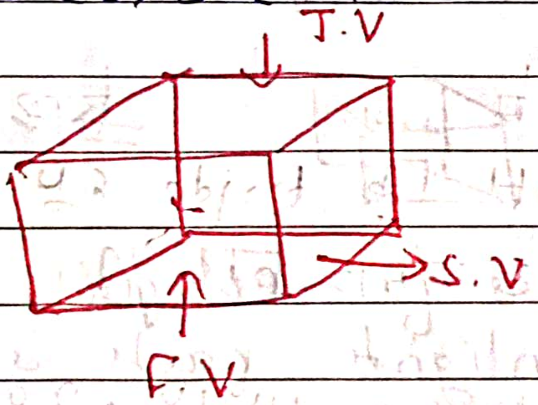
We can also form orthographic projections in such a way that more than one face of an object can be displayed. Such views are called Axnometric orthographic projection

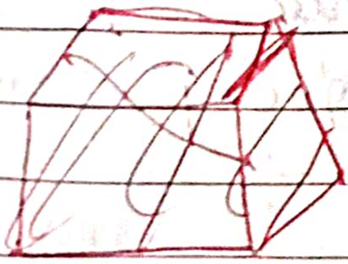
They are 3 types:

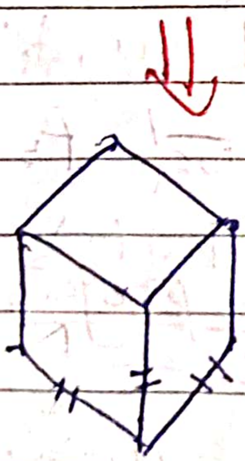
1. Isometric
2. Diametric
3. Trimetric

1. Isometric ⇒ In this the projection are aligned in such a way that all edges will appear shortened (equal) by same distance. In this projection we can see more than one face.









2. Diametric ⇒ The direction of projection is in such a way that the edges parallel to only two principle axes are equally shortened.

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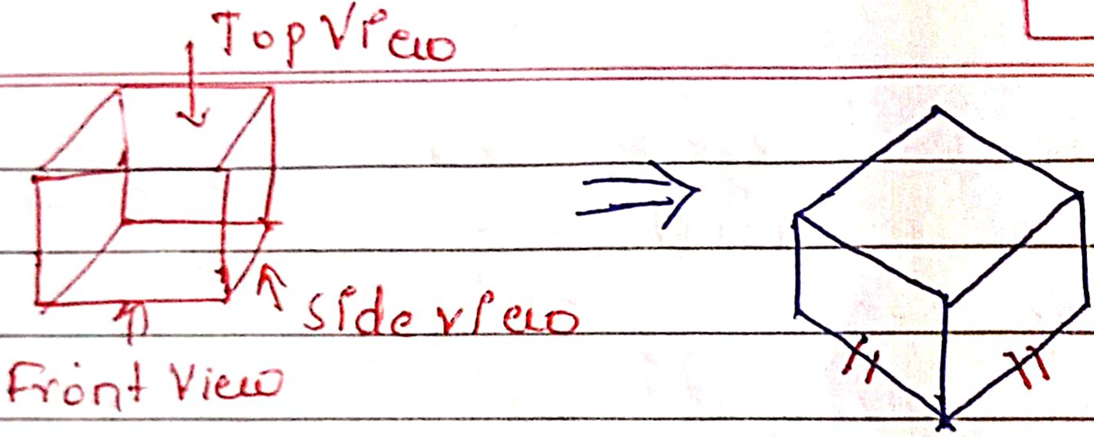
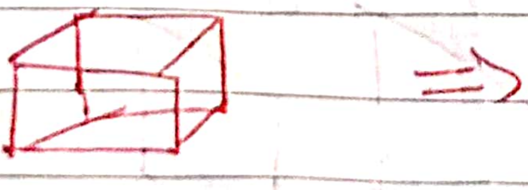


2. ~~Ab~~ **Trimetric projection** → The direction of projection is in such a way that none of the 3 edges are equally shortened



Types of oblique projection → When sunlights casts shadows on the ground. We have 2 types of oblique projection are :-

(I) Cavalier

(II) Cabinet

1) Cavalier

Fig. 5.3.7

An oblique projection is obtained by projecting points along parallel lines that are not perpendicular to the view, plane such a projection appears in nature when sunlight casts shadows on the ground. We have two types of oblique projections.

- Cavalier
- Cabinet

- *Cavalier Projection :*

Cavalier projection is a special case of oblique projection. This occurs when the projection vector forms an angle of 45° with the z-axis. The lines which are parallel to the z-axis are projected with no change in length.

Consider an object whose edges are parallel to axes and front face of the object is parallel to view plane xy. (See Fig. 5.3.8).

