Homework 8

May 1, 2024

- You are required to use Jupyter notebook to finish this quantitative exercise. You may refer to QuantEcon for help. Use university computer lab if you do not have a personal computer.
- This homework must be finished independently!
- You must submit your solution before the end of May 8. Submit your notebook file (**the .ipynb file**) to the following URL `https://yunbiz.wps.cn/c/collect/ckF6isEcWiP`.

The stochastic heterogeneous consumer's problem

Consider the following problem

$$\max_{c} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \ln c_t,$$

subject to

$$c_t + a_{t+1} = (1+r) a_t + w(s_t),$$

where s_t is a two-state Markov chain with state space $\{\text{unemployed}, \text{employed}\}$ and transition matrix

$$P = \begin{bmatrix} 0.6 & 0.4 \\ 0.05 & 0.95 \end{bmatrix},$$

and

$$w(s_t) = \begin{cases} 0.1, & s_t = \text{unemployed} \\ 1, & s_t = \text{employed} \end{cases}.$$

Let $\beta = 0.95$. Use a grid of 2500 points for a uniformly distributed over $a_t \in [-1.9, 15.0]$. Impose a tolerance of 10^{-7} for convergence.

1. (partial equilibrium) Let $r = 0.05$.
   (a) Solve for the policy function $a_{t+1} = g(a_t, s_t)$ and plot.
   (b) Solve for the stationary density of asset a and plot.

2. (general equilibrium of a pure credit model) A stationary equilibrium is an interest rate r, a policy function $g(a, s)$, and a stationary distribution $\lambda(a, s)$ for which g solves the household's optimum problem, λ is induced by P and g, and the loan market clears $\sum_{a,s} \lambda(a, s)\, g(a, s) = 0$. Use bisection method to solve for the equilibrium interest rate r. (Hint: let $r_{\text{min}} = 0$ and $r_{\text{max}} = 1/\beta - 1$. Calculate the aggregate credit $\psi = \sum_{a,s} \lambda(a, s)\, g(a, s)$ for $r_0 = (r_{\text{min}} + r_{\text{max}})/2$. If $\psi > 0$, then set $r_{\text{max}} = r_0$, otherwise set $r_{\text{min}} = r_0$. Repeat until convergence.)