Algorithms and Imperative Programming

COMP26120

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Introduction

This is a two-semester practical introduction to algorithms and data structures, concentrating on devising and using algorithms, including algorithm design and performance issues as well as 'algorithmic literacy' - knowing what algorithms are available and how and when to use them.

To reflect the emphasis on practical issues, there are two practical (laboratory) hours to each lectured hour. Lectures serve to motivate the subject, orient students, reflect on practical exercises and impart some basic information. A range of practical applications of algorithms will also be presented in the lectures. Other information resources will be important, including a set textbook, which will provide essential support.

The course-unit starts with a 5-week primer on the C programming language, enabling students to become competent programmers in this language as well as in Java (and, possibly, in other languages). This teaching is supported by an on- line C course and extensive laboratory exercises.

There is a follow-up course unit on Advanced Algorithms in the Third Year. This presents the foundational areas of the subject, including (1) measures of algorithmic performance and the classification of computational tasks by the performance of algorithms, (2) formulating and presenting correctness arguments, as well as (3) a range of advanced algorithms, their structure and applications.

Aims

- To make best use of available learning time by encouraging active learning and by transmitting information in the most effective ways.
- To give students a genuine experience of C.
- To make students aware of the importance of algorithmic concerns in real-life Computer Science situations.
- To emphasise practical concerns, rather than mathematical analysis.
- To become confident with a range of data structures and algorithms and able to apply them in realistic tasks.

Additional reading

Algorithm design: foundations, analysis and internet examples - Goodrich, Michael T. and Roberto Tamassia

Contents

1 Keys and total orders

It is often important to be able to implement some kind of comperator between data items. This is often achieved in the from of a total order relation, which has the following three properties:

Reflexive property $k \leq k$

Antisymmetric property $(k_1 \leq k_2 \wedge k_2 \leq k_1) \implies k_1 = k_2$

Transitive property $k_1 \leq k_2 \wedge k_2 \leq k_3 \implies k_1 \leq k_3$

A comperator that has the above three properties defines a linear ordering relationship between data items. This means there will be a smallest item k_{min}, where $k_{min} \leq K$ for all K in the collections of data items.

2 Trees

2.1 Definition

A tree is an abstract data type for hierarchical storage of information. Each element in a tree has a parent element, and zero or more children elements. The node at the top of the tree is called the root.

A sub-tree is the tree consisting of all the descendents of a child of a tree, including the child itself.

A tree is said to be *ordered* if a linear ordering relation is defined for the children of each node, that is to say that if we wanted to, we could apply this relation to sort the children into an ordered list.

A binary tree is one where each node can have a maximum of two children. A binary tree is *proper* if each node as two (or zero) children. At each level of a binary tree, the number of nodes in that level is at most 2^d where d is the level of the tree (starting from 0).

The depth of a node is the number of ancestors of the node excluding the node itself.

2.2 Tree algorithms

2.2.1 Depth of a node

The depth of a node in a tree is the number of ancestors of the node, excluding the node itself.

```
int depth() {
  if(parent == null) {
    // We've got no parent; we are the root!
    return 0;
  } else {
    return parent.depth() + 1;
  }
}
```

This algorithm runs in $O(n)$ time and space, since it's dependent on the depth of the tree, and is recursive.

Note that recursive algorithms always use space proportional to the number of times they have recursed, since (unless tail recursion is used), each recursion will use a stack frame.

2.3 Height of a tree

A simple way to find the height of the tree, would be to iterate over every node (maybe using a tree traversal algorithm mentioned in section **??**), and find the depth of each, keeping track of the maximum depth. This would be a $O(n^2)$ algorithm.