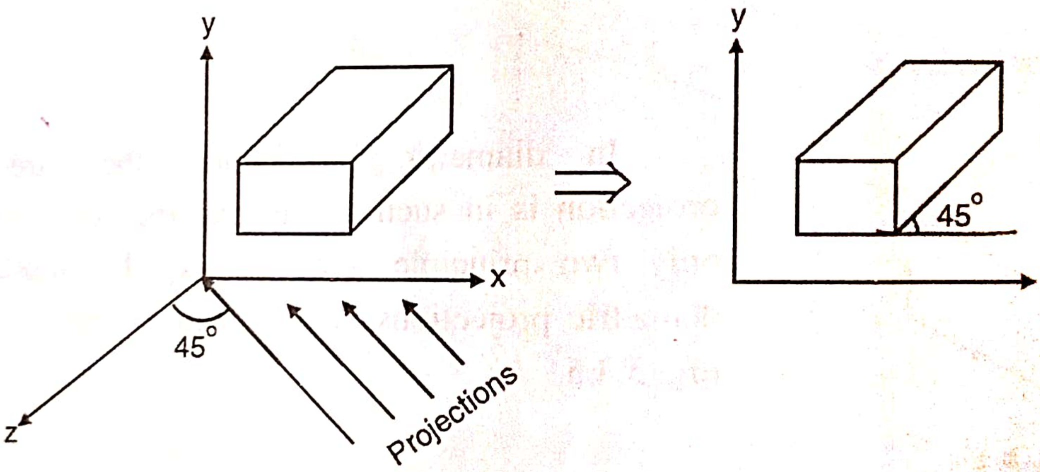


Fig. 5.3.8

As the projections are not perpendicular to view plane, we will not get only one face of the object. But as the angle of projection is 45°, we will get z-axis values as it is but x and y-axis values gets reduced. So the object will look like an elongated object.

- ***Cabinet Projections :***

In cabinet projections we are having only half the actual z distance along the projected axis. It uses a projection vector that forms an angle of approximately 26.6° with the z-axis. (See Fig. 5.3.9).

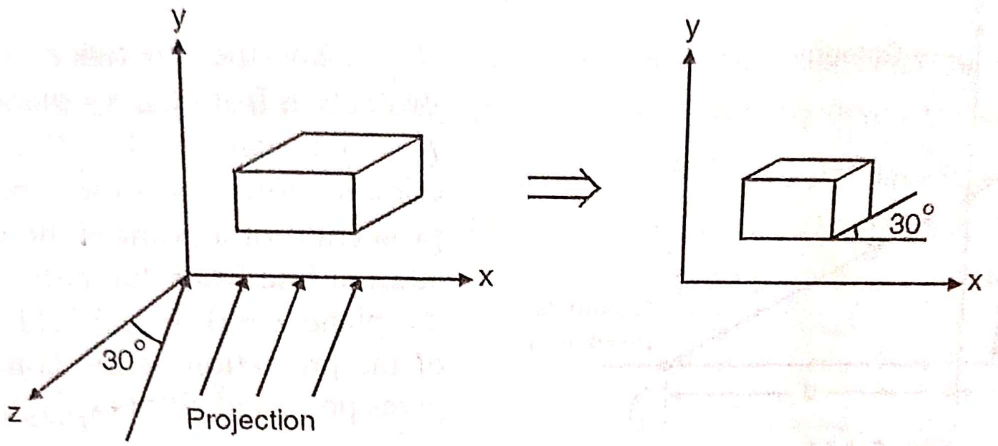


Fig. 5.3.9

As the projections are making angle of 26.6° (≈ 30°), the z-axis values get half of its original values. Cabinet projections appear more realistic than cavalier projections because of this reduction in the length of edges which are perpendicular to view plane.

5.3.2 Perspective Projection : ✓ m.Imp

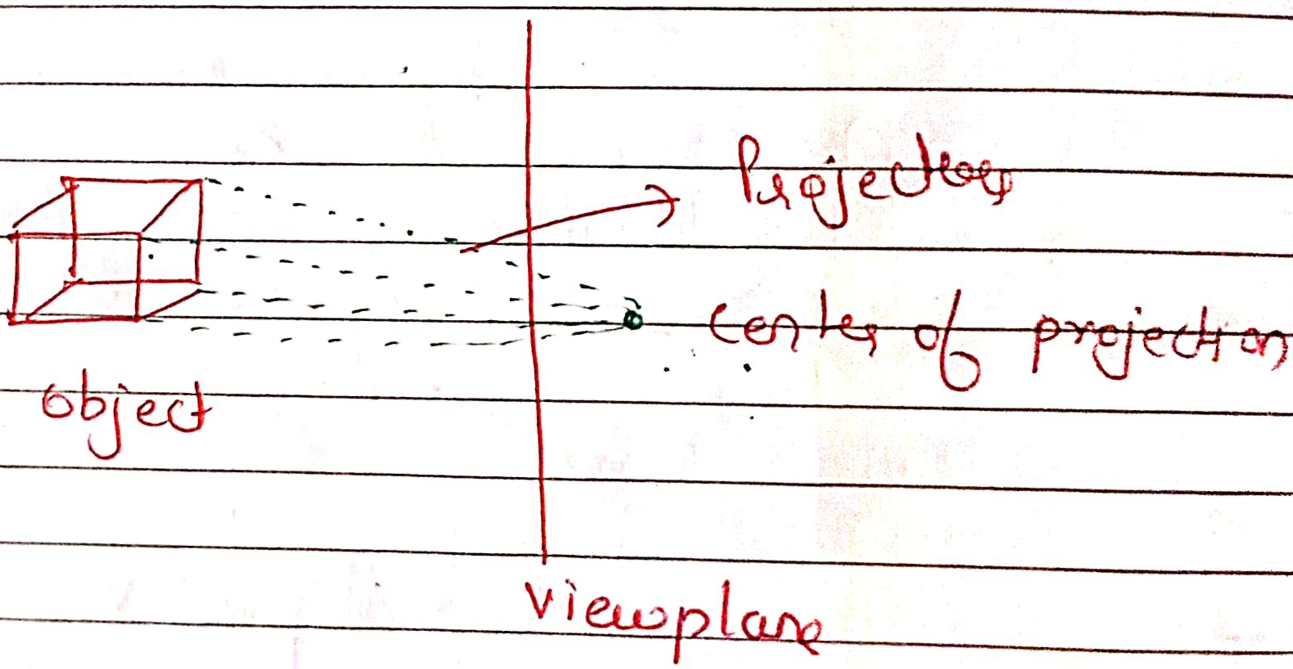
➤➤➤ [RGTU : Dec. 2002, Dec. 2003, June 2004 !!!]

