

Marchés de Taux d'Intérêts

Produits Fixed Income et Dérivés



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2 Octobre / 16 Octobre 2019

Introduction

- Taux d'intérêts – définitions et notions actuarielles
- Produits monétaires et obligataires
- Produits Dérivés simples (FRAs, Swaps, FX Swaps, XCcy Swaps)
- Sensibilité / Risk Management
- Produits Dérivés optionnels

Introduction

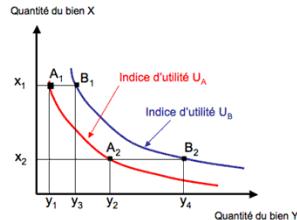
- La Monnaie :
 - (i) une unité de mesure de la valeur;
 - (ii) un moyen d'échange;
 - (iii) une réserve de valeur, permettant à ses utilisateurs de reporter leur pouvoir d'achat dans le temps.
 - Un actif parmi d'autres ?



Introduction

Qu'est-ce qu'un taux d'intérêt ?

« loyer de l'argent », i.e. le prix sur le marché de la Monnaie



Préférence
temporelle
(Fonction d'utilité)

Risque de Crédit

Taux
d'Intérêt

Offre &
Demande
(Liquidité)

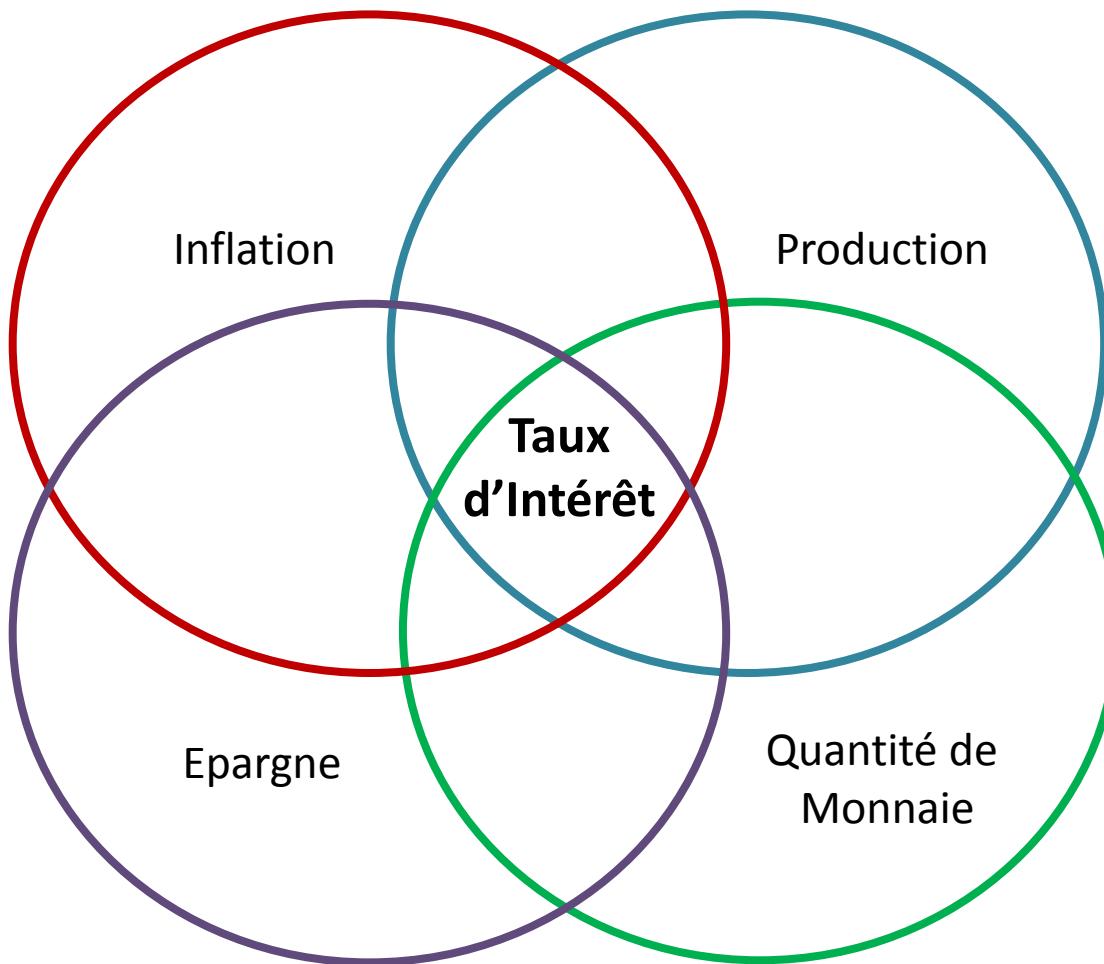


Introduction

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Qu'est-ce qu'un taux d'intérêt ?



Sous-jacent des dérivés de taux d'intérêt : de véritables opérations de prêt/emprunt

Taux d'intérêts – définitions et notions actuarielles

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- Un seul taux ?

GRAB

97) Change Country		98) Feedback		18:38:17		Treasury & Money Markets: France									
Money Rates		Interbank		1) EURIBOR Fix		2) EUR LIBOR Fix		EUR Swaps			BTANS/OATS				
0/N DEPO	0.000	T/N	0.07000	1M	0.12300	1M	0.09214	2Y	0.45	0.47	2Y	0.26	+0.00		
0/N LEND	0.750	1M	0.11000	2M	0.17200	2M	0.13286	3Y	0.63	0.65	3Y	0.41	+0.00		
1 Week	0.250	3M	0.16000	3M	0.21800	3M	0.16929	4Y	0.85	0.89	4Y	0.72	+0.02		
Floater REF		6M	0.26000	6M	0.32200	6M	0.25571	5Y	1.11	1.13	5Y	1.08	+0.03		
5 Year	1.04	+0.04	9M	0.34000	9M	0.41900	1Y	0.43929	7Y	1.55	1.59	7Y	1.42	+0.03	
10 Year	2.33	+0.05	1Y	0.42500	1Y	0.50100			10Y	2.06	2.09	10Y	2.26	+0.04	
									20Y	2.68	2.72	20Y	3.09	+0.05	
									30Y	2.74	2.78	30Y	3.47	+0.05	
Spot FOREX		EUR FRAs		EONIA Swap		Futures			REPO Rates						
€/USD	1.3445	1x4	0.2240	1W	0.0940	4M	0.1100	BUND 1st	140.63	T/N	0.07				
€/GBP	0.8447	3x6	0.2290	2W	0.0940	5M	0.1110	French OAT	133.63	1W	0.07				
€/JPY	133.8293	6x9	0.2570	3W	0.1000	6M	0.1110								
		9x12	0.2930	1M	0.0580	9M	0.1070								
T-Bills		1x7	0.3250	2M	0.1150	1Y	0.1180	Stock Indices							
		3x9	0.3410	3M	0.1110	2Y	0.1750	CAC 40							
		6x12	0.3800					SBF-250							
		12x18	0.4860					DJ EUR STOXX 50							
		18x24	0.6250					DJIA							
30) Economic Releases (ECO)															
Date	Time C	A M	Event	Period	Survey	Actual	Prior	Revised							
31) 11/12	08:30 FR		Bank of France Bus. Sentiment	Oct	97	99	97	--							
32) 11/14	07:30 FR		GDP QoQ	3Q P	0.0%	--	0.5%	--							
33) 11/14	07:30 FR		GDP YoY	3Q P	0.3%	--	0.4%	--							
34) 11/14	08:45 FR		Current Account Balance	Sep	--	--	-3.1B	--							
Australia 61 2 9777 8600 Brazil 5511 3048 4500 Europe 44 20 7330 7500 Germany 49 69 9204 1210 Hong Kong 852 2977 6000															
Japan 81 3 3201 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000 Copyright 2013 Bloomberg Finance L.P.															
SN 296242 H312-1401-0 12-Nov-13 18:38:17 CET GMT+1:00															

