Mobile Systems

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Introduction

Now that the mobile telephone has evolved into a powerful computer, the mobile dimension of computing is a vital part of Computer Science. This unit will give insights into many issues of mobile systems, including wireless communication networks, the processing of speech, music and other real-time signals, the control of bit-errors and maximising battery life. The techniques and software which underlie commonplace applications of mobile computing systems, including smart-phones, tablets, laptop computers, MP3 players and GPS satellite navigation, will be addressed.

Aims

Computing is becoming increasingly mobile. This unit will give insights into the issues of mobile systems, covering mobile communications, real-time signals such as speech, video and music, codecs, and maximising battery life.

- Commonplace examples of mobile computing systems: mobile phones; MP3 players; laptop computers; PDAs; GPS satellite navigation.
- Real-time signals
- Analogue and digital signals; time and frequency domain representations; sampling, aliasing, quantization; companding; real-time computation.
- Coding, decoding and compression
- GSM speech coding; MP3 music, JPEG image and MPEG video coding & decoding; error correcting codes; communications coding schemes.
- Mobile communication
- Transmitting real-time information over wireless networks; principles of cellular and ad-hoc networks; Coding of multimedia signals to increase the capacity of radio channels; to minimise the effect of transmission errors.
- Maximising battery life
- May be addressed at many levels including: chip design; signal coding and processing; medium access control; transmit power control.

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1 Course intro

A smartphone is a mobile phone running a mobile operating system, with advanced capabilities with regard to computing power and connectivity. They are much more advanced than traditional feature phones, and have lots of features, including cameras, multimedia functionality, GPS, touch screens etc.

There are three main mobile operating systems in use today:

Android

Founded in 2003 by Andy Rubin and backed by Google. It's mostly free and open source, and holds a very strong position in the market.

iOS

Introduced in 2007 by Apple, this is a closed source operating system. The first iOS phones were very groundbreaking in terms of their technology, and were the first to feature touch screens, which are now ubiquitous in the market.

Blackberry, Symbian, Palm OS etc could be listed here too.

Windows Phone

Version seven was released in 2010, previous versions were terrible (imho).

1.1 What's in a smartphone?

Modern smartphones are jam packed with technology, containing cameras, multimedia, GPS, high resolution touch screens, motion sensors, bluetooth, RFID, NFC and even more stuff besides. They let you talk over multiple different networks (2G, 3G, 4G etc), and access data using the same methods.

Of course, the millions of apps available for them is also a massive attraction.

Smartphones Operating Systems need to be very advanced in order to step up to the tasks required of them by users. They need to be multitasking, to run lots of different apps, and interface with the typical hardware a normal computer might use (DMA, standard (ish) input devices etc). However, they also need to have real-time elements in, since they must be able to drive sound, IO, radio communications, digital signal processing etc

2 Signals in mobile systems

Signals such as speech and music arrive at the device as physical, analogue quantities that vary in a continuous manner over time. If we plot a graph of voltage over time, then we get a waveform for that signal. We can convert analogue signals to *discrete time signals* by sampling them at set intervals (discrete points in time). This produces a list of numbers from — inf to inf.

2.1 Generating waves

In order to create a wave with a period of T seconds, you can use the formula:

$$y=\sin(\frac{2\pi x}{T})$$

Of course, since $f = \frac{1}{T}$, the frequency of the wave is the reciprocal of the time period.

2.2 Sampling waves

We can work out the frequency of a wave, by finding how many complete cycles it undergoes in one thousand samples, and multiplying that by the sample rate. For example, if a wave has ten

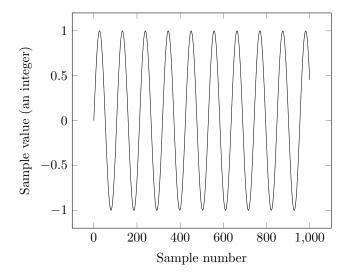


Figure 1: 1000 samples of an analogue sin wave of frequency 300Hz sampled at 30,000Hz.

cycles from 1000 samples at 30,000Hz, then its frequency will by $\frac{10}{1000} \times 30,000 = 300Hz$. This is visible in Figure 2.2.

Not all waves are so easy to analyse, rarely will a wave be at just one frequency, instead, they are usually 'noisy' and will be composed of many waves added together. Many waves (that aren't just noise) will have a discernible frequency that you can extract. The 'extra' waves on top of this frequency aren't necessarily bad, they might add harmonics, or texture to a sound.

2.2.1 The Sampling Theorem

Also known as Nyquist criterion.

The Sampling Theorem states that if a signal has all of its spectral energy below BHz, and is sampled at FHz, where $F \geq 2B$, then it can be reconstructed exactly from the samples and nothing is lost.

In other words, if you are sampling a signal at *less than half* of the maximum frequency of the signal, then you won't be able to fully reconstruct the signal from the samples, and you will get distortion.

Since music is usually sampled at 44.1kHz, we can accurately sample music with frequencies up to around 22kHz. Speech rarely has frequencies above 3.4kHz, so the sample rate for it can be much lower, as we will see later on in the course.

2.2.2 Aliasing

If we sample at less than twice the frequency of the wave, then we will get distortion, known as aliasing. The first lab on this course relates to aliasing. Even if we don't want frequencies higher than half our sample rate, we need to filter them out anyway, since otherwise, we will get aliasing when we sample them.

To be precise, if a wave of frequency f is sampled at a frequency of below 2f (lets call this F, then the sampled output will be a wave of frequency F - f.

For example, if we sample a 6kHz wave at a frequency of 10kHz, when we will get a wave of frequency 10-6=4kHz.

This is bad, because if you have harmonics in a musical note that are higher than half the sampling frequency, then these harmonics will be out of tune post sampling, and will go down when it's supposed to go up.

It may be hard to work out exactly why aliasing occurs, but Figure 2, showing a 6kHz wave sampled at 8kHz makes it easier to see:

2.3 Decibels

Sound can be measured in decibels, which is a logarithmic ratio. The formula is as follows:

$$dB = 10 \times log_{10} \left(\frac{s_1}{s_2} \right)$$

If s_1 is twice as loud as s_2 , then it will be $10 \times \log_{10}(2) = 3 dB$ louder than s_2 .