In real world, the objects which are away from the viewer, appear smaller. Perspective

Perspective projection => In real world, the objects which are away from the viewer appear smaller. Perspective projection preserves this property.

In P.P, the lines of projection are not parallel. Here all projection lines are ending at a single point called Center of Projection

Ex-: In real world this center of projection is human eye



→ Projector lines are meet at center of projection shading (Dark & Light Portion) | Illumination → रोशनी उजाला | Intensity (तीव्रता) Quantity of illumination

Basic Illumination model / ~~Shading Highting model~~ ⇒ We concentrate on the location and qualities of the light that falls on the objects and the way in which object interacts with it. A model for the interaction of light with a surface is called an illumination model. (Intensity किसी Point पे कितनी है)

(1) **Light emitting source** ⇒ Many object which are available in nature may produce or emit light. These objects are called as light emitting source.

Ex - Moon, Sun, lamp, stars etc

- Light emitting Source
  - Point Source
  - Distributed Source

Point Source ⇒ When the surface of the object which we want to illuminate is bigger that we are calling that point source.

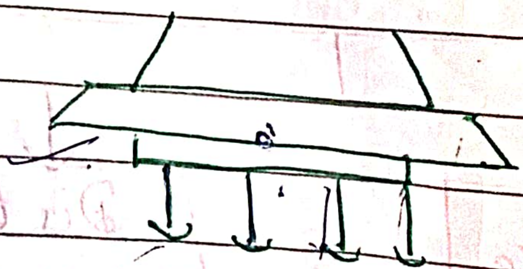
Ex => (Sun) all direction में फैलाता (किरणें light)

Ex Candle, sun

fig
candle

Distributed Source => When the surface of light emitting source is greater than the surface of object then we are referring it as distributed light source.

Ex Tubelight



fig

(किरणें Path एक direct में distribute होती हैं light निकलती है

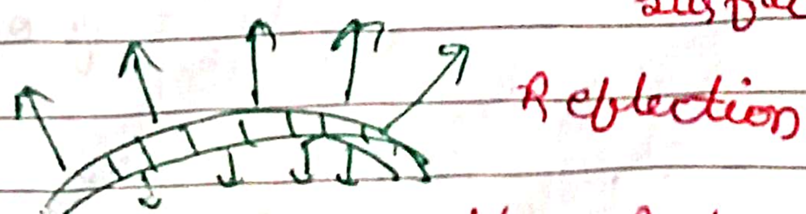
Illumination

Ambient light => It is called as Background light (Indirect).

The light which is reflected from somewhere is called as Background or ambient light.

$$\boxed{I_{amb} = K_a I_a}$$

Diffuse illumination => When the light falls on any surface, it can be absorbed by the surface, while the rest will be reflected or retransmitted. In D.I, incoming light is not is reflected in a single direct^n. but is scattered almost in all direction. The part of incoming light will be absorbed by the surface. The light which is not absorbed will be reflected randomly in all direction. (Rough & Grainy surface)



(dull & non-shiny surface)

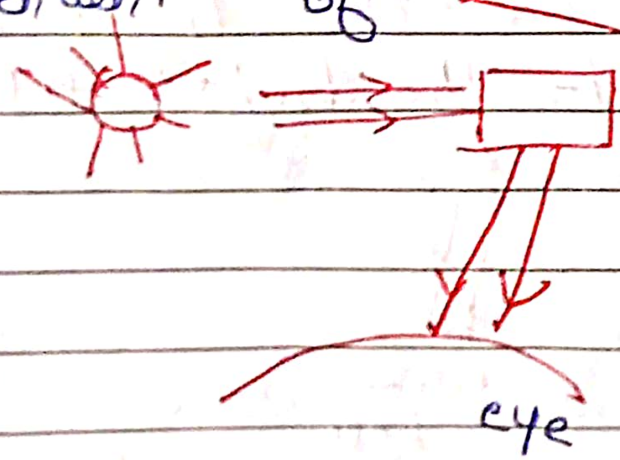
Ambient Illumination ⇒ It is called Background light (Indirect).

→ The light which is reflected from somewhere is called Ambient Illumination.

→ The amount of ambient light incident on each object is constant for all surface & over all direction.

→ It is a very simple model, not very realistic

→ Independent of < Surface of nature / Viewer location



$$\boxed{I_{amb} = K_a I_a}$$

K_a ⇒ Coefficient of surface ambient reflectivity

I_a ⇒ Ambient light Intensity

Diffuse illumination ⇒

→ object illuminated by only ambient light have equal intensity everywhere

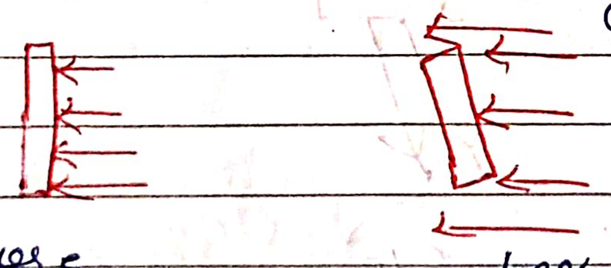
→ If there is a light source then diffⁿ object should have diffⁿ intensities based on distance & orientation w.r.t light source & viewer's position