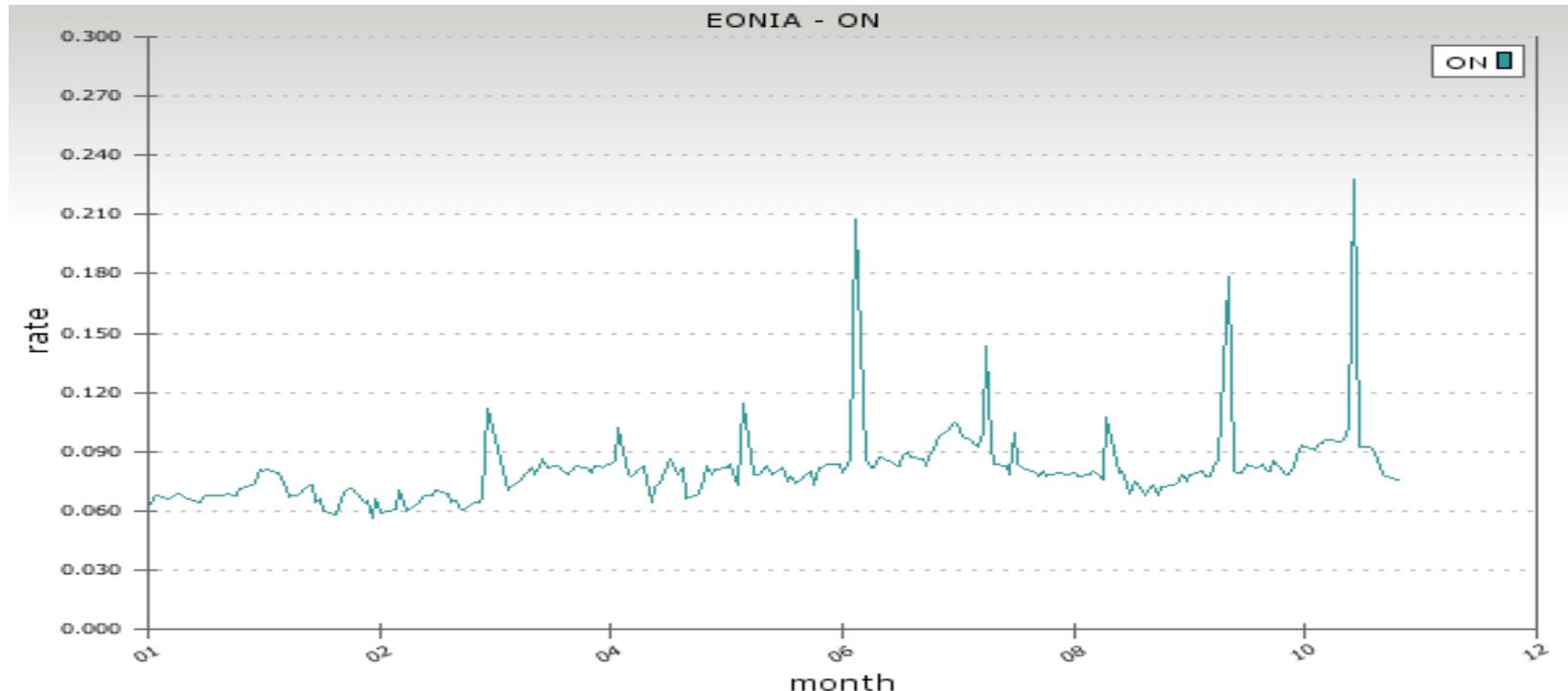
Source : Bloomberg

Références de taux, conventions

- **EONIA**

Eonia® (Euro OverNight Index Average) is the effective overnight reference rate for the euro. It is computed as a weighted average of all overnight unsecured lending transactions in the interbank market, undertaken in the European Union and European Free Trade Association (EFTA) countries.

Eonia® is computed with the help of the [European Central Bank](#).



EONIA - Panel

[EMMI](#) » [EONIA®](#) » Panel Banks

Panel Banks

The panel of banks contributing to Eonia® consists of 28 banks:

- Banks from EU countries participating in the euro from the outset.
- Banks from EU countries not participating in the euro from the outset.
- Large international banks from non-EU countries but with important euro zone operations.

The panel of contributing banks currently consists of 28 **CONTRIBUTORS**.

Austria

Erste Group Bank AG

Belgium

Belfius

Finland

Nordea - Pohjola

France

BNP-Paribas - HSBC France -

Natixis/BPCE - Crédit Industriel et

Commercial CIC - Société Générale

Germany

BayernLB - Deutsche Bank - DZ Bank

- Norddeutsche Landesbank

Girozentrale - Landesbank Baden-

Württemberg Girozentrale -

Landesbank Hessen Thüringen

Girozentrale

Greece

National Bank of Greece

Ireland

Bank of Ireland

Italy

Intesa Sanpaolo - Monte dei Paschi di
Siena - UniCredit

Luxembourg

Banque et Caisse d'Épargne de l'État

Netherlands

ING Bank

Portugal

Caixa Geral De Depósitos (CGD)

Spain

Banco Bilbao Vizcaya Argentaria -

Banco Santander - CECABANK -

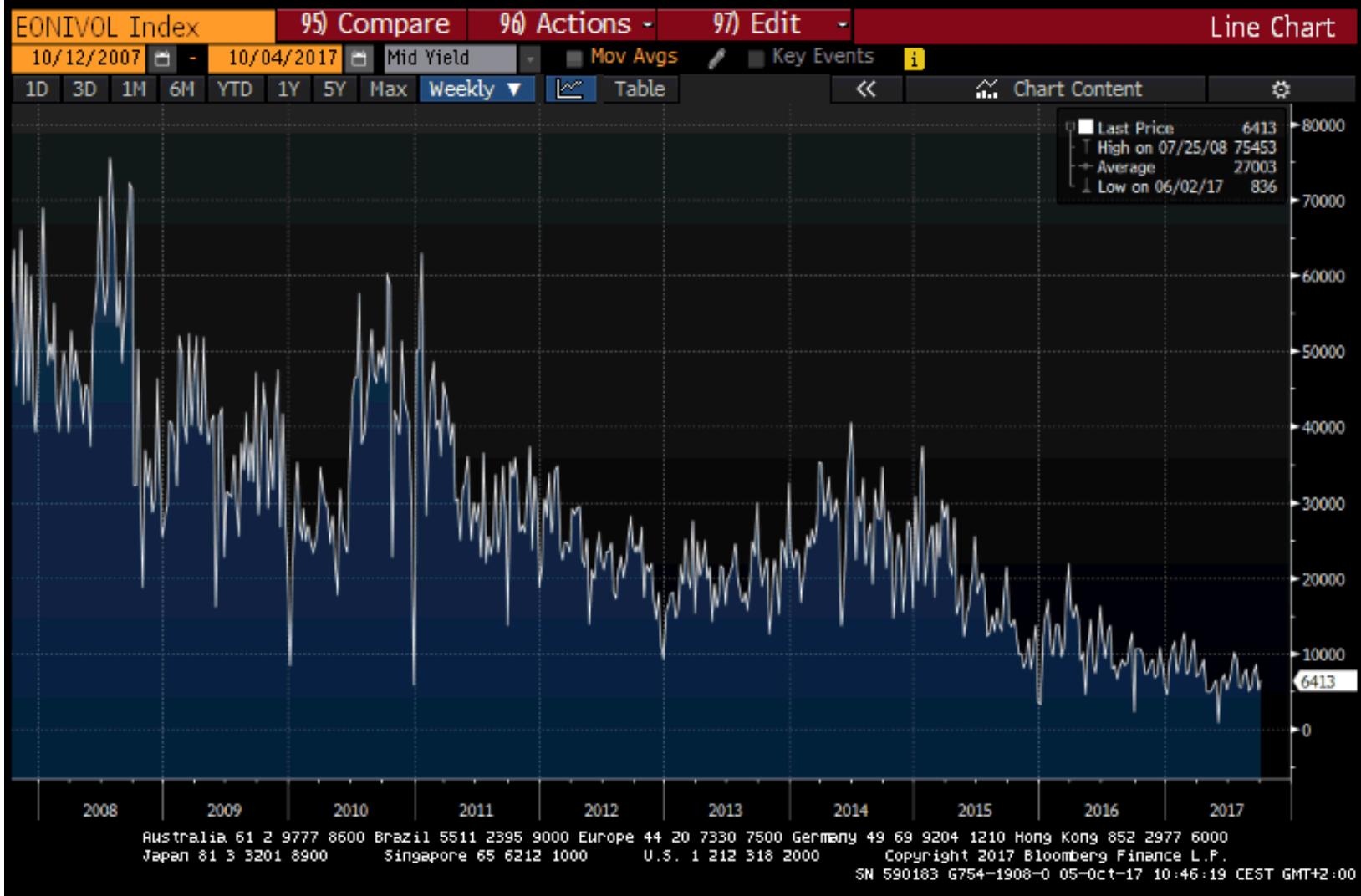
CaixaBank S.A.