The following table shows the power ratio $(\frac{s_1}{s_2})$ against the equivalent decibels:

Power ratio	Decibels
$\frac{1}{2}$	-3
1	0
2	3
4	6
10	10
100	20
1000	30
10^{5}	50
10^{10}	100

3 Frequency Domain Processing

Instead of processing waves, and parts of waves, how about we turn them into frequencies and process those instead? Frequencies are a lot easier to deal with in many ways since they are easier to understand and smaller to store.

3.0.1 Sinusoids

A sinusoid is a sine wave delayed by D seconds, and is given by the formula:

$$y = M \times cos(2\pi F(t-D))$$

Figure 3.0.1 shows two sinusoids; one where $D = 30^{\circ}$ and another where D = 0.

Figure 3.0.1 is a simple sine wave, but any wave (such as the one in Figure 3.0.1) that is periodic can be found to have a fundamental frequency of $\frac{1}{T}$, where T is the period of the wave. A recording of a voice, or musical instrument may be found to have a similar waveform to that of Figure 3.0.1 as long as we 'zoom in' to a segment short enough that any gradual change in frequency is negligible. This is called **psudo-periodicity**.

3.0.2 Fourier Series

Any periodic wave of frequency F can be written as:

$$x(t) = A_0/2 + A_1 cos(2\pi Ft) + B_1 sin(2\pi Ft)$$

$$+ A_2 cos(2\pi 2Ft) + B_2 sin(2\pi 2Ft)$$

$$+ A_3 cos(2\pi 3Ft) + B_3 sin(2\pi 3Ft)$$

$$+ \dots$$

This is sometimes called its 'cos and sin' form, but we could re-write it as:

$$x(t) = \frac{M_0}{2} + \sum_{k=1}^{\infty} M_k \cos(2\pi(kF)t + \phi_k)$$

Here, each time the summation iterates, we are adding the next harmonic of the sound. The fundamental frequency F is the first harmonic when k = 1.

Because the harmonics get higher and higher, we can't sample the whole series because otherwise the frequency of the harmonics will start to become higher than half the sampling frequency and we will get aliasing (see Section 2). In that case, we only go up to $k = \frac{N}{2} - 1$, where $F(\frac{N}{2} - 1) \le \frac{Fs}{2}$. If we took Fs samples every second, then the formula would be:

$$x(n) = \frac{M_0}{2} + \sum_{k=1}^{\frac{N}{2}-1} M_k \cos\left(2\pi(kF)\frac{n}{Fs} + \phi_k\right)$$
Where $n = 0, 1, \dots, \text{size}(x)$

The more harmonics we compute, the closer we can get to reconstructing the wave from them afterwards.

3.0.3 Discrete Fourier Transform (iDFT)

The DFT converts samples of length n into n/2-1 samples and a 'dc term'. They are arranged in a length n array, but the second half is the same as the first half but reversed. The 0^{th} element of the array is the DC term, but the i^{th} element is the i^{th} harmonic.

The algorithm to convert from the time domain into the frequency domain is N-point DFT. To do the opposite, we can use the N-point inverse DFT algorithm.

DFT is used to:

- Perform spectral analysis on a signal to find out what components are present.
- Convert into the frequency domain before applying signal processing (and converting back again afterwards). For example, you could filter out noise from a sine wave, since you could remove all frequencies with an amplitude below a certain level.

3.1 Way files

Signals are stored in many different ways. One 'easy' format, is the .wav format. It is just a list of binary numbers, each representing a single value of the wave at a discrete time.

4 Encoding and storing signals

There are a variety of different ways to encode sound. The two main types of coders for talking over the phone are *waveform* coders, and *parametric coders* (sometimes called vocoders).

Waveform coders

Waveform coders 'operate on' the sound waves directly, and aim to change the wave so that it is transmitted in an optimal way. They are simple to understand and implement, but can't achieve *really* low bit rates. They aim to preserve the exact shape of the wave.

Parametric coders

Parametric coders try and model human speech by exploiting how we produce sound with our vocal chords and mouth shape. They don't try and preserve the exact wave shape, but instead try and describe *perceptually significant features* as sets of parameters. This is more complicated and harder to implement, but achieves lower bit rates.

Techniques that fall into both types of coder will be discussed in this section.

Humans can hear frequencies between 20Hz and 20kHz, and have a dynamic range of 120dB. We can represent six decibels per bit of information using uniform quantisation (see section 4.2), so we need about $\frac{120}{6}$ bits to properly represent the sample.

Dynamic range is the ratio between the loudest sound we can hear, and the most quiet sound we can hear.

4.1 Bitrates in different settings

Sometimes, 20 bits per sample is too much, so different bit rates are used in different applications:

CD's

Unfortunately, 20 bits per sample was too much for CD's (you wouldn't be able to fit enough songs on each CD), so instead, 16 bits item per sample was adopted as the standard. In order to 'lose' these four bits, we make the quiet bits of the songs louder (since we probably wouldn't be able to hear them anyway).

As a consequence of this, a song on a CD plays at a rate of $16 \times 44100 \times 2 = 1411$ kbit/s.

Landlines

Landlines are band-limited to a range of frequencies from 50Hz to 3.4kHz. Obviously, this is much, much less than the dynamic range that humans can hear, but we only usually talk in frequencies covered by the range provided. This means that the sound will lose its 'naturalness' but not intelligibility (supposedly).

The sample rate for telephone quality speech is 8kHz, with 8bits. This gives a bitrate of 64kbit/s. In order to use just 8bit per sample, we need to use non-uniform quantisation (like in the (second?) lab), such as μ -law quantisation.

Mobile Telephones

Mobile phones use a bit rate that changes based on factors such as the signal quality, but has a minimum value of 4.75kbit/s, and a maximum value of 12.2kbit/s. AMR encoding (see Section ??) is used to achieve such low bitrates.

4.2 Quantisation

When we've sampled a wave, if we leave all of our readings as floating point numbers, then we could be using a lot of space to store each sample. We could slightly reduce the quality of our sound by *quantising* it. This is when you map the continuous values onto a range of integers, where the range of integers spans a power of two (so you can use as few bits as possible).

You can see that the third wave in Figure 5 has been quantised into five integers, since there are five discrete values in the waveform (0, 1, 2, 3, 4) (requiring four bits), and there will be an extra bit to indicate the sign. If whoever made the diagram was trying to be as efficient as possible, they might have had either one less value (so that the numbers could be represented in three bits), or made full use of the four bits for the value, and used values from 0-7.

