Homework 5&6

April 12, 2024

- You are required to use Jupyter notebook to finish this quantitative exercise. You may refer to QuantEcon for help. Use university computer lab if you do not have a personal computer.

- This homework must be finished independently!

- You must submit your solution before the end of Apr 17. Submit your notebook file (**the .ipynb file**) to the following URL `https://yunbiz.wps.cn/c/collect/c5aBdAmTdJp`.

The Stochastic Neoclassical Growth Model

Consider the following social planner's problem

$$\max_{c_t, k_{t+1}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \ln c_t,$$

where $\mathbb{E}$ is the conditional expectation operation, β is the discount factor, and the resource constraint is given by

$$c_t + k_{t+1} = z_t k_t^\alpha + (1 - \delta) k_t,$$

where z_t is the productivity. Pick $\delta = 0.1$, $\alpha = 1/3$, $\beta = 0.95$. Suppose the productivity z_t takes values in a 5-point Markov chain

$$z_t \in \{0.9792, 0.9896, 1, 0000, 1.0106, 1.0212\}$$

with transition matrix :

$$\Pi = \begin{pmatrix} 0.9727 & 0.0273 & & & \\ 0.0041 & 0.9806 & 0.0153 & & \\ & 0.0082 & 0.9836 & 0.0082 & \\ & & 0.0153 & 0.9806 & 0.0041 \\ & & & 0.0273 & 0.9727 \end{pmatrix}.$$

1. Write down the recursive formulation of this problem.

2. Use value function iteration to solve this problem numerically under the following conditions. Define $k^* = \left[\frac{\alpha\beta}{1-\beta(1-\delta)} \right]^{\frac{1}{1-\alpha}}$. Use a grid of 2500 points for k uniformly distributed ± 30 percent of k^*. Impose a tolerance of 10^{-7} for convergence.

3. Calculate the limiting distribution of k. Plot the probability density of the limiting distribution.