→ A point on a diffuse surface appears equally bright from all viewing position because it reflects light equally in all direction

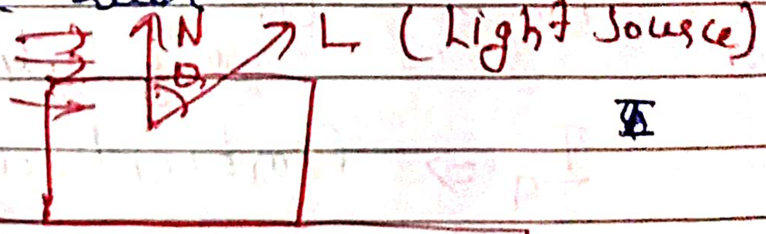
→ Intensity is independent of position of viewer

→ The intensity of point on a diffuse depend on the orientation of the surface w.r.t the light source & the distance to the light source.

→



→ Diffuse reflection model follow Lambert's Cosine rule



$$\boxed{\begin{aligned} I_{L.diff} &= K_d I_p \cos\theta \\ &= K_d I_p (N \cdot L) \end{aligned}}$$

Coefficient of reflection or reflectivity (K)

⇒ The ratio of light reflected from the surface to the total incoming light is called as Co-efficient of reflection or reflectivity.

$$\boxed{K = \frac{I_R}{I_S}}$$

(I_R = Intensity of reflected light

I_S = Intensity of light from sun)

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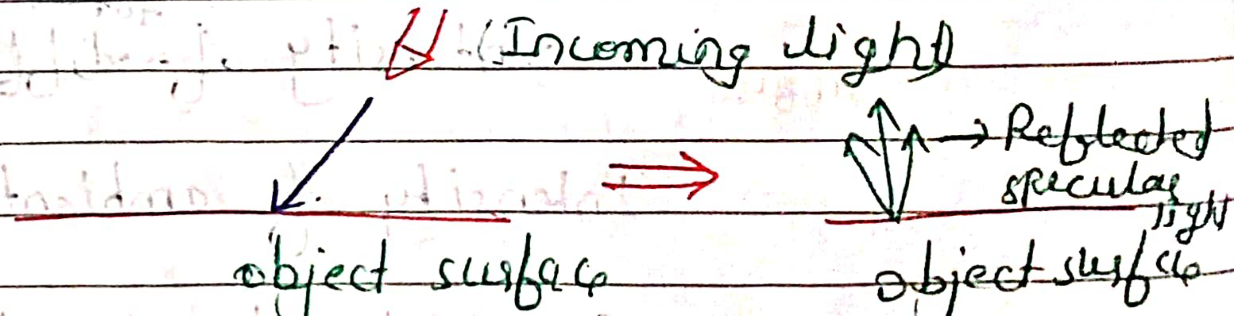
→ (shiny surfaces)

Specular Reflection ⇒ Specular reflection is a type of reflection which occurs at the surface of mirror

[When we look at an illuminated shiny surface, such as polished metal we see a highlight, or bright spot, at certain viewing directions. This phenomenon is called a Specular reflection]

Dependent on -: (I) Light Source position

(II) Viewer's position



Light

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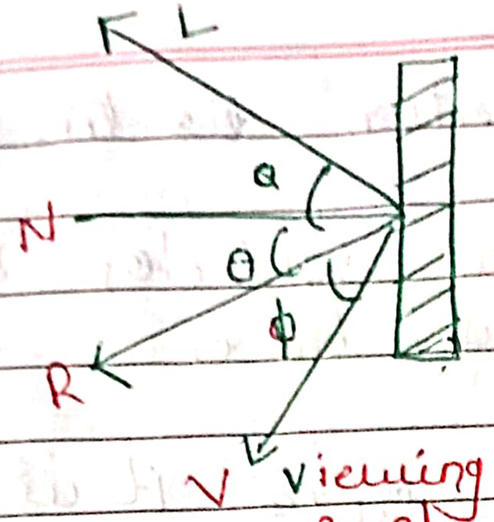

fig - 15.A

In S.R light come in, strike the surface and then bounce back. The angle which the reflected beam make with the surface normal is called angle of ~~Incidence~~ Reflection

It is same in magnitude as that of "angle of incidence".

L ⇒ It points in the direction of light source.
N ⇒ It is the surface normal.
R ⇒ The vector R points in the direction of the reflected light
V ⇒ It is the unit vector pointing to the viewer from the surface position
θ = angle of incidence (b/w L & N)
θ = angle of reflection (b/w N & R)
φ = viewing angle relative to the

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specular reflection direction R.

→ For an ideal reflector this angle ϕ should be 0.

→ For a shiny surface it is having narrow reflection range

→ For a rough surface has wider reflection range.

$$I_{spec} = I_p \, K_s \, (\cos\phi)^n$$

I_p = Light Intensity

K_s = Specular reflectivity

ϕ = the angle b/w V & R.

$\cos\phi$ = V.R

$$\boxed{I_{spec} = I_p \, K_s \, (V.R)^n}$$

$$I = I_{amb} + \sum_{K} (I^{K}_{diff} + I^{K}_{spec})$$ (At K (multiple) light source)

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Colour Model → A color model is a method for explaining the properties or behaviour of color within some particular context.

