Homework 5

April 7, 2024

- You are required to use Jupyter notebook to finish this quantitative exercise. You may refer to QuantEcon for help. Use university computer lab if you do not have a personal computer.
- This homework must be finished independently!
- You must submit your solution before the end of Apr 17. Submit your notebook file (**the .ipynb file**) to the following URL https://yunbiz.wps.cn/c/collect/c5aBdAmTdJp.

The Stochastic Neoclassical Growth Model

Consider the following social planner's problem

$$\max_{c_t, k_{t+1}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \ln c_t,$$

where $\mathbb{E}$ is the conditional expectation operation, β is the discount factor, and the resource constraint is given by

$$c_t + k_{t+1} = z_t k_t^{\alpha} + (1-\delta) k_t,$$

where z_t is the productivity. Pick $\delta = 0.1$, $\alpha = 1/3$, $\beta = 0.95$. Suppose the productivity z_t takes values in a 5-point Markov chain

$$z_t \in \{0.9792, 0.9896, 1, 0000, 1.0106, 1.0212\}$$

with transition matrix :

$$\Pi = \begin{pmatrix} 0.9727 & 0.0273 & & & \\ 0.0041 & 0.9806 & 0.0153 & & \\ & 0.0082 & 0.9837 & 0.0081 & \\ & & 0.0153 & 0.9806 & 0.0041 \\ & & & 0.0273 & 0.9727 \end{pmatrix}.$$

1. Write down the recursive formulation of this problem.
2. Use value function iteration to solve this problem numerically under the following conditions. Specifically, use a grid of 17820 points for k uniformly distributed ± 50 percent of the steady state value of k (expectation of the limiting distribution). Impose a tolerance of 10^{-7} for convergence. Use any optimization strategies you can to acclerate your code. Report how long your programs take to achieve the optimal. Plot the policy functions you get.