Quantisation produces noise (known as quantisation noise), since errors are introduced in the process. If there are many bits per sample (e.g. 16), then this error will be small, but if you only used say, five bits, then the error would be noticeable. As a consequence of this, we have to work a trade-off between storage capacity or bandwidth and quality.

There are two types of quantisation, uniform and non-uniform:

Uniform quantisation

A simple type of quantisation, each binary number represents a voltage, where incrementing the binary representation will correspond to an increase in the voltage of ΔV , called the quantisation step-size.

In practice, sometimes, vowel sounds like 'f' and 's' can be mixed up.

Band limiting is when a sound is Fourier transformed, and frequencies above or below a certain frequency are stripped. It might be hard to decide on a value for Δ , because different sounds will span different ranges of amplitude as shown in Figure 6.

One solution to this, is to adjust the value of Δ on a sample-to-sample basis. This uses up extra bandwidth though, so is not a preferred solution.

Non-uniform quantisation

Non-uniform quantisation is when the quantisation step-size is not the same between different quantised values, as evidenced in Figure 7.

Non-uniform quantisation is implemented like so:

- 1. Apply accurate uniform quantisation.
- 2. Apply a 'compranding' formula, such as μ -law.
- 3. Uniformly quantise again using fewer bits.
- 4. Transmit.
- 5. Reverse the process of steps three and two with an 'expander'.

A 'comprander' will increase the value of small samples, and decrease the value of large samples, and an 'expander' just does the reverse. Even though we only apply two uniform quantisations, the comprander and expander mean the overall effect is one of non-uniform quantisation.

Compranding is just a coding technique similar to compression. It doesn't increase the quality of the sound or anything (though it might do if the alternative was to use the same number of bits with uniform quantisation.)

4.2.1 Quantisation error

Quantisation Error is produced when we quantise a wave, and is random in the range of $\pm \frac{\Delta}{2}$. Since the error is random, the error is heard as white noise, and is spread evenly across all frequencies. The Mean Square Value (MSV) of the noise, is $\frac{\Delta^2}{12}$.

We can calculate the Signal to Quantisation Noise Ratio (SQNR), which is how loud the signal is in comparison to the noise (hence it is given in decibels - see Section 2.3). To do this, we use the formula:

$$\mathrm{SQNR} = 10 \log_{10} \left(\frac{\mathrm{MSV~of~signal~power}}{\mathrm{MSV~of~noise}} \right)$$

We can use this formula to calculate the dB/bit for sending telephone data (speech, text etc):

$$\begin{aligned} \operatorname{SQNR} &= 10 \log_{10} \left(\frac{A^2/2}{\Delta^2/12} \right) \\ &= 10 \log_{10} \left(\frac{\left(\frac{2^{2 \times bits}}{4} \Delta^2 \right)/2}{\Delta^2/12} \right) \\ \text{The maximum amplitude of a sine wave that} \\ \text{has been quantised with a step size of } \Delta \text{ is} \\ &= 10 \log_{10} \left(\frac{\frac{2^{2 \times bits}}{8} \Delta^2}{\Delta^2/12} \right) \\ &= 10 \log_{10} \left(\frac{\left(\frac{2^{bits}}{2} \Delta \right)^2/2}{\Delta^2/12} \right) \\ &= 10 \log_{10} \left(\frac{\left(\frac{2^{bits}}{2} \Delta \right)^2/2}{\Delta^2/12} \right) \\ &= 10 \log_{10} \left(1.5 \times 2^{2 \times bits} \right) \\ &= 10 \log_{10} \left(1.5 \times 2^{2 \times bits} \right) \\ &= 10 \log_{10} \left(\frac{2^{2 \times bits - 2} \Delta^2}{\Delta^2/12} \right) \\ &= 10 \log_{10} \left(\frac{2^{2 \times bits - 2} \Delta^2}{\Delta^2/12} \right) \\ &= 1.8 + (6 \times bits) \end{aligned}$$

This only strictly applies for uniformly quantised sine waves, but also holds fairly well for speech and music.

If an 8 bit uniform quantisation scheme is designed for loud talkers (so they will use all eight bits), then it will have a SQNR of $(6 \times 8) + 1.8 = 49.8$. If another talker is much quieter, and talks 30dB quieter, then the samples will be encoded using only 3 bits:

$$6 \times bits - 1.8 = 49.8 - 30$$
$$6 \times bits = 48 - 30$$
$$6 \times bits = 18$$
$$bits = 3$$

Since the quantisation noise for three bits is much lower (19.8dB) for the quietly talking person, they will probably be able to hear the noise over the phone.

For the loudest CD quality (16 bit) music, the maximum SQNR value is 97.8dB, whereas for the most quiet sounds, it is -22.8dB (which means the noise is louder than the actual sound). Obviously this isn't ideal, so we need to apply DRC (Dynamic Range Compression) improve the balance.

Remember, since the SQNR is calculated by $\frac{MSV \text{ of signal power}}{MSV \text{ of noise}}$ it has an inversely proportional relationship with the amount of noise. I.e. will go up as the noise decreases.

4.3 Differential encoding

A sample of audio isn't just random data, and since the data represents a wave, consecutive values are likely to be relatively close together. If that is the case, then maybe we could encode the data as the difference between one sample and the next. If the differences are easier to transmit than the raw data, then we could save resources such as bandwidth. Such an encoder is shown in Figure 8.

This is known as Adaptive Differential PCM, and can be made to work at bitrates of 16-32kbit/s. However, for use in mobile telephones, we need a bit rate that is at least four times lower than this...

PCM stands for Pulse Code Modulation, and is basically the same as what we mean by quantisation.

4.4 Linear Predictive Speech Coding

LPC is a type of parametric encoder. Working on the principle that voiced speech is created by the vocal chords opening and closing at frequencies that change over time, the coder can send the fundamental frequency of the noise as well as parameters (called coefficients here) modelling the shape of the mouth over the network to recreate the speech at the receiver.

The sender derives the parameters at a rate of 50hertz, which include how loud the speech is, the coefficients modelling the vocal tract, and the fundamental frequency of the sound. Figure 9 shows how these parameters are used to reconstruct the speech at the receiver.

Voiced speech is normal speech, unvoiced speech is whispering. If normal speech is sampled, then some of it will be classed as unvoiced, since only vowel sounds actually require the use of vocal chords (try it!).