1. RGB Color Model | Cubical Color Model

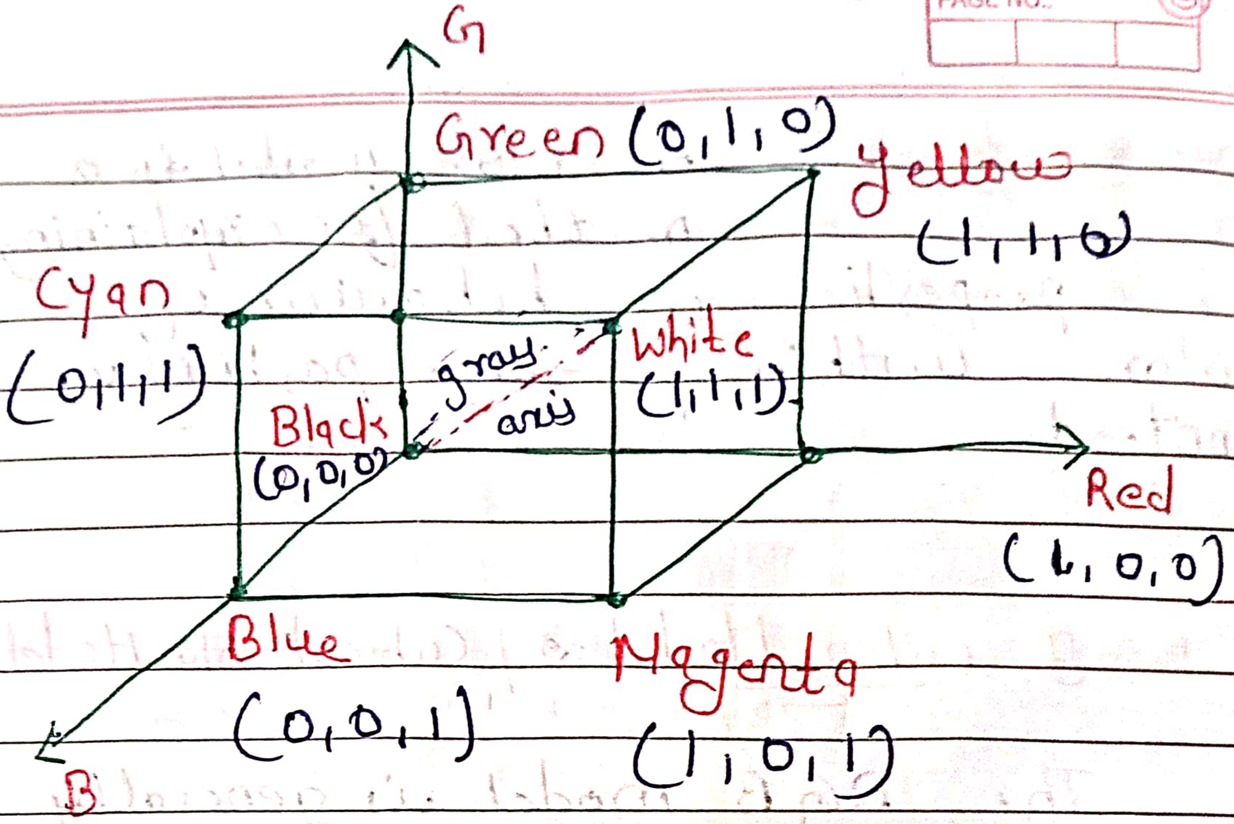
⇒ The RGB model is generally used in computer Graphics. It corresponds to Red, Green and Blue intensity setting of a color monitor.

We can represent this model with the unit cube defined R, G, and B axes.