Other EU Banks

Barclays

EONIA – Volumes déclarés

GRAB



EONIA – Volumes déclarés

Evolution of excess liquidity and EONIA volumes

(EUR billions)



Sources: EMMI and Bloomberg.

Note: The period under review is indicated by a light blue background.

Fin de l'EONIA : ESTER

- EONIA as it stands will no longer meet the criteria of the EU Benchmarks Regulation (June 2016) and will therefore see its use restricted as of 1 January 2020
- 13 September 2018 : the **Working group on euro risk-free rates** announced its recommendation that ESTER be used as the risk-free rate for the euro area.
- As of the start date, 2 October 2019, the ECB will publish the €STR at 08:00 CET on each TARGET2 business day. The €STR published on 2 October 2019 will reflect trading activity on 1 October 2019.
- Following a recommendation made by the working group on euro risk-free rates on 14 March 2019, as of 2 October for the trade date 1 October 2019 the European Money Market Institute (EMMI) will change the way it calculates the EONIA. The EONIA methodology will be redefined as the €STR plus a fixed spread, calculated using the methodology adopted by the EMMI as the difference between the underlying interest rate of the EONIA and the pre-€STR using daily data from 17 April 2018 to 16 April 2019. The ECB has calculated this spread as 0.085% (8.5 basis points).
- EONIA will be discontinued at the end of 2021.

https://www.ecb.europa.eu/stats/financial_markets_and_interest_rates/euro_short-term_rate/html/eurostr_overview.en.html

Nouvel indice de référence : ESTER

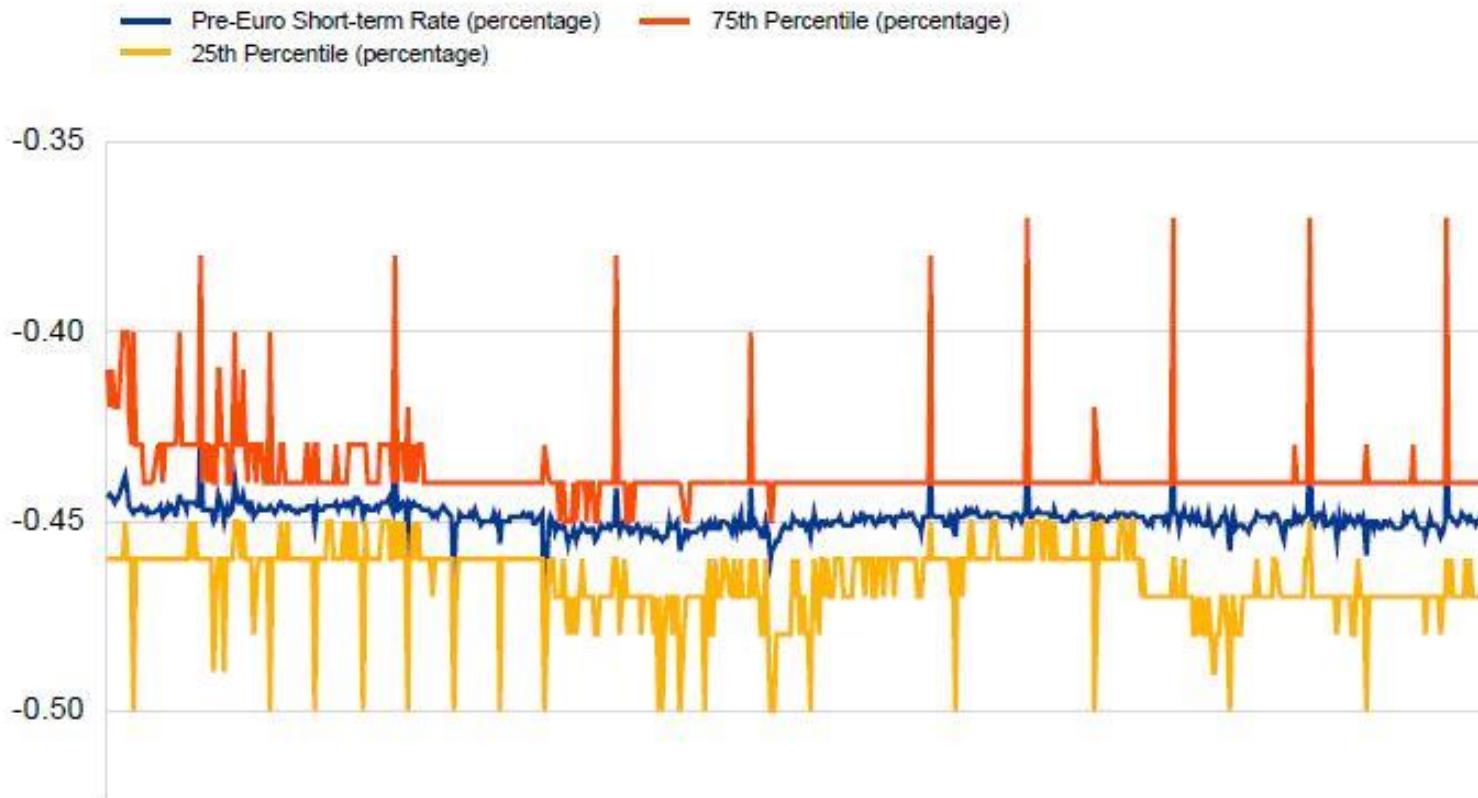
- ESTER is a rate which reflects the wholesale euro unsecured overnight borrowing costs of euro area banks. The rate is published for each TARGET2 business day based on transactions conducted and settled on the previous day (reporting date T) with a maturity date of T+1 and which are deemed to be executed at arm's length and thereby reflect market rates in an unbiased way.

ESTER - Methodology

- ESTER is calculated for each TARGET2 day as a volume-weighted trimmed mean rounded to the third decimal.
- The volume-weighted trimmed mean is calculated by:
 1. ordering transactions from the lowest rate to the highest rate;
 2. aggregating the transactions occurring at each rate level;
 3. removing the top and bottom 25% in volume terms; and
 4. calculating the mean of the remaining 50% of the volume-weighted distribution of rates.
- A pro rata calculation is applied to volumes that span the thresholds for trimming to ensure that exactly 50% of the total eligible volume is used in the calculation of the volume-weighted mean.

Pre - ESTER Data

Daily rate and percentiles



ESTER – List of Reporting Agents

- ABN AMRO Bank N.V.
- Allied Irish Banks plc
- Banca IMI S.p.A.
- Banca Monte dei Paschi di Siena S.p.A.
- Banco Bilbao Vizcaya Argentaria, S.A.
- Banco de Sabadell, S.A.
- Banco BPM Societa' per Azioni
- Banco Popular Español, S.A.
- Banco Santander, S.A.
- N.V. Bank Nederlandse Gemeenten
- Bankia, S.A.
- Banque fédérative du crédit mutuel
- Bayerische Landesbank
- Belfius Banque SA
- BNP Paribas
- BNP Paribas Fortis SA
- BPCE
- Caisse des dépôts et consignations - section générale
- Caisse Fédérale de Crédit Mutuel
- CaixaBank, S.A
- Cassa Depositi e Prestiti Societa' per Azioni
- Commerzbank Aktiengesellschaft
- Coöperatieve Rabobank U.A.
- Crédit Agricole Corporate and Investment Bank
- Crédit Agricole S.A.
- Crédit Lyonnais
- DekaBank Deutsche Girozentrale
- Deutsche Bank Aktiengesellschaft
- Deutsche Bank Privat- und Geschäftskunden Aktiengesellschaft
- Deutsche Postbank AG
- Dexia crédit local
- DZ Bank AG Deutsche Zentral-Genossenschaftsbank
- HSBC France
- HSH Nordbank AG
- ING Bank N.V.
- ING Belgique SA
- ING-DiBa AG
- Intesa Sanpaolo S.p.A.
- KBC Bank NV
- Kreditanstalt für Wiederaufbau
- La Banque Postale
- Landesbank Baden-Württemberg
- Landesbank Hessen-Thüringen Girozentrale
- Natixis
- Norddeutsche Landesbank -Girozentrale-
- Nordea Bank AB
- NRW.BANK
- Piraeus Bank, S.A.
- Société Générale
- UniCredit Bank AG
- UniCredit Bank Austria AG
- UniCredit, Societa' per Azioni

Key ECB interest rates, market reference rates and excess liquidity

(left-hand scale: percentages; right-hand scale: EUR billions)



Source: ECB.