A better approach is to use a recursive definition, and start from the root of the tree. We can find the height of all of the child nodes, and return that plus one.

```
int height() {
  if(children.size() == 0) return 0;
  else {
    int max = 0;
    for(Tree child : children) {
      int childHeight = child.height();
      if(childHeight > max) max = childHeight;
    }
    return max + 1;
  }
}
```

2.3.1 Tree traversal

There are two different traversal schemes for trees; pre-order and post-order. Each visits the elements in the tree in a different order. The following code shows two different map functions, iterating in pre-order and then post order.

```
void mapPreOrder(Tree* root, void (*action)(Tree*)) {
  action(root);
  for(int i = 0; i < root->numChildren; i++) {
    root->children[i].mapInOrder(action);
  }
}
void mapPostOrder(Tree* root, void (*action)(Tree*)) {
  for(int i = 0; i < root->numChildren; i++) {
    root->children[i].mapInOrder(action);
  }
  action(root);
}
```

You can also iterate in-order if your tree is a binary tree, you visit the left child first, call the function on the current node, and then visit the right child.

All the traversal algorithms take $O(n)$ time.

2.4 Tree datastructures

There are two *main* ways to store binary trees in memory; using a list of nodes (in a heap style), or by using a linked datastructure, having nodes point to other nodes.

2.4.1 Using a vector based datastructure for trees

In the vector (list/array/whatever you want to call it) style, the root of the tree is stored at the start of the array. To calculate the index of the left child of a node, you multiply the index of the current node by two. To find the index of the right child of a node, you multiply its index by

two and add one. This numbering function is known as *level numbering*, and can be implemented like so:

```
int left(int n) { return 2 * n; }
int right(int n) { return (2 * n) + 1; }
```

The running times of a vector-backed binary tree are good. Iteration can be done in $O(n)$ time with a low overhead, swapping elements is $O(1)$ as is replacing them.

2.4.2 Using linked nodes to form a tree datastructure

The trouble with using a vector datastructure, is you need to initialise an area of memory equal to $2^{depth} \times sizeof(Tree)$. This means that for deep trees, you will be wasting very large amounts of memory. This is a rather extreme case of a memory-speed trade off.

In a linked data structure, each node points to all of its children. A really simple example of a binary tree one could be:

```
public class Tree<T> {
  public Tree<T> left, right;
  public T value;
}
```

If wanted to represent general (i.e. not binary) trees, we would have to modify the datastructure so that we could have any number of children:

```
public class Tree<T> {
  public List<Tree<T>> children;
  public T value;
}
```

3 Priority Queues

A priority queue is a datastructure capable of ordering items based on some associated key. The two most important operations implemented on it are `insertItempriority, item` and `removeMin()`.

Priority queues are the basis of some sorting algorithms, for example heap sort, smooth sort, selection sort and insertion sort. The different type of sort depends on how the priority queue is implemented. To sort a list using this method, add all the items to the priority queue in any order, fill up the array again in the order that the elements come out of the queue.

```
public <T implements Comparable<T>> List<T> sort(List<T> list) {
  PriorityQueue<T> pQueue = new PriotityQueue<T>();
  while(!list.isEmpty()) {
    pQueue.add(list.remove(0));
  }
  while(pQueue.isEmpty()) {
    list.add(pQueue.poll());
  }
  return list;
}
```

3.1 Heaps

Priority queues are often implemented using a heap as the backing datastructure. A heap is a binary tree that stores a collection of values. The type of values stored by the heap must have a total order relationship, since otherwise, the heap cannot be constructed correctly. The two properties that make heaps different from normal binary trees are:

Heap order property:
For every node v in the tree, the value of v is greater than or equal to the value of its parent. The only exception is the root, since that doesn't have a parent.

This means that if you start from the root, and move towards any leaf, then the nodes you encounter will be in a non-decreasing order. It also means that the minimum key is at the root.

Complete binary tree property:
This means that if heap has a depth of d, then all levels of the tree from 0 to $d-1$ must be completely full (i.e. they have 2^{level} nodes in), and the last level is filled up from left to right with the remaining elements.

There are no operations on a heap run that in worse than $O(log(n))$ time. The operations are performed in a time proportional to the height of the tree rather than the number of its elements.

3.1.1 Insertion

To store a new element in the heap, we add a new node to the heap at the next available position (the last position in the tree). If the heap is backed by an array, then the index of the array will be $n+1$, where n is the current heap size.

