Tick values and regulation

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Outline

1. Tick value, tick size and spread
   - Tick value
   - Tick size
   - Large tick asset and spread
2. The model with uncertainty zones
   - Simplified version
   - Comments and intuitions about η
3. Implicit spread and volatility per trade
   - Setup
   - Regression design
   - Cost analysis
4. Predicting consequences of tick value changes
5. Application: The Tokyo Stock Exchange Pilot Program

Outline

Notion of tick value

- Exchange rule: There exists a price grid for orders.
- **Tick value**: smallest price increment. Dimension: currency of the asset.
    - Subject to changes by the exchange.
    - In some markets, the spacing of the grid can depend on the price.
    - eg: stocks trading on Euronext Paris have a price dependent tick scheme. Stocks priced 0 to 9.999€ have a tick value of 0.001€ but all stocks above 10€ have a tick of 0.005€.

Notion of tick size

- In practice: the tick value is given little consideration. What is important is the **tick size**.
- **Tick size**: qualifies the traders' aversion to price movements of one tick.
- In the US, The SEC has required a whole report about the tick size issue in the context of high frequency trading and a special roundtable was devoted to this topic on 5 February 2013.
- In Europe, one of the main conclusion of the Foresight report is the crucial need for proper tick sizes to regulate high frequency trading.
- BUT: No quantitative guidelines for tick sizes analysis.
- In 2014: Japan pilot experiment.
- In 2016: US pilot experiment.

What is a large tick asset ?

- Notion of tick size is ambiguous in general. However, we can identify large tick assets.
- From Eisler, Bouchaud and Kockelkoren: *Large tick stocks are such that the bid-ask spread is almost always equal to one tick, while small tick stocks have spreads that are typically a few ticks.*
- This leads to the following questions:

Issues

- Small tick assets: spread is a good proxy for the tick size.
  - If spread $\simeq$ 1 tick $\Rightarrow$ How to quantify the tick size ?
- In the literature: there exist special relationships between the spread and some market quantities. BUT:
  - Not valid for large tick assets: spread bounded by 1.

  How to extend these studies in the large tick case?
- Tick value change $\Rightarrow$ What happens to the microstructure?
- Can we define an optimal tick value?

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Spread theory for small tick assets

Madhavan, Richardson, Roomans economic model

- p_{i+1}: ex post *true or efficient price* after the i-th trade (all transactions have the same volume), ε_i: sign of the i-th trade. The MRR model is defined by:

$$p_{i+1} - p_i = \xi_i + \theta\varepsilon_i,$$

with ξ_i an independent centered shock component (new information,...) with variance v^2.

Spread theory for small tick assets (2)

MRR model (2)

- Market makers cannot guess the surprise of the next trade. So, they post (pre trade) bid and ask prices a_i and b_i given by

$$a_i = p_i + \theta + \phi, \quad b_i = p_i - \theta - \phi,$$

  with ϕ an extra compensation claimed by market makers, covering processing costs and the shock component risk.
- This rule ensures no ex post regrets for market makers (if $\phi = 0$ the traded price is on average the right one).
- If $\phi = 0$, the ex post average cost of a market order with respect to the efficient price $a_i - p_{i+1}$ or $p_{i+1} - b_i$ is 0.

Spread theory for small tick assets (3)

MRR model (3)

- We can compute several quantities:
- The spread $S = a - b = 2(\theta + \phi)$.
- The variance per trade of the efficient price

$$\sigma_1^2 = E[(p_{i+1} - p_i)^2] = \theta^2 + v^2 \sim \theta^2$$

(the news component being negligible, see Wyart *et al.*).

- Therefore:

$$S \sim 2\sigma_1 + 2\phi.$$

Market making strategy

The Wyart et al. approach

- Market makers: patient traders. Send limit orders $\Rightarrow$ delayed execution. Pocket the spread. $\exists$ volatility risk.
- Market takers: impatient traders. Send market orders $\Rightarrow$ immediate execution. Pay the spread. No volatility risk.
- Wyart *et al.*: consider a simple market making strategy. Its average P&L per trade is

$$P\&L = \frac{S}{2} - \frac{c}{2}\sigma_1,$$

with c depending on the assets but of order $1 \sim 2$.
- P&L = cost of a market order (on average).

Market maker vs market taker

Wyart *et al.*:

- On electronic markets, any agent can choose between market orders and limit orders.
  $\Rightarrow$ Both types of orders will have the same average (ex post) cost = 0.
  $\Rightarrow$ Market makers' P&L = 0 (if not so, another market maker comes with a slightly tighter spread).
- Therefore:

$$S \sim c\sigma_1.$$

- This relationship is very well satisfied on market data.