Notes: MLF – marginal lending facility; MRO – main refinancing operations; Pre-€STR – preliminary version of the euro short-term rate (€STR). The period under review is indicated by a light blue background.

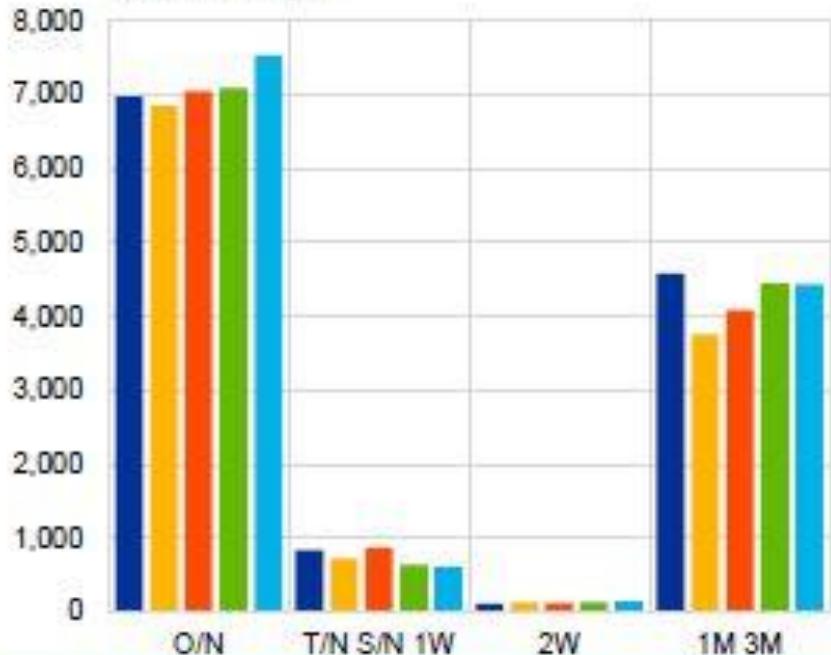
Unsecured Money Markets – Maturities

Volume per maturity bucket

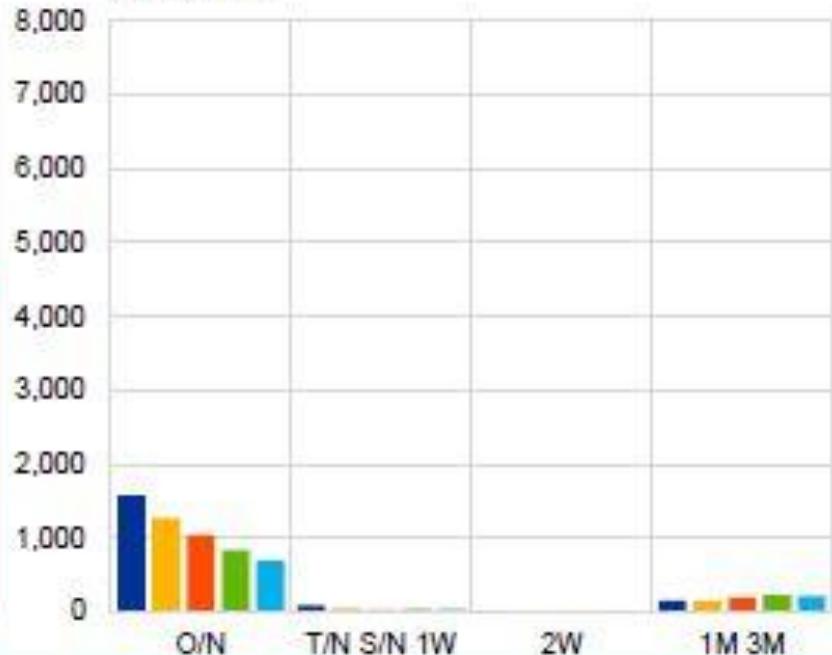
(EUR billions)

- █ H2 2016
- █ H1 2017
- █ H2 2017
- █ H1 2018
- █ H2 2018

a) Borrowing



b) Lending



Source: ECB (MMSR).

Note: O/N – overnight; T/N – tomorrow/next; S/N – spot/next.

Références de taux, conventions

- **Euribor**

Euribor® is the rate at which Euro interbank term deposits are offered by one prime bank to another prime bank within the EMU zone, and is published at 11:00 a.m. (CET) for spot value (T+2).

Maturity	Today	Previous day
1 Week	0.098 (12/11/2013)	0.098 (11/11/2013)
2 Weeks	0.102 (12/11/2013)	0.102 (11/11/2013)
3 Weeks	0.12 (31/10/2013)	0.117 (30/10/2013)
1 Month	0.123 (12/11/2013)	0.122 (11/11/2013)
2 Months	0.172 (12/11/2013)	0.173 (11/11/2013)
3 Months	0.218 (12/11/2013)	0.218 (11/11/2013)
4 Months	0.267 (31/10/2013)	0.265 (30/10/2013)
5 Months	0.309 (31/10/2013)	0.306 (30/10/2013)
6 Months	0.322 (12/11/2013)	0.322 (11/11/2013)
7 Months	0.381 (31/10/2013)	0.38 (30/10/2013)
8 Months	0.418 (31/10/2013)	0.417 (30/10/2013)
9 Months	0.419 (12/11/2013)	0.417 (11/11/2013)
10 Months	0.489 (31/10/2013)	0.489 (30/10/2013)
11 Months	0.519 (31/10/2013)	0.518 (30/10/2013)
12 Months	0.501 (12/11/2013)	0.502 (11/11/2013)

Euribor fixing

Quote: EURIBOR01

Menu ▾ Q | Search | Related ▾ Trade ▾ T T ↻ [] v

09:01 21OCT14 EMMI EURIBOR RATES UK67516 EURIBOR01

EURIBOR RATES ACT/360 AT 11H00 BRUSSELS TIME 21/10/2014	21/10 09:01 GMT
ACT/ 360	VALUE DATE 23/10/14
<EURIBORSWD=> 1WK -0.018	FIXING ALERTS <FIXALERT>
<EURIBOR2WD=> 2WK -0.008	=====
<EURIBOR3WD=> 3WK	EURIBOR [Euro Interbank Offered Rate]
<EURIBOR1MD=> 1MO 0.008	is the rate at which Euro interbank
<EURIBOR2MD=> 2MO 0.046	term deposits within the Euro zone are
<EURIBOR3MD=> 3MO 0.082	offered by one Prime Bank to another
	Prime Bank. It is computed as an average
<EURIBOR4MD=> 4MO	of daily quotes provided for eight
<EURIBOR5MD=> 5MO	maturities by a panel of 26 of the most
<EURIBOR6MD=> 6MO 0.186	active Banks in the Euro zone.
	It is quoted on an act/360 day count
<EURIBOR7MD=> 7MO	convention, and is fixed at 11:00am[CET]
<EURIBOR8MD=> 8MO	displayed to three decimal places.
<EURIBOR9MD=> 9MO 0.258	=====
<EURIBOR10MD=> 10MO	
<EURIBOR11MD=> 11MO	Historical recap and Panel - <EURIBOR>.
<EURIBOR1YD=> 12MO 0.339	=====
EUREPO <EUREPO> EONIA <EONIA>. Please see <EURIBORLEGACY> for EURIBOR changes,	
Composite displays: [a/360] see <EURIBOR=>, [a/365] <EURIBOR365>,	
EONIA SWAP INDEX <EONIAINDEX>, Panel Bank Rates <0#EURIBORCONTS>	

Panel Euribor

EMMI EURIBOR RATES		UK67516		EURIBOR05	
EURIBOR PANEL BANK DATA 3 MONTH					
BANK	3 MO	BANK	3 MO	BANK	3 MO
BLFIUS BRU	0.09	BCEE LUX	0.07		
DEU BK FFT	0.16	ING AMS	0.07		
DZ FFT	0.08	CGD LIS	0.09		
BBVA MAD	0.08	NORDEA HELI	0.06		
SAN MAD	0.09	BARC FFT	0.03		
CECA MAD	0.08	DANSK COP	0.13		
NATIX PAR	0.09	BTMU LON	0.07		
CASA PAR	0.13	JPMCB LON	0.04		
HSBC PAR	0.08	CAIXA BCN	0.08		
BNPP PAR	0.05	POHJOL HEL	0.08		
SOC GN PAR	0.17				
NBG ATH	0.08				
INTESA MIL	0.09				
MPS MIL	0.08				
UNICR MIL	0.09				

Références de taux, conventions

- **Libor**

Every contributor bank is asked to base their bbalibor submissions on the following question:

“At what rate could you borrow funds, were you to do so by asking for and then accepting inter-bank offers in a reasonable market size just prior to 11 am?”