After we've inserted the new element we must check that the heap properties aren't violated. The *complete binary tree* property will be fine since we added the element at the end of the array (which is always the last element in the current level, or the first element in a new level), but the *heap order* property may be violated.

To maintain the *heap order* property, we need to 'bubble' the newly inserted item up the heap until the property is restored. This is achieved by comparing the key of the newly inserted node n with its parent p. If $k(u) > k(p)$, then they are swapped around (which is literally caused by indexes the two items in the array). In the worst case, you would insert the (new) smallest element into the heap, and the element would bubble all the way up to the top, taking $log(n)$ time. This is called 'up heap bubbling'.

3.1.2 Removal

If we want to remove the smallest element from the heap (as is required for a priority queue), then that will be the root element of the heap. However, if we just removed that element, then we would have two binary trees not one (a left and a right tree).

Instead, we move the last element in the array into the first element, and return the original first element (the last element is deleted). The *complete tree* property is now satisfied again. Now we have the element previously at the bottom of the heap at the top, so we need to move it into a position that will satisfy the *heap order* property.

To restore the *heap order* property, we must 'down heap bubble' the root element. If the root element is the only node, then the property is already satisfied. If the only child is on the left, then let s be the left child, and otherwise, let s be the smallest (direct) child of the root node.

If $k(root) > k(s)$ then the heap order property is restored by swapping r and s. You continue down the tree, swapping what was the route node until no violation of the heap order property

occurs. Since we may have to move all the way down the tree from root to leaf, the runtime of this is also $O(log(n))$.

3.1.3 Heap sort

Since insertion and removal are both $O(log(n))$, and we're inserting n elements, and then removing n elements, then the runtime of a heap sort will be $n \times n\log(n)$. Since constant factors are removed from the Big-O notation, then the runtime is $O(n\log(n))$. This is a large improvement from selection and insertion sort that take n^2 time. If the sequence was implemented as an array, then the sort can be done in place too!

4 Dictionaries and Hash Tables

In a dictionary, the user assigns keys to elements, and the keys can be used to look up elements in the datastructure (also known as a map in other languages such as `java`). Dictionaries can be unordered, or ordered. When keys are unique, then the dictionary is referred to as an associated store.

If the keys are unique, then a good way to implement the dictionary is a hash table. Here, the key is hashed into an integer using a hashing function. The result is used as the index of an array of 'buckets'.

Each bucket stores a collection of values that have the same hashed key. Ideally, each bucket will hold one element. This is easy to ensure for some data, for example, if we had a Hash Table of the integers from 0-50, then we could use the integer as the index to an array of fifty elements. In general though, it is not possible to ensure that no collisions will occur, and one bucket will probably end up with multiple elements. In this case, we need a collision resolution strategy.

If we have a collision resolution strategy, then we don't need to have an array that is as big as the number of items we're going to hold. We could just rely on the collision resolution strategy to let us have a smaller array and therefore need less contiguous memory space. Even if we do have the memory to spare, its wasteful to have a massive array if we're only going to store a few items in there.

If our hash function produces integers as indexes that are greater than the number of buckets, then we can either take the modulus of that number by the size of the array, or use the MAD method:

$$hash = |ak + b|modN$$

Where N is a prime, and a, b are random non-negative integers such that $amodN \neq 0$. The MAD (Multiply Add and Divide) method means the chance of a collision is $\frac{1}{N}$.

4.1 Collision handling

There are different collision handling schemes, and each has its own benefits and drawbacks. Here are just three:

Separate chaining:
: This is most often implemented using a Linked List for the buckets. Basically, when you want to add an item to the bucket, you just append it to the existing Linked List of items.

Open addressing:
: In this strategy, if the index of the hash value is already occupied, then the buckets are examined in some sequence (the easiest is linear probing, which just looks at consecutive elements in the array), until a free slot is found.

Double hashing:
Here, a second hashing algorithm is used to find an alternative slot when the first shows a collision. If we did j attempts to find a new slot, the new index would be $hash1+(j \times h(key))$, where h is the second hashing algorithm.