4.4.1 LPC-10

LPC-10 was a coder once used by the military that had a bit rate of 2.4kbit/s. Each 20ms frame has the following components:

37 bits Ten filter coefficients
1 bit Voiced/unvoiced decision
8 bits Gain (the amplitude)
8 bits Fundamental frequency

MIPS is Millions of Instructions Per Second

This made for a total of 56bit per frame. Although it's easy to understand and implement, and requires little processing power (according to Wikipedia¹, it requires 20MIPS and 2kB of RAM).

¹http://en.wikipedia.org/wiki/FS-1015

4.4.2 Codebook Excited LPC

Codebook Excited LPC (CELP) is a way of further reducing the bandwidth used by LPC. The principle is that there are a number of pre-configured parameters that the receiver and sender have stored in a 'codebook', and the sender just sends the index of the one that is most similar to the sound that it is trying to produce.

It finds the most similar sample using 'analysis by synthesis', which involves trying all the entries in the codebook one by one, and sending whichever is the closest to the one we want.

CELP is used in commercial mobile telephones, and is utilised by the **Adaptive MultiRate** (AMR) coder for rates at 12kbit/s or lower.

4.5 Comfort noise

When a two way conversation is going on, and neither party are speaking, nothing will be played through the speakers to either party (since we don't transmit quiet sounds since that wastes power). To mitigate this, the phones will generate and insert 'comfort noise' which is psudorandom background noise that sounds like whatever is going on at the other end of the phoneline. Each phone will determine if its owner is silent, and transmit the characteristics of the background noise using very few bits if they are so that the other phone can recreate a similar background noise for the other person.

4.6 Error correction, detection & prevention

Mobile systems are affected by errors in the transmission of data, often in the form of noise affecting radio reception. There are many ways of avoiding/mitigating errors, the most important ones looked at in this course are; Forward Error Correction (FEC) and error detection and retransmission (Automatic ReQuest).

4.6.1 Forward Error Correction

We can build redundancy into the transmission by appending check bits or some other form of coding that will bloat the size of the transmission, but result in less errors.

We can use block coding or convolutional coding to implement this. **Block coding** is when you process the data in blocks (well duh), which requires you to encode the whole block at the transmitter and decode it again at the receiver. You can't use a block unless it's been fully decoded. **Convolutional coding** on the other hand can yield usable bits soon as just a few of the transmitted bits arrive at the receiver. Convolutional coders treat data as a stream, whereas block coders deal with data in chunks.

Here are some examples of the above:

Repeating bits:

We could simply send bits a few times, instead of just once and take the mode of them at the receiver. If we sent three bits for every bit, then if there was one bit error, then it would be fine since we could take the majority of the received bits, and the one error bit would be ignored.

The number of repeats must be an odd number, otherwise we could end up with a half and half split.

Parity

As a simple (and not very effective) way of detecting bit errors, we could append a parity bit to the end of blocks. The parity bit would be 0 if the block had an even number of ones, and 1 if it had an odd number of ones. If we have an n bit number, we can calculate the parity using n-1 XOR operations; $b_0 \oplus b_1 \oplus \cdots \oplus b_n$.

When the receiver gets the bits, the parity of all of them (including the parity bit) must be computed. If the parity is 1, then some bit error definitely occurred, whereas if it is 0, then the data may be correct (but there could be two or more errors as well).

Parity checking is a block code, since you need to wait for the whole block to arrive before checking the parity.

Hamming distance

The Hamming distance between two binary numbers is the number of bits that are different. This is obtained by XORing the two numbers together and counting the number of ones. This is useful if we want to know how many bit errors it takes to convert one number to another.

You have a set list of code words that you can use, each with a minimum hamming distance from it to any other. Then, you can detect and even errors by comparing your received code to the allowed ones.

Figure 4.6.1 shows an implementation.

Hamming codes are block codes, since you need the whole block to calculate the distance between the received block and the ones in the codebook. It is common practice to introduce check bits onto code words to make the distance larger.

The notation for Hamming Codes is (m+r,m), where m is the number of message bits, and r is the number of check bits. The r bits are appended to the end of the code words, and are derived from the value of the m bits. For example, with a (7,4) hamming code, we could make:

- $r_0 = m_0 \oplus m_1 \oplus m_2$
- $r_1 = m_0 \oplus m_1 \oplus m_3$
- $r_2 = m_0 \oplus m_2 \oplus m_3$

This would give a *syndrome table* (i.e. correction table) of:

r_0	r_1	r_2	Correction to
0	0	0	None
0	0	1	r_2
0	1	0	r_1
0	1	1	m_1
1	0	0	r_0
1	0	1	m_2
1	1	0	m_3
1	1	1	m_0

Interleaving:

Since bit errors in radio links often occur in bursts (e.g. caused by a car turning on), we could transform one dimensional blocks of data into two dimensional matrices, and transfer them column by column. This way, if there is a 'bursty' bit error, then it will affect multiple rows, but the bit error correction (such as) the repeating method, or hamming codes could detect the error since only one or two bits may be wrong, not all of them.

4.6.2 CRC checks

A Cyclic Redundancy Check is a block code for detecting bit errors. If we find the value of the number we're transmitting in decimal and divide it by a number such as seven, we can append the remainder onto the message. Then at the receiver, we can divide the received bits again and if we get a different remainder, then we know that a bit error has occurred.

If the bit errors happened to add or subtract the number that you were dividing by, then the remainder would be the same and you would not be able to detect the errors. If we make the number large, then this is unlikely.

Real CRC checks use $Galois\ Field\ Arithmetic^2$ to do the maths, since binary numbers can be expressed as polynomials:

Summing:

To calculate the sum of two binary numbers, we can just XOR their bits. Subtracting is the same:

$$1001 \oplus 0111 = 1110$$

Long division:

This isn't explained particularly well by the lecture slides (mainly because multiplication isn't explained, which is helpful to understand for division) and I haven't got time to explain it before the exam, sorry! Ask Barry:)

There are three CRC generators used in practice:

• CRC-8-ATM: $x^8 + x^2 + x + 1$

• CRC-16-IBM: $x^{16} + x^{15} + x^2 + 1$

• CRC-32-IEEE: Quite long...

A generator of order r can detect all error bursts of length $\leq r$.