(Source : BBA)

- Autres fixings : Eonia Swaps, ISDAFIX

Calcul des intérêts

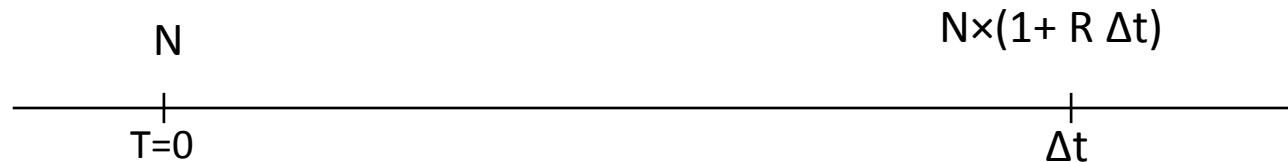
Sur une seule période

$$\text{flux d'intérêts} = N \times R \times \Delta t$$

Où :

- N est le notionnel qui porte intérêts,
- R est le taux d'intérêts, exprimé annuellement, ce taux est fixé au début de la période et payé en fin de période (intérêts post-comptés),
- Δt représente la durée sur laquelle on calcule les intérêts, exprimé en fraction d'année si le taux utilisé est un taux annuel.

Calcul des intérêts



Intérêts post comptés, calculés sur une période simple.



Calcul des intérêts

– Business Day Convention

Section 4.12. Business Day Convention.

(a) “Business Day Convention” means the convention for adjusting any relevant date if it would otherwise fall on a day that is not a Business Day. The following terms, when used in conjunction with the term “Business Day Convention” and a date, shall mean that an adjustment will be made if that date would otherwise fall on a day that is not a Business Day so that:

(i) if “Following” is specified, that date will be the first following day that is a Business Day;

(ii) if “Modified Following” or “Modified” is specified, that date will be the first following day that is a Business Day unless that day falls in the next calendar month, in which case that date will be the first preceding day that is a Business Day; and

(iii) if “Preceding” is specified, that date will be the first preceding day that is a Business Day.

Calcul des intérêts

– Day Count Convention (fractions d'année) :

- Act/360 (Money Market)

Nombre de jours exacts / 360 jours

- 30/360 (Bond Basis)

1 année = 12 mois de 30 jours

Nb de jours exact si ≤ 30 , sinon 30

- 30E/360

Idem 30/360 mais le 31 n'existe jamais

- Act/Act

Nombre de jours exacts / 365 jours ou 366 si année bissextile

- Act/ 365 Fixed

Nombre de jours exacts / 365 jours

- Exemple :

Fixing Euribor 3M du 13/11/13 : 0,218%

Période d'application du taux :

J+2 à J+2+ 3 mois -> du 15/11/13 au 15/02/14 mais le 15/02/14 est un samedi

Convention modified following -> du 15/11/13 au 17/02/14 soit 94 jours

La base des fixings Euribor est A/360

Pour un notionnel de 100 000 000 EUR le flux réglé le 17/02/14 est de :

$$100\ 000\ 000 \times 0,218/100 \times 94/360 = 56\ 922,22 \text{ EUR}$$

RECOMMENDED MARKET CONVENTIONS FOR THE EURO

- **Euro money markets**

- Day count basis: actual/360
- Settlement basis: spot (two day) standard
- Business days: TARGET operating days should form the basis for euro business days

- **Euro Swap Markets**

- Floating day count basis: actual/360
- Fixed rate day count basis: 30/360
- Business days: TARGET operating days should form the basis for euro business days
- Fixing period: two day rate fixing convention
- Coupon frequency: annual

- **Euro Bond markets**

- Day count basis: actual/actual
- Quotation basis: decimals rather than fractions
- Business days: TARGET operating days should form the basis for euro business days
- Coupon frequency: annual
- Settlement dates: the standard for internationally traded cross-border transactions for the euro should remain on a T+3 business day cycle

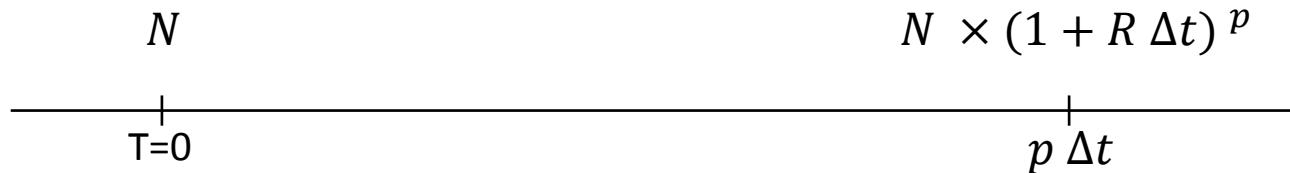
- **Euro foreign exchange markets**

- Settlement timing: spot convention, with interest accrual beginning on the second day after the deal has been struck
- Quotation: 'certain for uncertain' (i.e. 1 euro = x foreign currency units)
- Reference rate: the ECB (or NCBs) should be responsible for the publication of daily closing reference rates

Calcul des intérêts

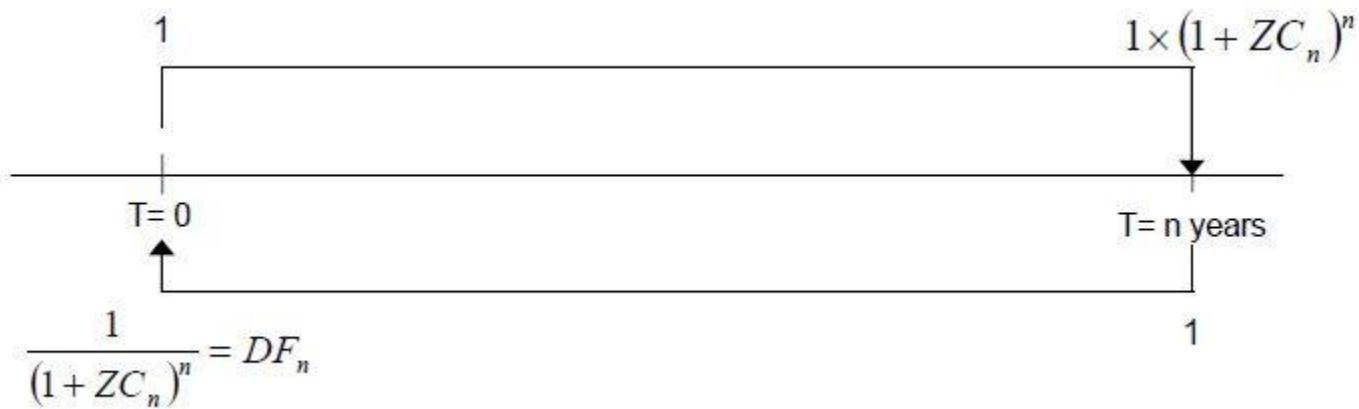
Intérêts composés (capitalisés) :

Les intérêts portent sur plusieurs périodes consécutives et sont capitalisés, en plus du montant notionnel, sur chacune d'entre elles.