~~Black~~ ~~Origin~~

The origin represent Black and the vertex with the co-ordinates (1,1,1) is white

Cartesian Co-ordinate System-

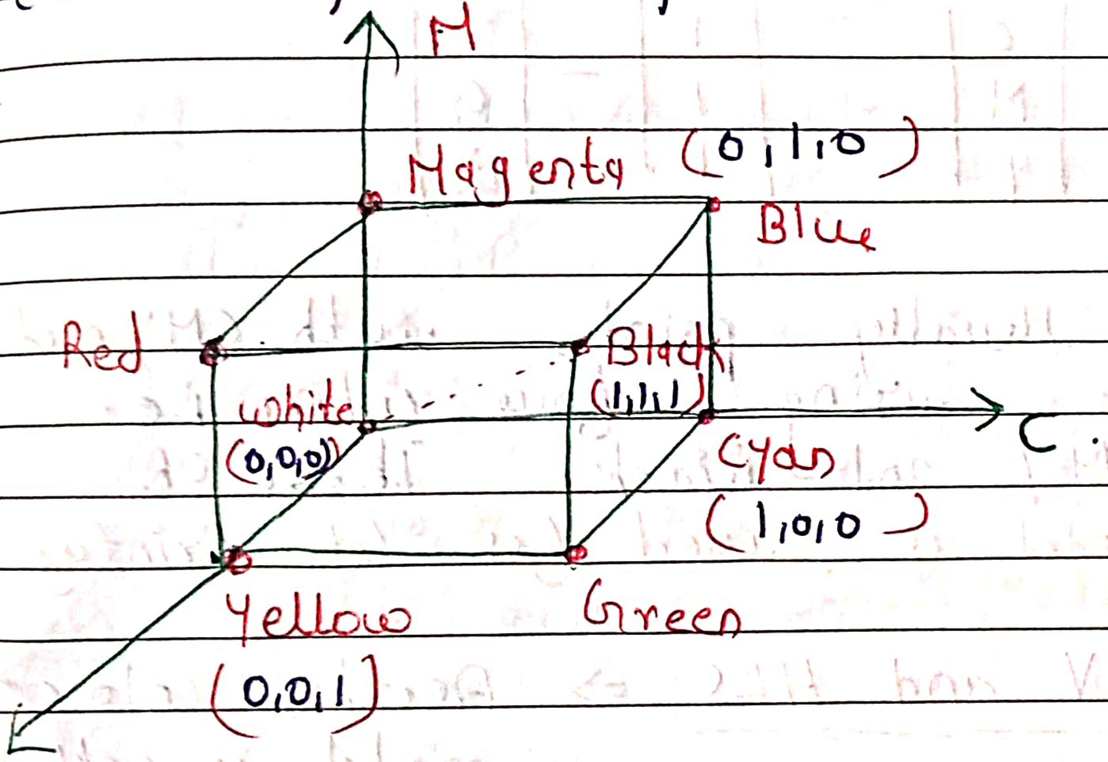


Other colors are generated by adding intensities of primary colors, such as yellow (1,1,0) is a combination of Red & Green.

It used in-:

(i) Color Monitors
(ii) Vedio Camera

→ CMY color model → Whenever the primary colors are Cyan, Magenta and Yellow in a color model, then it will be useful for describing color output on printer.



The Cyan can be formed by adding green and Blue color. It means there is no red component i.e Red color is absorbed or subtracted by the ink. Similarly magenta absorbs Green and yellow remove the blue color.

So by using this complementary relationship property b/w RGB & CMY make it easy to convert from one color model to the other.

$C = 1 - R$ $M = 1 - G$ $Y = 1 - B$

$$\begin{vmatrix} C \\ M \\ Y \end{vmatrix} = \begin{vmatrix} 1 \\ 1 \\ 1 \end{vmatrix} - \begin{vmatrix} R \\ G \\ B \end{vmatrix}$$

Ex Usually printers with CMY model are using four colors i.e CMY and Black. ~~These~~ OR Model is used for Color Printing

HSV and HLS => Another color model is called HSV model.

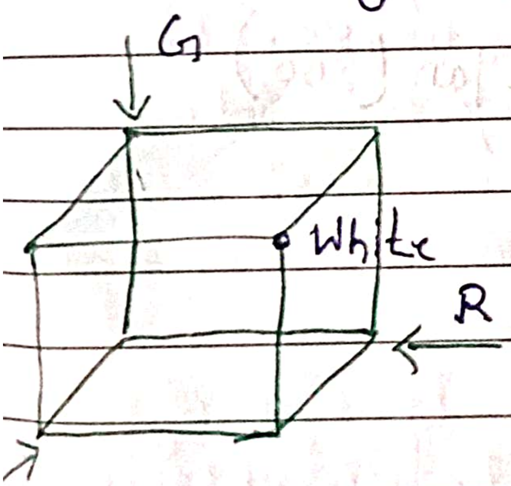
(i) H = Hue (It indicate pure color of light)

(ii) S = Saturation (It indicates how much white light is mixed with color) ~~Purity~~ Rang 0 To 1

iii) V = (Value = Brightness) ⇒ Value is the intensity of the maximum of the red, blue & green components of the color.

V = 0 = Black
V = 1 = White } Intensity of color (0 - 1)