4.6.3 Convolutional Coders

If we are processing streams of data, then we could have a 'rolling parity check'. When we encode, the previous n bits are XOR'd together to produce another stream of bits. We can then interleave the new parity stream with the normal stream.

At the decoder, the bits are valid if each of the parity bits (i.e. not the normal ones) is the XOR of the previous three normal bits. If the bit sequence is invalid, then we can select the valid sequence with the minimum Hamming distance to the received sequence and therefore have error correction.

With sequences of around 8 bits, that is fine, since there would be 256 valid sequences of 16 bits long each, however, for longer sequences (e.g. 1024 bits), then this would not be feasible.

Convolutional coders that encode bit streams are called rolling parity coders.

A soft decision decoder is where if we're unsure as to what the value of a bit is, then we could use previously known probabilities to determine what we could pick.

4.6.4 Miscellaneous FEC stuff

Well thought out use of FEC techniques in mobile systems increases the energy efficiency and effectiveness of the system. Transmitting at higher power is one way of reducing errors in the system, but 'loud' signals also cause lots of interference over a wider range and, of course, uses lots of battery. FEC lets us reduce the transmission power, yet overcome the bit errors that come as a result, which allows us to re-use different frequencies and reduce power consumption.

We could increase the bit rate by not encoding in binary, but using any 2^n number of states. If we could send the values $\{0, 1, 2, 3\}$, then we could send two bits per pulse, effectively doubling our bandwidth. This is limited by the noise in the channel.

The **Shannon-Hartley Law** says that the channel capacity C is equal to:

$$C = B \log_2(1 + \frac{S}{N}) \text{bit/s}$$

Where B is the bandwidth in Hz, and $\frac{S}{N}$ is the signal to noise power ratio.

http://en.wikipedia.org/wiki/Finite_field_arithmetic

5 Mobile networks

Since their inception, the desired use of cellular networks has changed from being solely about transmitting speech, to transmitting textual data, and now arbitrary packets. This has required them to evolve from 'circuit switched networks', where (at a fundamental level) a physical connection must be made between wires to facilitate data transfer, to 'packed switched networks'.

This evolution is really interesting, since circuit switching originated from landline phones, but packet switching was from digital networking such as Ethernet. As the technology has progressed, cellular data has moved towards the Internet's way of doing things.

However, mobile phones have moved on from being single feature devices, merely letting you call other users. Now they are *smart*phones, and this requires them to use a plethora of different technologies to be able to fulfil user's demand; GPS, bluetooth, radio etc to name a few.

5.1 Cellular networks

One reason why mobile phones can work so well is that they take advantage of *spatial multiplexing*. This divides an area into small areas called cells, each having a frequency band. The cells are different sizes based on their expected usage rates (cells will be more dense in areas with more demand). Frequency bands are re-used when a cell with the same frequency band is far enough away, and because of this, mobile phones must take care not to transmit their signals too loud and pollute the spectrum for nearby cells.

Breaking the spectrum up into frequencies, and mapping that onto physical cells is a nice concept - and it works very very well in reality, but it does have some drawbacks; for example, what happens when a user moves into a different cell while on the phone? Thankfully, the system supports transparent and seamless handovers as the user moves from cell to cell.

5.1.1 The evolution of cellular technologies

Cellular networks have gone through four iterations to date, with a fifth one coming in the future:

0G:

The zero'th generation of phones were merely radio telephones. They didn't use cells at all.

1G (1983):

This is when cellular mobile was created, all signals were analogue.

2G (1991):

Data is introduced, though it is very slow. In 1998, GPRS was introduced, which brought speeds from 56 - 114kbit/s and in 2003, EDGE was introduced which has speeds of up to 384kbit/s. These are called 2.5 and 2.75G respectively.

3G (2001):

This introduced better speech and faster data. 3G has multiple revisions; 3.5G (2007) is HSPDA with speeds of 1.8-7.2Mbit/s download and 384kbit/s upload, 3.75G (2010) is HSPA+ which has speeds of 56Mbit/s download and 22Mbit/s upload, and finally 3.9G which is still being created and launched, and is related to technologies such as WiMax and LTE.

4G (2011):

The specification for 4G is 1Gbit/s download and 100Mbit/s upload. However, the technology doesn't meet that yet (even now!). Henceforth, companies marked 3.9G technologies as 4G since they 'aspire' to meet the target speeds.

5G:

Due to be launched around 2021, though we've still not met the 4G speeds yet...

Cells are typically 0.1 – 35km in diameter. Obviously small cells are better since they will handle more users per unit area (assuming cell size is independent of user capacity), but users will need to transmit at a lower power (so the neighbouring cells using the same frequency aren't affected). This is actually a good thing sometimes, since it uses less battery, and we can use FEC to mask bit errors.

This isn't the whole story though. It wasn't just the marketing names and the speeds that changed, they also used completely different multiplexing techniques to let multiple users share the spectrum:

All of these use spatial multiplexing with cells except 0G.

0/1G:

Frequency Division Multiplexed Access (FDMA) was used, which is where each transmitter is given a different carrier frequency.

2G (in Europe - GSM):

Time Division Multiplexed Access (TDMA) was used so that each transmitter is given a regular time slot to send data in.

3G/2G (America only):

Code Division Multiplexed Access (CDMA) - each transmitter uses the same frequency band, but different codes are used to send data.

4G:

Orthogonal Frequency Division Multiplexed Access (OFDMA) is used by 4G, which utilises multi-input/multi-output (MIMO) antennae to transmit on multiple frequencies at once. Data is sent in packets.

5.1.2 2G in detail

2G implements the features shown in the next list. They are linked together using buffers shown in Figure 5.1.2.

TDMA:

Each TDM frame is 4.615ms long, and each mobile can send 114bit of data in every frame. Since there are 217 frames/s, the bit rate is $114 \times 217 = 24.7 \text{kbit/s}$. If FEC is used, then the data rate drops to 13kbit/s.

FDMA:

There are two 25MHz channels split into 0.2MHz bands. One channel is for base station to mobile and the other is for mobile to base station.

Speech Transmission:

The device captures the analogue voice, runs a low pass filter over it. The analogue wave is then sampled, quantised and packetised into blocks of around 20ms. Linear Predictive Coding is then used to reduce the bit rate to around 260 bit/block (the equivalent of around 1.5 bits/sample), which gives a bit rate of around 13kbit/s.

When the TDMA time slot happens, 114 bits are red from the buffer, modulated into a sinusoidal carrier wave, and transmitted via the antenna.