Discount factors / Taux Zéro Coupons

Taux Zéro Coupon : yield d'un investissement notionnel sur une période donnée, sans paiement d'intérêts intermédiaires



Discount Factor : la valeur aujourd'hui d'1€ reçu à une date future T
(prix d'un bond zéro coupon)

Discount factors / Taux Zéro Coupons

Notation (capitalisation annuelle) :

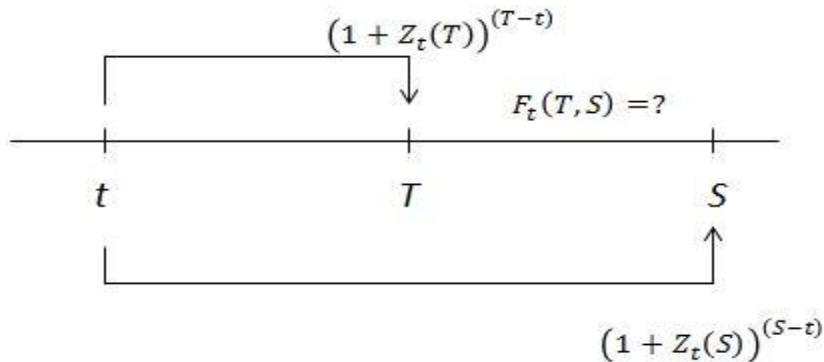
- $P_t(T)$: la valeur en t d'un bond zéro coupon maturant en T
- $Z_t(T)$: le taux zéro coupon de t à T

On a :

$$P_t(T) = (1 + Z_t(T))^{-(T-t)}$$

$$Z_t(T) = P_t(T)^{-1/(T-t)} - 1$$

Taux Forward



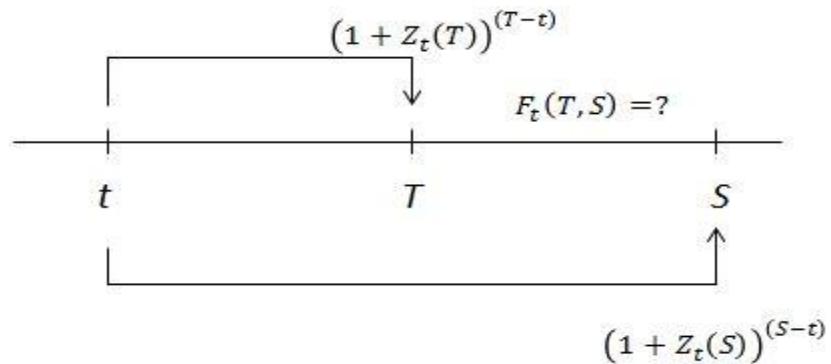
On doit nécessairement avoir :

$$(1 + Z_t(T))^{(T-t)} \times (1 + F_t(T,S))^{(S-T)} = (1 + Z_t(S))^{(S-t)}$$

Soit :

$$F_t(T,S) = \left(\frac{(1 + Z_t(S))^{(S-t)}}{(1 + Z_t(T))^{(T-t)}} \right)^{1/(S-T)} - 1$$

Taux Forward



Ou bien :

$$F_t(T, S) = \left(\frac{P_t(T)}{P_t(S)} \right)^{1/(S-T)} - 1$$

Cas des Euribors / Libors

La capitalisation se fait sur des périodes inférieures à un an (1, 3, 6 ou 12 mois), sans flux d'intérêts intermédiaires.

On peut donc réécrire :

$$P_t(T) = (1 + (T - t) \times L_t(T))^{-1}$$

$$L_t(T) = \frac{1}{(T - t)} \times (P_t(T)^{-1} - 1)$$

$$L_t(T, S) = \frac{1}{S - T} \times \left(\frac{P_t(T)}{P_t(S)} - 1 \right)$$

Par Yield

Définition : une obligation au pair est une obligation dont la valeur (somme actualisée des cash flows futurs) est égale à sa valeur faciale.

Soit :

$$\sum_{i=1}^N C_N \times P_t(t + i) + P_t(t + N) = 1$$

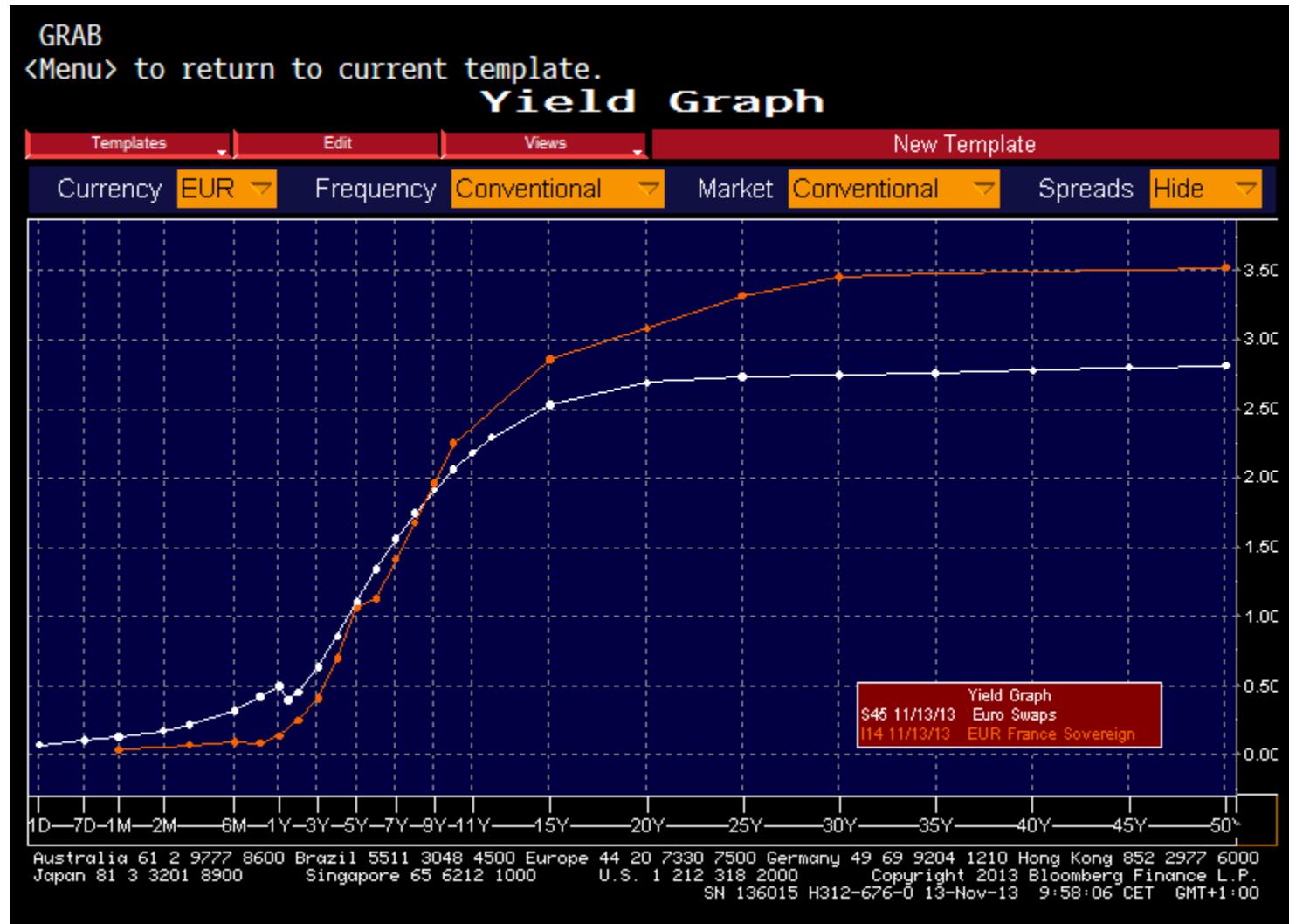
Où C_N représente le cash flow calculé à partir du taux à la valeur de marché d'une obligation maturant dans N années (par yield)

Pour un bond au pair Forward

$$\sum_{i=1}^{N-S} C_{S,N} \times P_t(t + S + i) + P_t(t + N) = P_t(t + S)$$

Où $C_{S,N}$ représente le taux à la valeur de marché (par yield) d'une obligation achetée en S et maturant en N .