Different collision resolution schemes exhibit different performance as the load factor changes. The load factor is the number of entries divided by the number of buckets.

If the load factor becomes too high, then we could *rehash* the table. This involves increasing the number of buckets, and re-inserting all the elements currently in the table so that they are distributed according to the new size of the array.

4.1.1 Probing

Probing is part of the open addressing collision resolution strategy. Linear probing is easiest, you just walk through the array one by one until you find an empty slot. Quadratic probing is harder, the next index to try and insert the element into is found using an equation such as $|index + j^2|modN$, where j is the j^{th} attempt at finding a slot.

Removing elements from a open addressed hash table is slightly more complicated, but can be resolved by marking elements as 'deactivated' rather than removing them.

5 Ordered Dictionaries

An ordered dictionary is a key value store that lets you iterate over the keys in order. This requirement means that we can't use a hash table to implement the dictionary, so we need to use something else.

5.1 Lookup tables

We could order the keys with their associated values in a list by some comperator and use the sorted nature of the keys to find the a specific pair. If we were silly)or we'd chosen a linked list to implement the list), then we would iterate through the whole array to find the pair we wanted. However, if we were clever, then we could implement a binary search.

5.1.1 Binary Search

A binary search is an efficient way of searching though a sorted list for an item. Basically, you start at index $\frac{n}{2}$, where n is the number of items in the list. If the index at that value is equal to the one you want, then you've found the correct index! If its less than the one you want, then you remove the bottom half of the list and start again, but if it's greater, then you do the same for the top half of the list.

```
1  public static <T extends Comparable<T>> int binarySearch(T[] input, T item,
2                                                            int start, int end) {
3    if(start > end) return -1;
4    else {
5      int middleIndex = start + ((end - start) / 2);
6      int comparison = item.compareTo(input[middleIndex]);
7      if(comparison == 0) {
8        return middleIndex;
9      } else if(comparison < 0) {
10       return binarySearch(input, item, start, middleIndex - 1);
11     } else {
12       return binarySearch(input, item, middleIndex + 1, end);
```

```
13      }
14    }
15  }
```

Since we must do at most $log(n)$ recursions of the above algorithm (we halve the input set every time), then the run time of the algorithm is $O(log(n))$. Formally, this is because if $T(n)$ is the computational cost of a binary search then:

$$T(n) = \begin{cases} c, & \text{if } n < 2 \\ T(\frac{n}{2}) + c, & \text{Otherwise} \end{cases}$$

6 Binary Trees

If you don't know what a binary tree is by this point in your degree then you might not have been listening in all of your lectures, or in fact, reading my notes properly.

You can use a binary tree as an ordered dictionary, since the keys can be stored in the tree format, and they can each link to their respective values. The runtime of a binary tree lookup is $O(n)$, as is insertion.

A perfectly balanced binary tree will exhibit a runtime of $O(log(n))$ for search and insert, but maintaining a balanced binary tree requires extra effort.

6.1 AVL Trees

AVL trees are self balancing binary trees. The idea is that the datastructure will re-order itself when it is mutated to stay close to the optimum shape. To do this, we add an additional property onto that of the binary tree; the *height balance property*:

For every internal node v of the tree T, the heights of its children can differ by at most 1.

From this, we can derive the following properties about AVL trees:

- Any subtree of an AVL tree is also an AVL tree itself.
- The height of an AVL tree that stores n items is $O(log(n))$.

Since the maximum depth of an AVL tree is $log(n)$, searching for an item in the tree is also an $O(log(n))$ operation.

6.1.1 AVL tree insertion

If our tree satisfies the height-balance property prior to us inserting a new element, then after the element has been inserted, the heights of the nodes from the root to the newly inserted node will increase by one. This could cause them to violate the height-balance property.