Receiving Speech:

First of all, the signal is demodulated and 114 bits are extracted into the receiver buffer. Then, at 20ms intervals, 490 bits are extracted from the same buffer and are decrypted and the interleaving is removed. Bit error correction is applied and FEC is removed to leave 260 bits, where we can then apply the LPC decoder to get 160 samples that are then converted back to analogue, amplified and sent to the speaker.

Binary Frequency Shift Keying (B-FSK):

Binary FSK is used to transmit data in a sinusoidal wave. A low frequency wave is sent for 1 and a high frequency one is for 0. It is simple, not very efficient, but has a constant amplitude. This is shown in Figure 5.1.2.

5.1.3 3G in detail

3G uses CDMA instead of TDMA to provide support for multiple users, but in many ways, it is similar to 2G. The main difference is the use of **CDMA**:

Remember, not all of the capacity of the network is there for the user since some is left for synchronisation and signalling.

Remember, a low pass filter keeps low sounds and attenuates (removes) higher frequencies greater than half the sampling frequency. Code Division Multiplexed Access is used to let multiple carriers transmit on the same frequency. Using this technique, each bit is turned into a sequence of chips that are transmitted instead, maybe:

```
1 = 1 \ 0 \ 1 \ 0 \ 1 \ 1 \ 0 \ 0 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \
```

In this manner, every bit becomes a psudo-random sequence of chips, which lets the receiver distinguish different carriers from each other using a cross correlation process. This requires a much larger bit rate than TDMA, but carriers can transmit at any time though. Another advantage of CDMA is that it has a soft capacity limit; the quality merely degrades when more people use it (since the background noise increases as other carriers transmit seemingly random signals).

Lets say that User A has a code of:

When we receive a signal, we should multiply it by the code for 1 (1 -1 1 -1 1 1 -1 -1 1) and sum the bits, and we get +10 for 1 and -10 for 0.

User B should use a different code:

```
1 = 1 \ 1 \ 1 \ 1 \ 1 \ -1 \ -1 \ 1 \ -1 \ -1

0 = -1 \ -1 \ -1 \ -1 \ 1 \ 1 \ 1
```

If we multiply a code received by B's code for 1, then we should get +10 or -10 again if B sent it. If we multiplied and added using A's code, then we would get 0.

If A transmitted a 1 and B transmitted a 0, then we'd get:

```
0 -2 0 -2 0 2 0 -2 0 2
```

If we multiply and add using A's code, we get +10, and if we do it with B's code, we get -10.

5.1.4 The details of 4G

As previously mentioned, 4G uses Orthogonal Frequency Division Multiplexed Access, but it also implements MIMO and packet switching:

OFDMA:

This is when the sender transmits on many sinusoidal carrier waves at the same time (for each receiver). Data is dispersed among them, so that if some waves are not received properly, then it can be obtained from the others.

Packet switching:

Only at 4G, has mobile telephony moved away from connecting wires to facilitate communication. Now, all speech and data is sent in packets and conveyed using the Internet Protocol (IP).

Multiple Input, Multiple Output:

The capacity of the channel is doubled by having two transmit and two receive antennas. This is also used by WiFi.

The idea of MIMO is that if more than one transmitter and receiver is used, then the capacity of the wireless link will be increased. The transmitter will modify the signal of each so that the sender can distinguish which sent what signal, and besides, each signal

will reach the transmitter via a slightly different path causing a different frequency loss and delay.

5.1.5 5G and the future!

With 4G still not ready (it doesn't meet the speeds that were expected for 2010 as of 2015), 5G development is more in the ideas stage rather than the implementation stage, however, the ideas are still really cool.

It could use BDMA (we'll come to that in a second), smart radio to use different parts of the spectrum based on current usage, IPv6 (as opposed to IPv4 which is the currently implemented, but outdated standard), the option of device to device communications (bypassing the base station entirely) and more cool stuff.

Beam Division Multiple Access is when you use multiple (e.g. 7) antennas spaced at equal intervals to make amplitude of the signal higher in some places than at others. See Figure 5.1.5 for a pictorial explanation (you might have to think back to A-level physics to understand why this happens).

The transmitter can change the amplitude and phase of the transmitted signal from each of the antennae to form the signal to be strongest at the receiver. Other receivers can be nulled out, or at least sent a reduced signal. This works both ways; a receiver that has seven antennae can target a specific transmitter. It's seen by many as a better concept than cellular multiplexing, more like having actual wires.

Group Cooperative Relaying is a technique where a separate carrier between the transmitter and receiver can boost the signal by repeating it and acting as a relay. This can be performed in two ways:

- Amplify and forward sends the raw signal on again, but also amplifies the noise.
- Decode and forward decodes the transmission, re-codes it again and sends it on. This has the benefit of reducing noise since the signal is forwarded on with the noise removed.

5G could also be really **energy efficient**; it is proposed that as load on the network decreases, some cells could be turned off, and the coverage area of other cells could be allowed to increase. The network can adapt to the current usage pattern in real time. Care must be taken though; since transmitting to base stations further away will require more battery power from the user's phone which is obviously a bad thing.

5.2 GPS

GPS stands for Global Positioning System, and it works by having a receiver calculate its position from timing signals sent by GPS satellites that are circling about 20200km above earth. They send messages that give the exact time and the position of the satellite, which allows the receiver to determine its distance from the satellite by the offset between the transmission time and received time. If at least four satellites can be 'heard' by the receiver, then its position can be accurately found.

6 Medium Access Control

If there are N independent stations each generating packets for transmission at essentially random times on a single channel, then we may have collisions between packets. If we assume that when two packets collide, the contents will be garbled, then our goal should be to reduce the rate of collisions so that packet loss is minimised.

We could use slotted or continuous time when deciding when to transmit packets; slotted time is when the time is divided into fixed slots that transmitters can transmit in. Continuous time

Smart radio is also known as cognitive radio and is a form of dynamic spectrum management.

means that transmitters can start to transmit packets at any time, not just at the start of the slot boundaries.

Carrier Sense is the term used to describe when a station can detect that the channel is currently busy before it starts to transmit (hence avoid having to re-send the packet again later when it was garbled this time).

We've already looked at the techniques used to allow multiple users to use the same network on mobile phones (FDMA, TDMA, CDMA, OFDMA etc), but one rather older technique called Pure ALOHA has a different approach entirely.

