Homework 3

March 22, 2024

- You are required to use Jupyter notebook to finish this quantitative exercise. You may refer to QuantEcon for help. Use university computer lab if you do not have a personal computer.
- This homework must be finished independently!
- You must submit your solution before the end of Mar 27. Submit your notebook file (**the .ipynb file**) to the following URL `https://yunbiz.wps.cn/c/collect/chggwKlUmxW`.

Optimal Growth Model in Recursive View

Consider a social planner who solves the following optimization problem:

$$\max_{c_t, k_{t+1}} \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\gamma}}{1-\gamma},$$

s.t.

$$c_t + k_{t+1} = k_t^{\alpha} + (1-\delta)\, k_t,$$

where $k_t \geq 0$, $c_t \geq 0$, k_0 is given. c is consumption, k is the capital stock, $\alpha \in (0,1)$ is a production function parameter and $\beta, \delta \in (0,1)$ are the subjective discount factor and the depreciation rate of capital respectively.

1. Write down the associate Bellman equation of the above maximization problem.
2. Use Envelope theorem to obtain the Euler equation.
3. Derive the steady state of capital k^*.
4. Let $\alpha = 1/3$, $\beta = 0.95$, $\delta = 0.1$, $\gamma = 1.5$. Use value function iteration to solve this problem numerically. Specifically, use a grid for k uniformly distributed in $[0.5 * k^*, 1.5 * k^*]$ with a total of **300** grid points. Impose a tolerance of 10^{-6} for convergence. Plot the value and policy functions you get.
5. Calculate the maximum of the Euler equation residual (absolute value).
   (a) Given the grid number 300, report and compare your results for different tolerance value: 10^{-4}, 10^{-6}, 10^{-8}.
   (b) Given the tolerance 10^{-6}, report and compare your results for different grid numbers: 150, 300, 600.
6. Given the fact that the value function is concave and the policy function is increasing, develop a technique that accelerates the execution of your code.