Alastair Hall

ECON61001:
Econometric Methods

Semester 1

Study Group Questions # 2

Please work on these questions in your study groups and submit your answers to Gradescope following the instructions in the *Weekly Assignments* folder on BB.

The questions refer to the *log-level* model considered in the Tutorial session that is,

$$log(w) \; = \; \beta_1 \, + \, \beta_2 x_2 \, + \, u$$

where the (observable) dependent variable is $y = log(w)$, x_2 is an observable explanatory variable, u is the unobservable error, $(\beta_1, \, \beta_2)$ are the regression parameters, and for ease of presentation we omit the observation index on the variables as it plays no role in our discussion here.

As in the Tutorial session, we use the following notation: for any variable a, the change in a is denoted by Δa and the percentage change in a is denoted by $\Delta\% a$ that is, if the value of a changes from a_0 to a_1 then: $\Delta a = a_1 - a_0$ and $\Delta\% a = 100\,\{(a_1 \, - \, a_0)/a_0\}$.

Recall from the tutorial that if x_2 is a continuous variable then for sufficiently small changes Δx_2, we have

$$\Delta w \; = \; \frac{\partial w}{\partial x_2} \Delta x_2. \tag{1}$$

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1. Consider the case where x_2 is a continuous variable but (1) does not necessarily hold. Show that if x_2 changes by Δx_2 then the resulting percentage change in w is given by

$$\Delta\% w \;=\; 100 \times \left(e^{\beta_2 \Delta x_2} - 1 \right). \tag{2}$$

We have the model:

$$\log(w) = \beta_1 + \beta_2 x_2 + u.$$

Let
$$\log(w_1) = \beta_1 + \beta_2 x_{2,1} + u$$
$$\log(w_0) = \beta_1 + \beta_2 x_{2,0} + u$$
$$\Rightarrow \log(w_1) - \log(w_0) = \beta_2 (x_{2,1} - x_{2,0})$$
$$= \beta_2 \cdot \Delta x_2$$

$$\Leftrightarrow \log\left(\frac{w_1}{w_0}\right) = \beta_2 \cdot \Delta x_2$$

$$\Rightarrow \frac{w_1}{w_0} = \exp(\beta_2 \cdot \Delta x_2)$$

$$\Leftrightarrow \frac{w_1}{w_0} - 1 = \exp(\beta_2 \cdot \Delta x_2) - 1$$

$$\Leftrightarrow 100\left(\frac{w_1 - w_0}{w_0}\right) = 100\left[\exp(\beta_2 \Delta x_2) - 1\right]$$

$$\Leftrightarrow \Delta\% w = 100 \times \left(e^{\beta_2 \Delta x_2} - 1\right).$$

2. What is the MacLaurin expansion for e^a?

We have the Taylor expansion of real function $f(x)$ at point $x = a$:

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \dots + \frac{f^{(n)}(a)}{n!}(x-a)^n + \dots$$

the Maclaurin expansion is one special case of the Taylor expansion when $a = 0$.

$$f(x) = f(0) + f'(0)\,x + \frac{f''(0)}{2!}x^2 + \dots + \frac{f^{(n)}(0)}{n!}x^n + \dots$$

For $f(x) = e^x$, we have the Taylor expansion for $x = a$:

$$e^x = e^a + e^a(x-a) + \frac{e^a}{2!}(x-a)^2 + \dots + \frac{e^a}{n!}(x-a)^n + \dots$$

For $a = 0$, we have Maclaurin expansion of e^x:

$$e^x = e^0 + e^0 x + \frac{e^0}{2!}x^2 + \dots + \frac{e^0}{n!}x^n + \dots$$

$$= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!} + \dots$$

3. Show that if Δx_2 is sufficiently small then the formula for $\Delta\%w$ in (2) implies that

$$\Delta\%w \;\approx\; 100\beta_2\Delta x_2.$$

From (2), we have $\Delta\%w = 100 \times (e^{\beta_2 \Delta x_2} - 1)$

To prove that $\Delta\%w \approx 100 \cdot \beta_2 \Delta x_2$ for when $\Delta x_2 \to 0$

we need to show that $(e^{\beta_2 \Delta x_2} - 1) \approx \beta_2 \Delta x_2$ when $\Delta x_2 \to 0$

Indeed, from the Maclaurin expansion of $f(x) = e^x$, we have

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!} + \dots \quad (\text{at } a = 0)$$

For x very small $\Rightarrow$ $e^x \approx 1 + x$

Similarly, when Δx_2 become very small,

$$e^{\beta_2 \Delta x_2} \approx 1 + \beta_2 \Delta x_2$$

$$\Leftrightarrow \quad e^{\beta_2 \Delta x_2} - 1 \approx \beta_2 \Delta x_2 \quad (\text{Q.E.D})$$

Then, for $\Delta x_2 \to 0$, we have $\Delta\%w \approx 100\beta_2 \Delta x_2$.