Homework 4

March 29, 2024

- You are required to use Jupyter notebook to finish this quantitative exercise. You may refer to QuantEcon for help. Use university computer lab if you do not have a personal computer.
- This homework must be finished independently!
- You must submit your solution before the end of Apr 5. Submit your notebook file (**the .ipynb file**) to the following URL `https://yunbiz.wps.cn/c/collect/cGHaoyR1jwb`.

Optimal Growth Model in Recursive View

Consider a social planner who solves the following optimization problem:

$$\max_{c_t, k_{t+1}} \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\gamma}}{1-\gamma},$$

s.t.

$$c_t + k_{t+1} = k_t^{\alpha} + (1-\delta) k_t,$$

where $k_t \geq 0$, $c_t \geq 0$, k_0 is given. c is consumption, k is the capital stock, $\alpha \in (0,1)$ is a production function parameter and $\beta, \delta \in (0,1)$ are the subjective discount factor and the depreciation rate of capital respectively. Denote the steady state of capital as k^*. Let $\alpha = 1/3$, $\beta = 0.95$, $\delta = 0.1$, $\gamma = 1.5$.

1. Use value function iteration to solve this problem numerically with the following optimization. Specifically, use a grid for k uniformly distributed in $[0.5 * k^*, 1.5 * k^*]$ with a total of **1000** grid points. Impose a tolerance of 10^{-6} for convergence. Report the computation time for your algorithm and compare it with the benchmark algorithm from your previous homework assignment.
   (a) concave value function only
   (b) increasing policy function only
   (c) both concave value function and increasing policy function
2. Redo Q1 with multigrid method (for example, 100+1000 girds). Afterward, report the computation time and compare it with your result in Q1.
3. (optional) redo Q1 and Q2 and attempt to optimize your code using `numba`. Afterward, report the computation time and compare it with your result in Q1 and Q2.