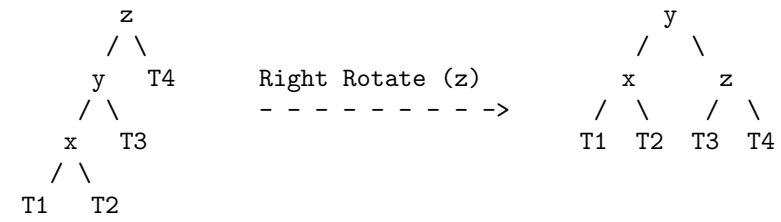
To restore the balance to the tree, we use a strategy called *search and repair*:

- Let z by the first node on the path from the new node to the root that is unbalanced.
- Denote y to be the child of z that has the larger height.
- Denote x to be the child of y that has the larger height.
- Since z is unbalanced, we need to perform *trinode restructuring*, which re-orders the tree but preserves the inorder traversal of the tree and runs in $O(1)$ time. There are four cases to this algorithm:

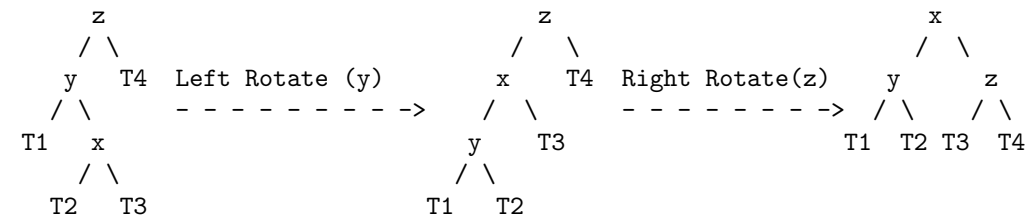
  Left-left case:

See Figure 1 for a description of where to put z, y and x.

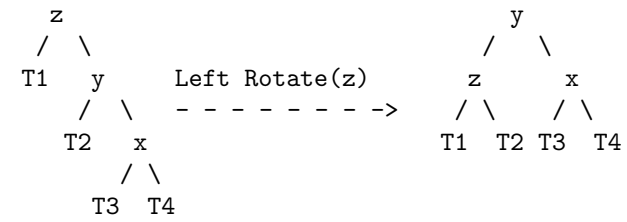
Choose any ancestor of z if there is a tie when choosing y or x.



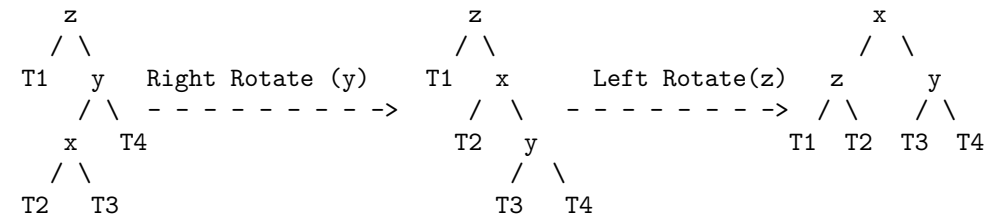
Left-right case:



Right-right case:



Right-left case:



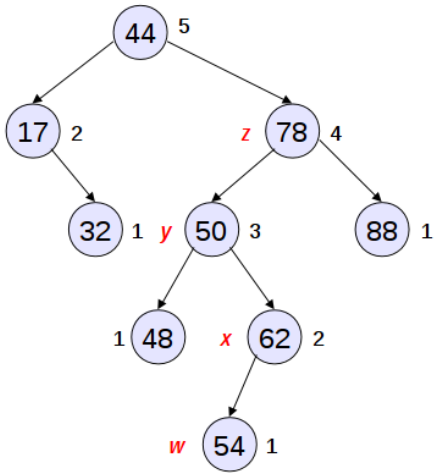


Figure 1: You can see the correct choices of z, y and x when searching an AVL tree after an insert

6.1.2 AVL tree removal

If we remove an external node, then the height balance property of the AVL tree will remain satisfied, but if we remove an internal node, then it might not be. In this case, the node is removed as normal, but we again apply the trinode restructuring after its been removed to restore the balance.