The HSV model representation is derived from the RGB color model



B fig: RGB color cube

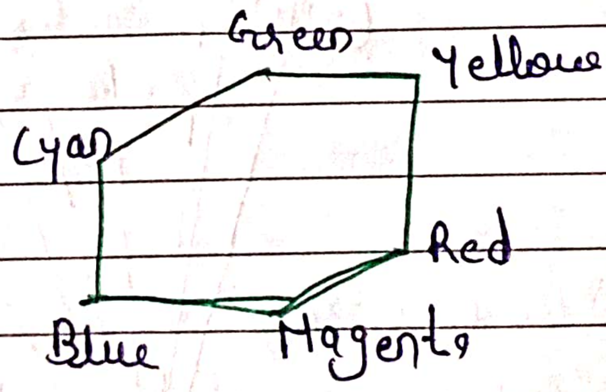
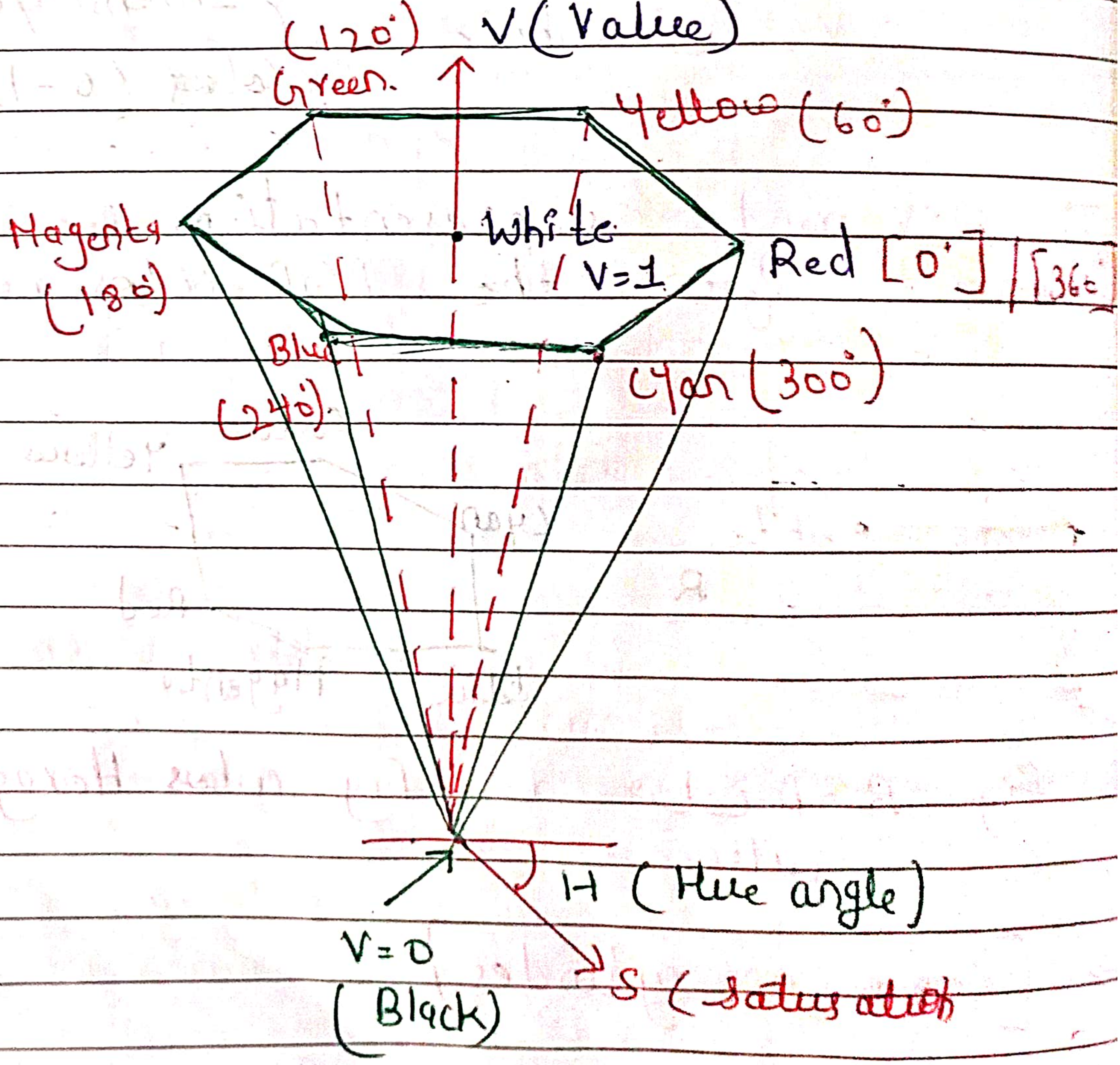


fig color Hexagon

⇒ [Cone or cylinder]

View the cube along its sides giving as a shape of Hexagon.

→ The top surface of Hexagon represent Pure Hue colors



→ Hue represent the angle with the vertical axis. All the 6 color which cover total 360° of color hexagon & are separated by 60°.

→ Saturation represent the ratio of purity of the Hue. Saturation is measured along the Horizontal axis. Saturation varies from 0 to 1.

→ Value is along the vertical axis through the centre of Hexagon.

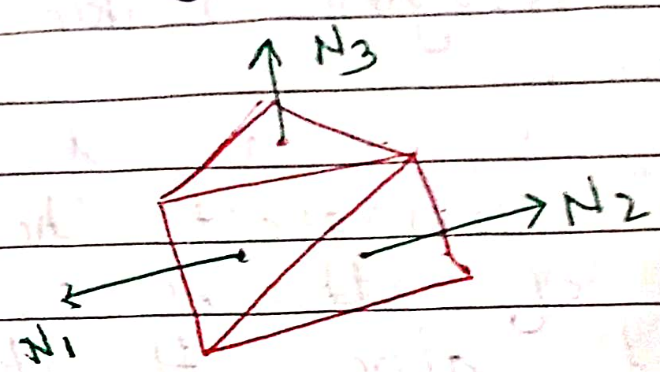
→ At the top of Hexagon, the colors have the max intensity.

Used -: It used for Image Processing appln.

Shading models -: { Determines where the lighting model is applied.

1. Flat / Constant Shading ⇒ In this type of shading method, for each polygon a single Intensity is calculated.

Then with the same intensity value all points on that surface of the polygon are displayed



Advantage

→ This shading techniques they display all the points in a polygon with a single color

Disadvantage - Discontinuity of colors can be observed in diff^n faces of the polygon

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Gouraud Shading :- It is an example of interpolation shading.