6.1 ALOHA

Pure ALOHA is when stations (this was the 70's) transmit packets whenever they have data. Success is detected by listening to the same channel that they transmit on, or by the receiver acknowledging the frame. If a frame is destroyed, then it is re-transmitted after a random delay. This is known as a **contention** system.

Since if the frame overlaps with any part of another frame, then the vulnerable period for the frame is $3 \times$ the length of the frame (see Figure 6.1).

Random delays, of course, avoid the repeated collisions that would occur if the transmitters just kept transmitting sequentially after one frame had failed

6.1.1 Slotted ALOHA

If we divide the time domain into slots, then fewer collisions will occur.

Figure 6.1.1 shows the difference in efficiency between pure and slotted ALOHA.

6.2 Carrier Sense Multiple Access

This is when a transmitter will check nobody is 'talking' on a channel before it transmits. It will then either:

1-persistent CSMA:

This is when the transmitter waits until the current frame ends before trying to send its own frame.

p-persistent CSMA:

The transmitter might speak at the next slot, the probability that it will do is given by the probability p.

Non-persistent CSMA:

If another transmitter is sending a frame, wait a random time before trying again.

Figure 6.2 shows a comparison between the different types of CSMA.

From the image in Figure 6.2, it would seem that low persistence is the way to go, since the efficiency is very high. However, if the probability of resending a frame is so low, then it might take ages for the frame to be sent, and so although the network is efficient overall, for any individual, it is very slow.

6.2.1 CSMA collision detection

We can save power and transmission capacity by having the transmitter monitor its own transmissions, and abort as soon as it detects a collision. This is implemented by Ethernet, and is present at the physical layer of the network stack. The result of the collision detection is used by the Medium Access Control (MAC) sub-layer, which is part of the data link layer. This makes the decisions about when to transmit and when to back off etc.

6.3 Wireless contention

A wireless transmitter cannot monitor its own transmissions, because it can be either transmitting or receiving, but not both concurrently. Even if a wireless transmitter could do that, it wouldn't matter because it only matters what is being received at the receiver, which is probably different from what is observed at the transmitter.

Because of this, wireless protocols place an emphasis on avoiding collisions:

Real channel sensing:

Sense the channel and see if it's free, start transmitting if so. If the receiver senses a transmission, then request a re-transmission after a back-off period.

Virtual channel sensing:

If a device wants to transmit, then it sends a short RTS (request to transmit) control frame. If the receiver is ready, it sends a CTS (clear to send) frame. The sending device then transmits the message and starts an ACK timer. When the receiving device has received the frame, then it will send an ACK, and if the sender doesn't receive the ACK within the timer period, then it will re-transmit. When other devices hear an RTS or CTS, they won't transmit for a period of time.

A NAV (Network Allocation Vector) flag is one that tells a device to assume the channel is busy and is set whenever a RTS or CTS frame is heard.

6.3.1 Wireless problems

Wireless comms have some interesting problems that wired comms don't have, mainly to do with the range of the radio:

Hidden Device Problem:

If A wants to transmit to B, but A is too far from C to hear any of C's messages, then it won't be able to hear RTS from C. Luckily, B will ignore A's RTS until it is finished talking to C.

Exposed Device Problem:

See the lecture notes for this one (lecture 7, slide 18).

6.3.2 Bluetooth

The base unit of a Bluetooth system is a pico-net. This consists of a master node, and up to seven slaves within ten meters. You can connect piconets together to form a 'scatternet' as shown in Figure 6.3.2.

The bluetooth designers decided to set a list of things that you could do over Bluetooth, such as file transfers, fax etc. This is in contrast to other protocols such as WiFi, where its up to the user/programmer to think of stuff to do. This makes Bluetooth more rigid.

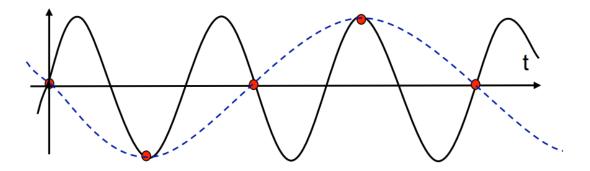


Figure 2: A $6 \mathrm{kHz}$ wave sampled at $8 \mathrm{kHz}$ has a post-sampling frequency of $2 \mathrm{kHz}$

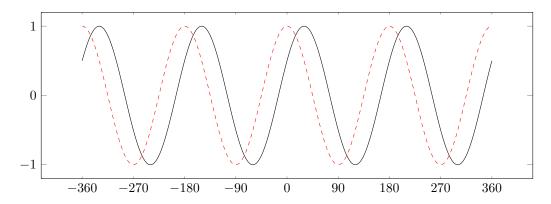


Figure 3: 1000 samples of a sinusoid where $D=30^\circ$ (black) and where D=0 (dotted red).

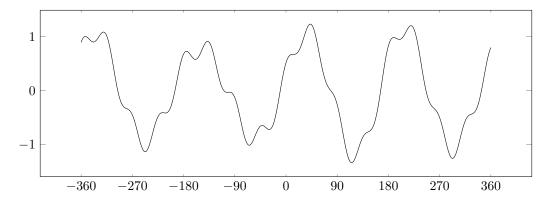


Figure 4: A complex wave with a frequency of $\frac{1}{180}=0.00\overline{5}\mathrm{Hz}$

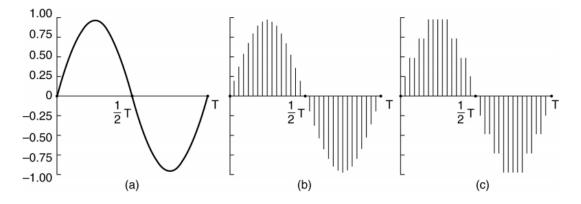


Figure 5: An analogue wave (a), after its sampled (b), and once it's been quantised (c)

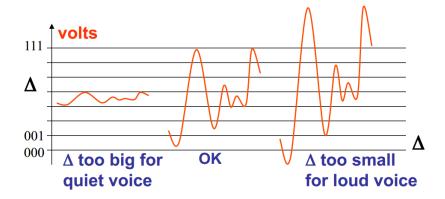


Figure 6: This waveform shows how a a bad choice for Δ can adversely affect the quality of the sound, when it is undergoing uniform quantisation.

