

# Homework 2

March 15, 2024

- You are required to use **Jupyter notebook** to finish this quantitative exercise. You may refer to **QuantEcon** for help. Use university computer lab if you do not have a personal computer.
- This homework must be finished independently!
- You must submit your solution before the end of Mar 20. Submit your notebook file (the .ipynb file) to the following URL <https://yunbiz.wps.cn/c/collect/cEJedXR0YTd>.

## Optimal Growth Model in Recursive View

Consider a social planner who solves the following optimization problem:

$$\max_{c_t, k_{t+1}} \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\gamma}}{1-\gamma},$$

s.t.

$$c_t + k_{t+1} = k_t^\alpha + (1 - \delta) k_t,$$

where  $k_t \geq 0$ ,  $c_t \geq 0$ ,  $k_0$  is given.  $c$  is consumption,  $k$  is the capital stock,  $\alpha \in (0, 1)$  is a production function parameter and  $\beta, \delta \in (0, 1)$  are the subjective discount factor and the depreciation rate of capital respectively.

1. Write down the associate Bellman equation of the above maximization problem.
2. Use Envelope theorem to obtain the Euler equation.
3. Derive the steady state of capital  $k^*$ .
4. Let  $\alpha = 1/3$ ,  $\beta = 0.95$ ,  $\delta = 0.1$ ,  $\gamma = 1.5$ . Use value function iteration to solve this problem numerically. Specifically, use a grid for  $k$  uniformly distributed in  $[0.5 * k^*, 1.5 * k^*]$  with a total of 500 grid points. Impose a tolerance of  $10^{-7}$  for convergence. Plot the value and policy functions you get.
5. How can you verify the numerical accuracy of your solution? Develop a method and apply it to assess your results.
6. Based on your evaluation method, how to improve the numerical accuracy of your solution?