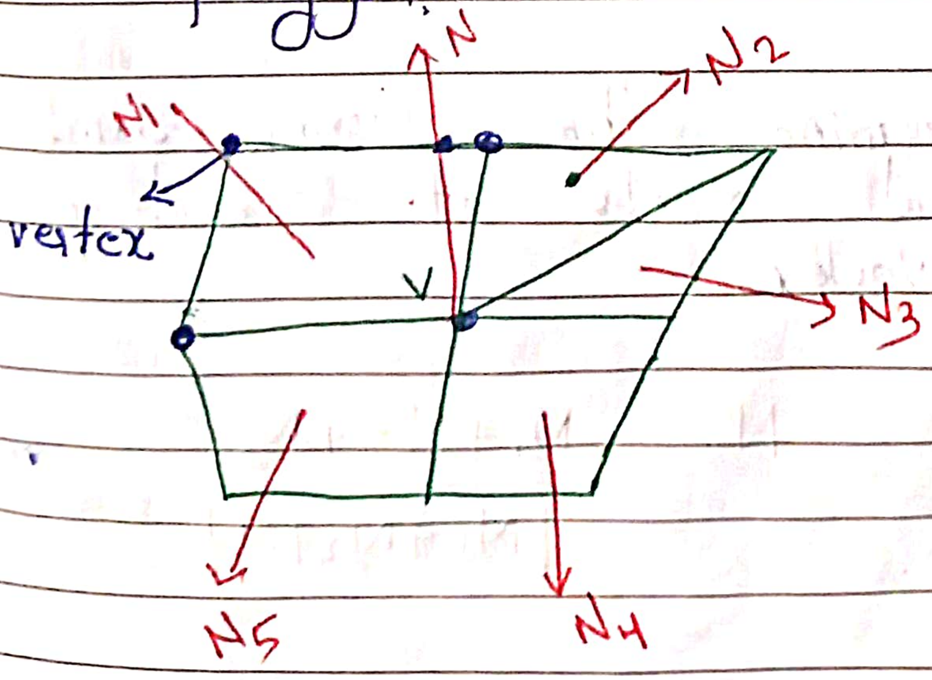
→ Intensity levels are calculated at each vertex & interpolated across the surface.

steps-:

1. At each vertex of polygon, determine average unit normal vector.

2. Calculate vertex intensity.

3. Linearly interpolate that vertex intensity over the surface of polygon.



To determine a normal vector N at vertex V in fig., we use the normal vectors the polygons that meet at vertex V

$$\boxed{N = \frac{N_1 + N_2 + N_3 + N_4 + N_5}{5}}$$

Advantage :- Removal of discontinuities associated with the constant shading model

Algo

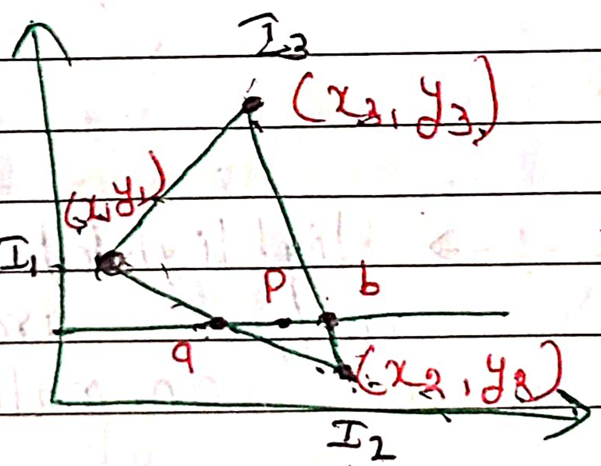
(1) Determine the average unit normal vector at each polygon vertex

$$N = \frac{N_1 + N_2 + N_3}{|N_1 + N_2 + N_3|}$$

$$= \frac{\sum_{i=1}^{n} N_i}{\left| \sum_{i=1}^{n} N_i \right|}$$

n = no of surface of polygon sharing that vertex

ii) By illumination we get Intensity of each vertex.



Intensity of q ⇒

$$I_q = \frac{y_q - y_2}{y_1 - y_2} I_1 + \frac{y_1 - y_q}{y_1 - y_2} I_2$$

Intensity of b ⇒

$$I_b = \frac{y_b - y_2}{y_2 - y_3} I_3 + \frac{y_3 - y_b}{y_1 - y_2} I_2$$

(iii) Linearly Interpolate

Intensity at point P is given by

$$\boxed{I_p = \frac{x_b - x_p}{x_b - x_a} I_a + \frac{x_p - x_a}{x_b - x_a} I_b}$$

~~Advantage~~ :-

Disadvantage :- → Highlighted on the surface are sometimes displayed with anomalous shape

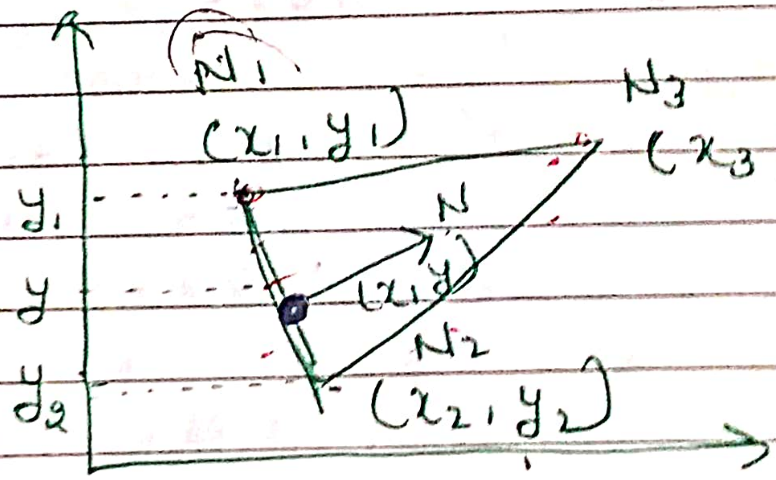
→ Bright & dark streaks appearing on the surface known as mach band

Phong Shading → Algo:

1. At each vertex of polygon, determine average unit normal vector.

2. Linearly interpolate the vertex normals

3. Calculate the pixel intensity of each scan line.



$$\boxed{N = \frac{y - y_2}{y_1 - y_2} N_1 + \frac{y_1 - y}{y_1 - y_2} N_2}$$

Advantages:

1. It is more accurate than Gouraud shading

Disadvantage:-

1. It requires calculation, hence greately increases cost of shading.