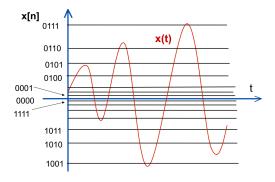


Figure 7: An example of non-uniform quantisation.

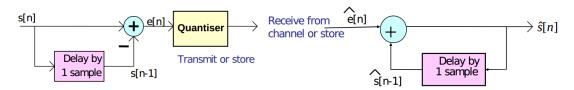


Figure 8: An example of how a differential encoder might both encode and decode audio samples.

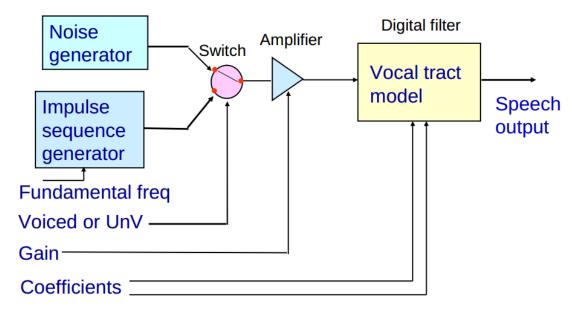


Figure 9: An implementation of a Linear Predictive Speech Decoder

```
public class HammingCode {
  private final HammingNumber[] numbers;
  int correctionDistance = Integer.MAX_VALUE;
  public HammingCode(HammingNumber[] numbers) {
    this.numbers = numbers;
   for(int i = 0; i < numbers.length - 1; i++) {</pre>
     for(int j = i + 1; j < numbers.length; j++) {</pre>
       int distance = numbers[i].hammingDistance(numbers[j]);
       if(distance < correctionDistance) {</pre>
         correctionDistance = distance;
     }
   }
  }
  public HammingNumber correct(HammingNumber input) {
   for(HammingNumber number : numbers) {
     int distance = input.hammingDistance(number);
     if(distance == 0 || distance < correctionDistance) {</pre>
       return number;
   }
   return null;
  public static class HammingNumber {
   private final boolean[] number;
   public HammingNumber(boolean[] number) {
     this.number = number;
   public int hammingDistance(HammingNumber other) {
     int distance = 0;
     for(int i = 0; i < number.length; i++) {</pre>
       if(number[i] != other.number[i]) distance++;
     return distance;
   }
   public String toString() {
     StringBuilder sb = new StringBuilder();
     for(boolean b : number) sb.append(b ? "1" : "0");
     return sb.toString();
   }
 }
}
```

Figure 10: A Java implementation of HammingCodes

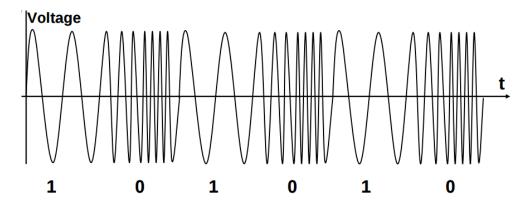


Figure 11: How the data 101010 might be encoded using BFSK

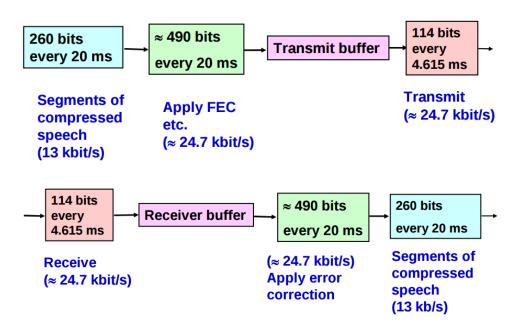


Figure 12: The sending and receiving buffers in a typical 2G implementation.

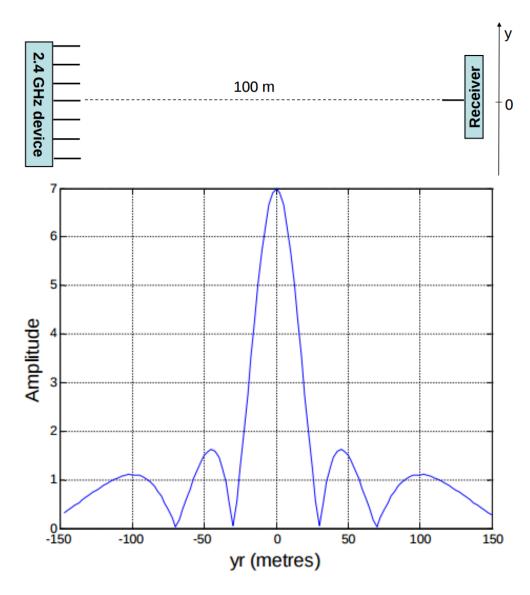


Figure 13: Here, you can see that as we adjust y, the intensity changes.

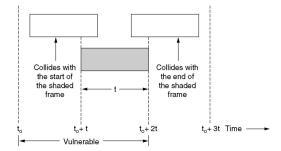


Figure 14: The vulnerable period for the shaded frame is $3\times$ its length.

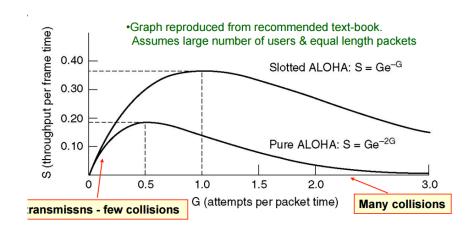


Figure 15: Pure ALOHA achieves 18% utilisation max, whereas Slotted ALOHA gets to 37%.

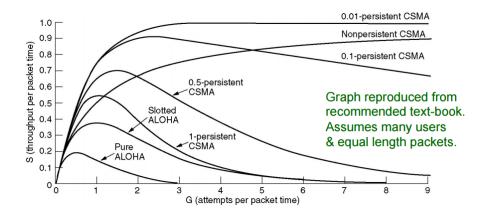


Figure 16: A comparison of channel utilisation versus load for various random access ALOHA protocols.

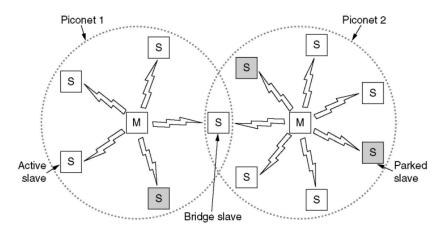


Figure 17: A bluetooth scatternet.