Once you have applied the trinode restructuring once, you need to carry on moving up the tree and checking the balance, since otherwise there could still be unbalanced nodes higher up the tree. Since the maximum number of times we'll need to restructure the tree is equal to the depth of the tree, removal is $O(log(n))$.

7 Problem solving

Three techniques for implementing algorithms are looked at in this course; divide-and-conquer, greedy strategies and dynamic programming.

7.1 Divide and Conquer

Divide and conquer is a design paradigm based on multi-branch recursion. This means that at each stage of the problem, we split the problem into parts that can be solved by the same algorithm. This has the effect of reducing the problem into something smaller every time. Eventually, we will have reduced the input set for any one problem into something much smaller, at which point it is significantly easier to solve the problem (since the size of the input set is small).

The divide and conquer technique is very popular, for example MergeSort, and QuickSort are both based on it, you can do integer multiplication with it, and even solve nearest neighbour problems (like the ones in `COMP24111`. You can even use it for the last lab in this course (ex13) to split up the input set of nodes into chunks and process them individually.

7.1.1 D&C Example

Lets imagine we want to find the largest and smallest numbers in a set of integers. We could do this in $O(n)$ very easily (see Figure 7.1.1), so the divide and conquer algorithm isn't really needed here, however, this problem is still a good demonstration of the technique.

```
public class Easy {
  public static void main(String[] args) {
    int[] input = new int[args.length];
    for(int i = 0; i < args.length; i++) {
      input[i] = Integer.parseInt(args[i]);
    }
    int min = input[0], max = input[0];
    for(int i : input) {
      if(i < min) min = i;
      if(i > max) max = i;
    }
    System.out.printf("Largest = %d\nSmallest = %d\n", max, min);
  }
}
```

Figure 2: Solving the largest/smallest integer problem is easy to do, even in $O(n)$ time.

To solve the problem using the Divide and Conquer strategy, we should split the input list into two at the start of the method, and run the algorithm recursively on the two sub-lists. Then, when the sublist length is either one or two, a pair should be generated of the maximum and the minimum, which is then returned. Once the method has two pairs of max/min elements from its sublists, then it should combine them by finding the smallest and largest from the two pairs. See Figure 7.1.1 for a simple implementation.

```
public class DivideAndConquer {

  private static class Pair { int min, max; }

  public static Pair findMinMax(int[] input, int start, int end) {
    if((end - start) <= 1) {
      Pair output = new Pair();
      output.max = (input[start] < input[end]) ? input[end] : input[start];
      output.min = (input[start] < input[end]) ? input[start] : input[end];
      return output;
    } else {
      Pair firstHalf = findMinMax(input, start, end - ((end - start) / 2));
      Pair secondHalf = findMinMax(input, end - ((end - start) / 2), end);
      if(firstHalf.min > secondHalf.min) firstHalf.min = secondHalf.min;
      if(firstHalf.max < secondHalf.max) firstHalf.max = secondHalf.max;
      return firstHalf;
    }
  }

  public static void main(String[] args) {
    int[] input = new int[args.length];
    for(int i = 0; i < args.length; i++) {
      input[i] = Integer.parseInt(args[i]);
    }
    Pair out = findMinMax(input, 0, input.length - 1);
    System.out.printf("Largest = %d\nSmallest = %d\n", out.max, out.min);
  }
}
```

Figure 3: We can also solve the largest/smallest integer problem using divide and conquer.

While the run time of the Divide and Conquer solution may not be significantly faster than the Easy one, there are other advantages, and disadvantages. Obviously, the divide and conquer one

uses more memory space, since we're using recursion, we add a new frame onto the stack for every recursive call. However, we could parallelise the divide and conquer solution much more easily (which can bring near-linear performance gains).

7.2 Greedy method

The greedy method is a heuristic for making the optimal choice at each stage of a problem. For example, if we wanted to give the correct change for $36p$, then we would first choose the highest coin that fits into that, which is a $20p$, and then choose the highest coin that fits into $36 - 20 = 16p$, which is $10p$, and then the highest that fits into $16 - 10 = 6$, which is $5p$ and then $1p$, since that's what we have left.