Courbe de taux (Term Structure)



Courbe de taux (Term Structure)

- Description de l'ensemble des taux zéro coupons (ou des discount factors)
-> permet d'obtenir la valeur de tout cash flow futur (ou de tout taux au pair d'un instrument de marché, spot ou forward)

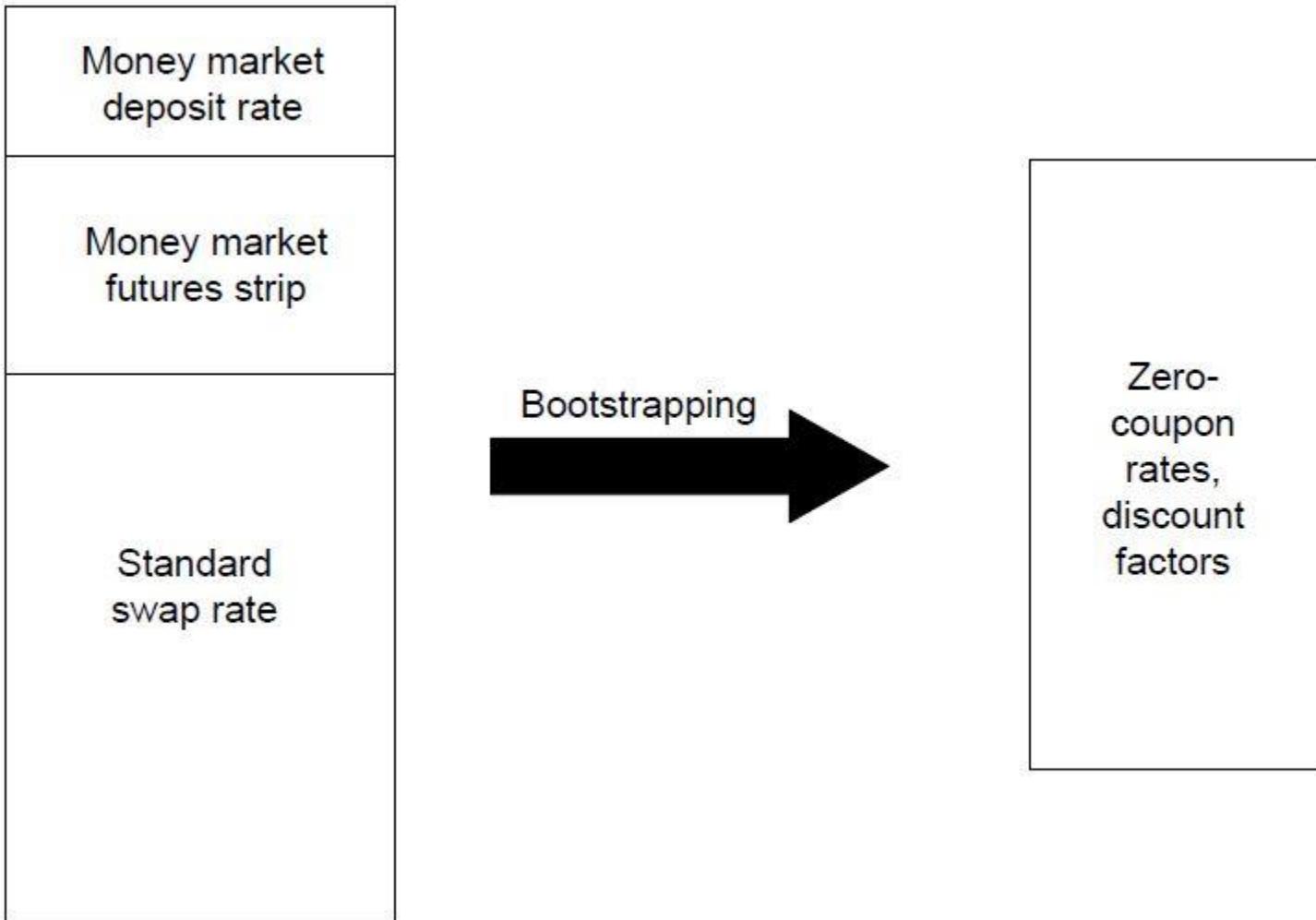
$$C_t = \{Z_t(T) | T \in [t, \infty[\}$$

ou bien

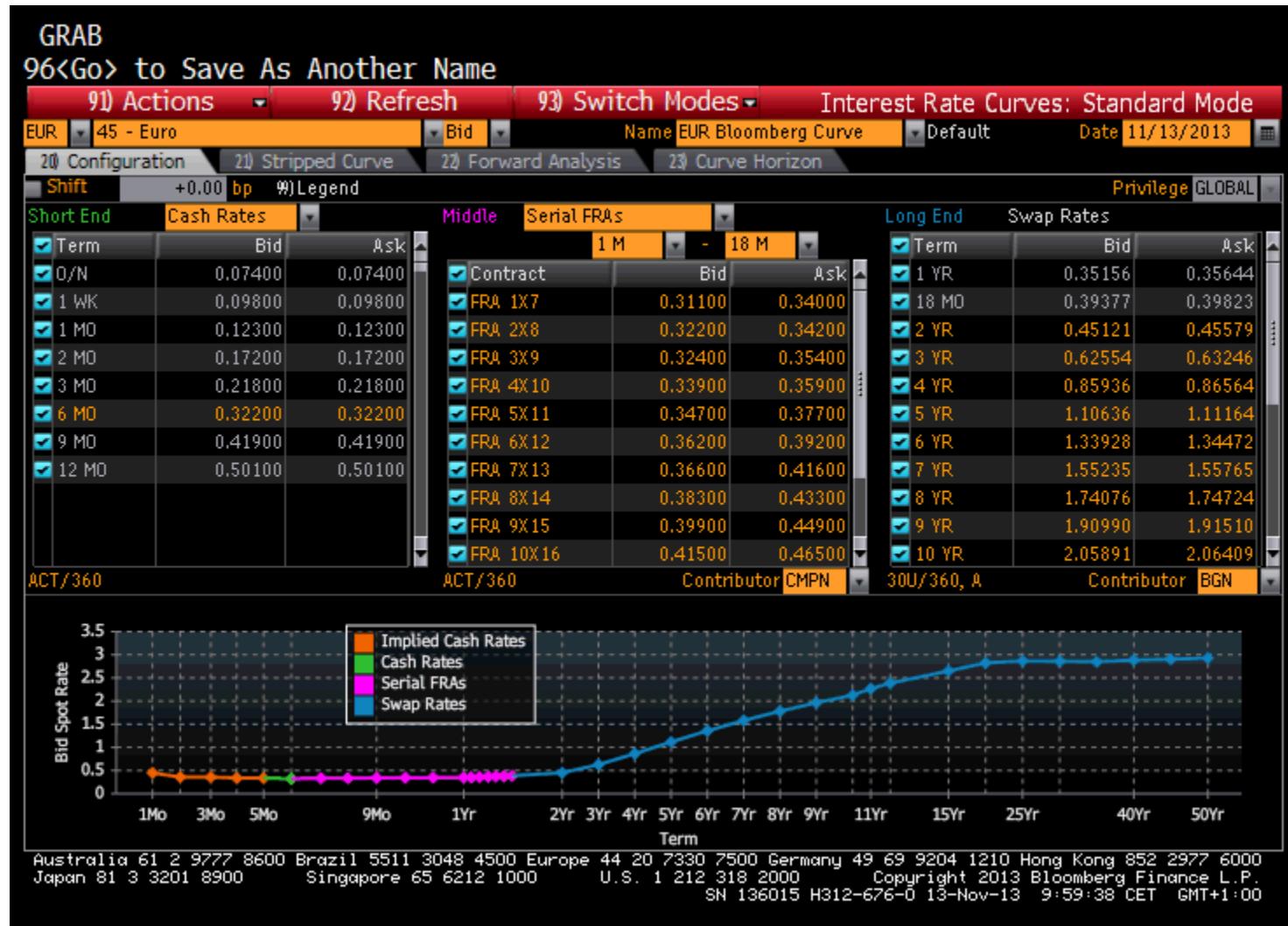
$$C_t = \{P_t(T) | T \in [t, \infty[\}$$

- Estimation de la courbe de taux zéro coupons : à partir des prix disponibles des instruments de marché => **bootstrapping**