Since greedy is a heuristic, it doesn't always work optimally. For example, if we had the coin values $10, 6$ and 1 and were trying to make 12, then greedy would pick $10, 1, 1$, but the optimal solution is $6, 6$.

An implementation of this is at the end of the document, Figure 8.

For some applications, a greedy strategy is always successful, but for others, it will perform poorly, or not find solutions at all. Greedy-applicable problems include path finding, job scheduling, knapsack problems etc.

7.3 Dynamic programming

In general, the greedy method fails for the coin problem, and many other problems too! Dynamic programming can always produce optimal solutions to some of the problems that greedy will fail at.

Lets solve the coin problem with dynamic programming. To do this, we build up a solution from the bottom and word towards a complete solution.

If we had the coins $9, 1, 5$ and 6 and we were trying to make the number 11, then the greedy method would use $9, 1, 1$ as the coins. With dynamic programming, we build up an array of sub-problem solutions and use those results to form better solutions:

Coins	1	2	3	4	5	6	7	8	9	10	11
1	∞	∞	∞	∞	∞	∞	∞	∞	1	∞	∞
$1, 9$	1	2	3	4	5	6	7	8	1	2	3
$1, 9, 5$	1	2	3	4	1	2	3	4	1	2	3
$1, 9, 5, 6$	1	2	3	4	1	1	2	3	1	2	2

The efficiency of DP is the number of rows in the table multiplied by the number of columns. Sometimes, you can save CPU time by not computing all of the subproblems, but that might be problematic, since you could need them later.

Dynamic programming can be applied to path finding, text similarity tests, knapsack problems etc.

8 Misc figures

These were too big to go in other places, so I shoved them at the end.

```java
import java.util.TreeSet;
import java.util.TreeMap;
import java.util.Map.Entry;

public class Greedy {

  public static int solve(int change, TreeMap<Integer, Integer> coins) {
    while(change > 0) {
      boolean foundCoin = false;
      for(Integer coin : coins.descendingKeySet()) {
        if(coin <= change) {
          change -= coin;
          coins.put(coin, coins.get(coin) + 1);
          foundCoin = true;
          break;
        }
      }
      if(!foundCoin) {
        return change;
      }
    }
    return change;
  }

  public static void main(String[] args) {

    int change = Integer.parseInt(args[0]);
    TreeMap<Integer, Integer> coins = new TreeMap<Integer, Integer>();
    for(int i = 1; i < args.length; i++) {
      coins.put(Integer.parseInt(args[i]), 0);
    }

    int remaining = solve(change, coins);
    System.out.println(remaining + " remaining!");
    System.out.println("Used coins: ");
    for(Entry<Integer, Integer> e : coins.entrySet()) {
      System.out.printf("%d - %d\n", e.getKey(), e.getValue());
    }
  }

}
```

Figure 4: An implementation of the greedy money counter.

8.1 Tractable and Intractable Problems

Divide and Conquer and Greedy Strategies give us fast (polynomial time) algorithms, and Dynamic Programming gives us fast algorithms, but also algorithms that work on intractable problems.

A problem is considered tractable if we can produce a polynomial ($O(1), O(n), O(n^x)$) time algorithm to solve it. Algorithms are intractable if they are exponential, such as $O(2^n), O(n^n), O(n!)$.

Exponential algorithms often arise from exhaustive enumeration (i.e. checking all the possible solutions), or systematic searches of partial solutions using backtracking. If we wanted to colour a graph (that is give each node a colour, but make sure that no adjacent nodes had the same colour), we could use both techniques.

Exhaustive enumeration is the most simple approach to graph colouring; you start with k colours,

and enumerate all the allocations of the colours on the nodes, and for each, you check to see if its valid. Checking an allocation takes $O(E)$ time, but there are k^N number of allocations, so the algorithm takes $O(E \times k^N)$ time.

E is the number of edges, N is the number of nodes.

9 Graphs

Graphs are a very important datastructure in Computer Science. Thankfully, the lecture notes for this course do a fantastic job of defining them, and giving all the terminology related to them, so see the slides from Lecture 6 of the second semester of the course. I can focus on the fun bits!