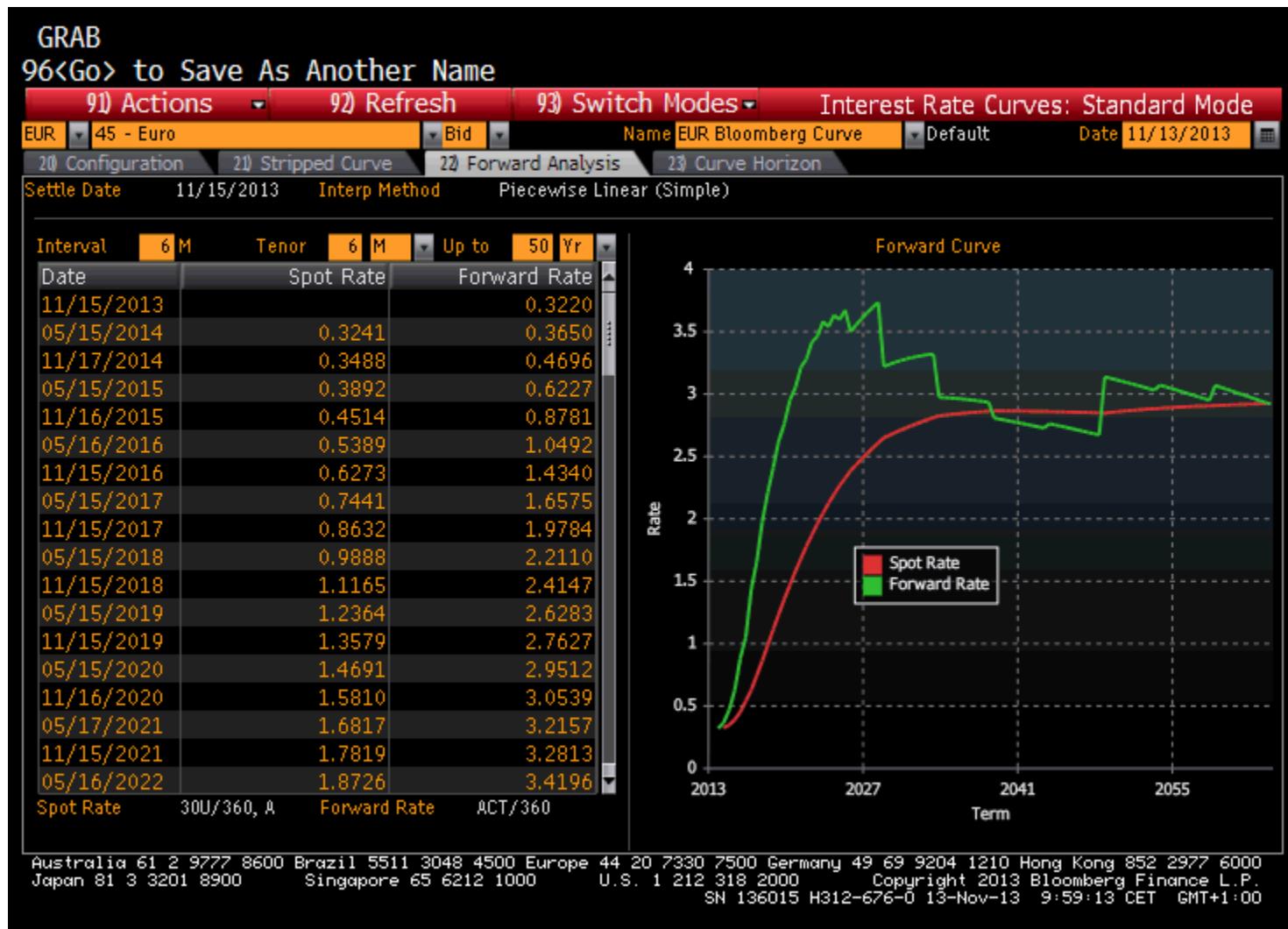
Courbe de taux (Term Structure) – Swaps



Courbe de taux (Term Structure) – Swaps



Courbe de taux – Forwards



Bootstrapping – Exemple simple

- Soient M obligations de valeur B_t^i , maturant respectivement en $1, 2, \dots, M$, et détachant un coupon Y_{ij} à chaque période $1, 2, \dots, M$

On a donc :

$$\begin{aligned} B_t^1 &= \begin{pmatrix} Y_{11} & \cdots & Y_{1M} \\ \vdots & \ddots & \vdots \\ Y_{M1} & \cdots & Y_{MM} \end{pmatrix} P_t(t+1) \\ B_t^M &= \end{aligned}$$

Si la matrice est non singulière, alors on connaît $P_t(t+i), i = 1 \dots M$,

Dans le cas général (M bonds, N périodes), si le système est sur-déterminé ($M > N$), alors, s'il n'y a pas de solution, c'est qu'une opportunité d'arbitrage existe.

Dans la réalité, le système est souvent sous-déterminé, impliquant que plusieurs courbes de discount peuvent être utilisées de façon cohérentes avec le principe de non-arbitrage.

Courbe de taux - Modélisation / interpolation

Exemples

- Interpolations de type polynomiales : Splines Cubiques

$$0 = \tau_0 \leq \tau_1 \leq \dots \leq \tau_k = T_M$$

$$\bar{P}(T) = \sum_{j=0}^{k-1} G_j(T) I_j(T)$$

$$I_j(T) = \begin{cases} 1, & \text{si } T \geq \tau_j \\ 0, & \text{sinon} \end{cases}$$

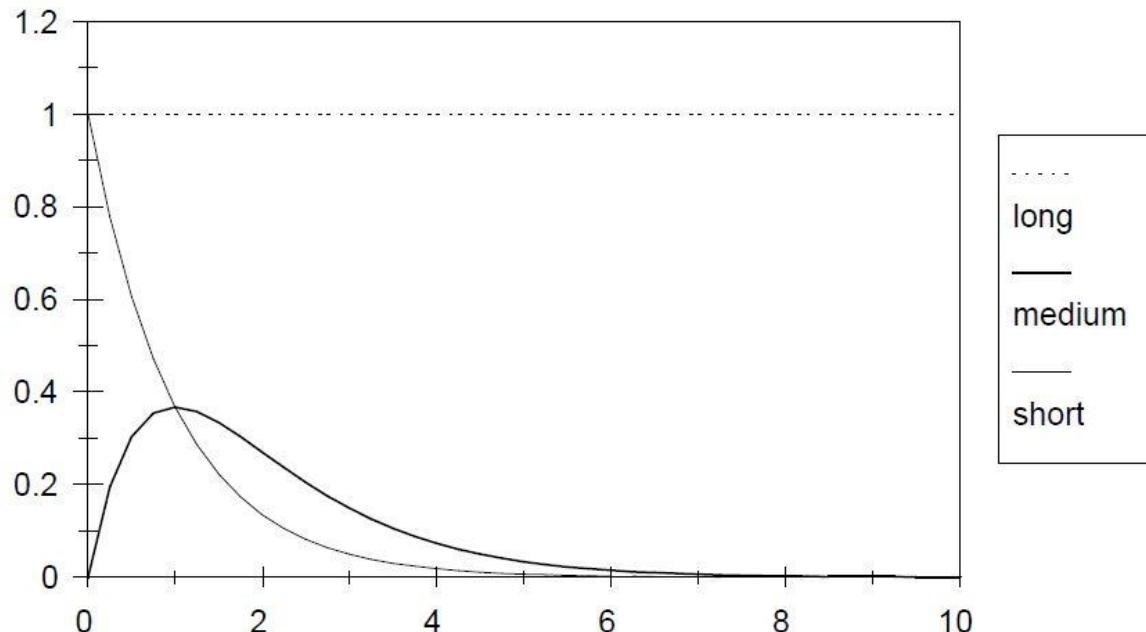
$$G_j(T) = \alpha_j + \beta_j(\tau - \tau_j) + \gamma_j(\tau - \tau_j)^2 + \delta_j(\tau - \tau_j)^3$$

Courbe de taux - Modélisation / interpolation

- Modélisations paramétriques : Nelson-Siegel

Forme paramétrique du taux forward

$$\bar{f}(\tau) = \beta_0 + \beta_1 e^{-\tau/\theta} + \beta_2 \frac{\tau}{\theta} e^{-\tau/\theta}$$





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