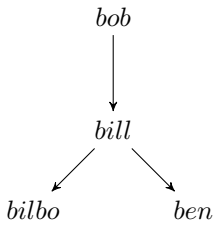
9.1 Representing graphs

The two datastructures used to represent graphs on the course are adjacency lists and adjacency matrices:

Adjacency List:
This is a list of each node in the graph, and the nodes that it connected to via an edge, maybe looking like this:

Node	**Outlist**
Bob	Bill
Bill	Bilbo, Ben
Bilbo	
Ben	

For the graph:



We could implement the adjacency list as an array of linked lists for fast random access to any one node, and sequential access to the outlist (which is a common access pattern as we might see later in the graph algorithms section).

Adjacency Matrix:
This is a two dimensional array, where each dimension is indexed by the nodes, and the values are 1 for an edge between them, and 0 for no edge. For the same graph as above:

	Bob	Bill	Bilbo	Ben
Bob	0	1	0	0
Bill	0	0	1	1
Bilbo	0	0	0	0
Ben	0	0	0	0

Obviously, an adjacency matrix takes up more memory space than an adjacency list, but it is also probably faster, since there aren't any pointers involved (if the array is in contiguous areas of memory). Adjacency matrices can also be wasteful in terms of memory space, especially if the graph is sparse (i.e. it doesn't have many edges) since most of the array will be zeros! If the graph is undirected, then the matrix will by symmetrical too, which is obviously very wasteful in terms of memory space. One massive advantage of adjacency matrices is that you can do matrix arithmetic on them, so you can effectively add graphs together, multiply them etc.

Remember, what is faster almost always depends on your use case!

9.2 Algorithms

Graph algorithms are really important. Not only do they crop up in loads of interview questions, but they also can be applied to problems that you might never of imagined they would be related to.

9.2.1 Depth and Breadth First Search and Priority Search

All three of these searches use the same algorithm, its just the backend datastructure that changes. The algorithm is as follows:

```
public void search(Graph graph, Node startNode) {
  Datastructure dt = new Datastructure();
  Set<Node> discovered = new HashSet<Node>();
  dt.push(startNode);
  while(!dt.isEmpty()) {
    Node n = dt.pop();
    // If the node has not been seen before
    if(!discovered.contains(n)) {
      discovered.add(n);
      for(Node out : n.outlist) {
        dt.push(out);
      }
    }
  }
}
```

For a **depth first search**, you use a **stack**, for a breadth first search, you use a **queue** and for a priority search. you use a **priority queue**.

The complexity of a depth first search is $O(N + E)$ for an adjacency list, and $O(N^2)$ for an adjacency matrix. This is because adjacency lists are more suited to random access of a iterable list of a nodes outlist than an adjacency matrix is.

9.2.2 Dijkstra's

Dijkstra's algorithm is a shortest path algorithm for graph structures. It performs in $O(E + Vlog(V))$ time (if used with a min priority queue), and is implemented as follows (taken from Wikipedia):

```
function Dijkstra(Graph, source):

  dist[source] = 0           // Distance from source to source
  prev[source] = undefined   // Previous node in optimal path initialisation

  for each vertex v in Graph:// Initialisation
    if v | source            // Where v has not yet been removed from Q (unvisited nodes)
      dist[v] = infinity     // Unknown distance function from source to v
      prev[v] = undefined    // Previous node in optimal path from source
    end if
    add v to Q               // All nodes initially in Q (unvisited nodes)
  end for

  while Q is not empty:
    u = vertex in Q with min dist[u]  // Source node in first case
    remove u from Q
```

```
17
18      for each neighbour v of u:          // where v is still in Q.
19        alt = dist[u] + length(u, v)
20        if alt < dist[v]:                 // A shorter path to v has been found
21          dist[v] = alt
22          prev[v] = u
23        end if
24      end for
25    end while
26
27    return dist[], prev[]
28
29  end function
```

9.2.3 Dijkstra's vs Floyd