Outline

Notations

- X_t : non observable efficient price, continuous Itō semi-martingale (almost no other assumption on X).
- α: tick value. a: ask, b: bid. $m = \frac{a+b}{2}$: midpoint.
- t_i: time of the i^{th} transaction with price change.
- P_t: observable price. P_{t_i}: transaction price at time t_i.
- $U = 2\eta\alpha < \alpha$: uncertainty region around m.
- $U_k = [0, \infty) \times (d_k, u_k)$ with

$$d_k = (k + 1/2 - \eta)\alpha \text{ and } u_k = (k + 1/2 + \eta)\alpha.$$

Notations

- t_i: transaction times: i^{th} exit time of an uncertainty zone:

$$t_{i+1} = \inf\left\{t > t_i, X_t = X_{t_i}^{(\alpha)} \pm \alpha(\frac{1}{2} + \eta)\right\},$$

where $X_{t_i}^{(\alpha)}$ the value of X_{t_i} rounded to the nearest multiple of α.

- $P_{t_i} = X_{t_i}^{(\alpha)}$ (i-th new transaction price).


Price
Time
101.5
101
100.5
100
99.5
99
0
50
100
150
200
250
300
$2\eta\alpha$
α

Estimation of the efficient price

Efficient price

We have the following very useful property:

- $$X_{t_i} = P_{t_i} - \alpha(\frac{1}{2} - \eta) sign(P_{t_i} - P_{t_{i-1}}).$$
- $$\hat{X}_{t_i} = P_{t_i} - \alpha(\frac{1}{2} - \hat{\eta}) sign(P_{t_i} - P_{t_{i-1}}).$$

t_i: transaction times,
P_t: last traded price, X_t: efficient price.

Estimation of η

- A continuation is a price variation whose direction is the same as the one of the preceding variation.
- An alternation is a price variation whose direction is opposite to the one of the preceding variation.
- N^c = # continuations. N^a = # alternations.
- Estimator $\widehat{\eta}$:

$$\widehat{\eta} = \frac{N^c}{2N^a}.$$

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Bund and DAX, estimation of η

Day	η (Bund)	η (FDAX)	Day	η (Bund)	η (FDAX)
1 Oct.	0.18	0.41	18 Oct.	0.16	0.33
5 Oct.	0.15	0.37	19 Oct.	0.13	0.37
6 Oct.	0.15	0.37	20 Oct.	0.13	0.33
7 Oct.	0.15	0.38	21 Oct.	0.15	0.33
8 Oct.	0.15	0.41	22 Oct.	0.11	0.33
11 Oct.	0.14	0.36	25 Oct.	0.12	0.31
12 Oct.	0.14	0.36	26 Oct.	0.14	0.31
13 Oct.	0.14	0.32	27 Oct.	0.14	0.32
14 Oct.	0.16	0.35	28 Oct.	0.14	0.32
15 Oct.	0.16	0.35	29 Oct.	0.14	0.34

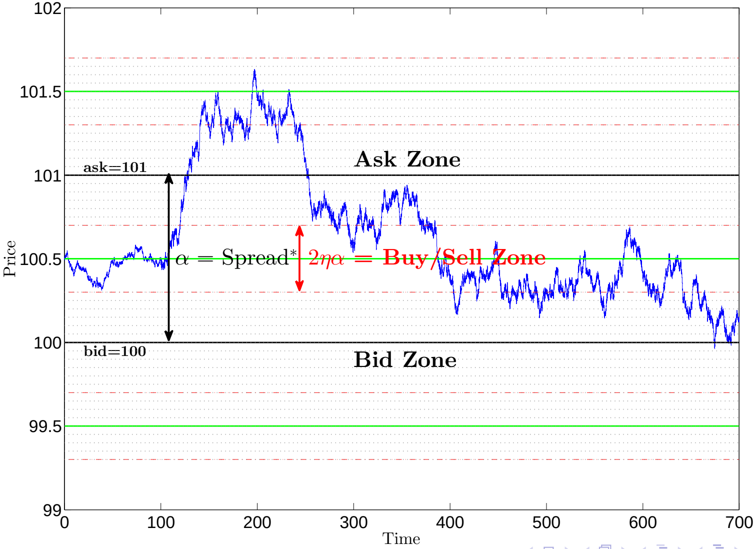
Intuitions about η

- $\eta \iff$ Distribution of high frequency tick returns.
- η **small** $\Rightarrow$ Uncertainty zone small $\Rightarrow$ Strong mean reversion in the observed price $\Rightarrow$ Decreasing signature plot, significant ACV of tick returns $\Rightarrow$ Tick size **large**.
- $\eta \sim 1/2 \Rightarrow$ the last traded price can be seen as a sampled Brownian motion $\Rightarrow$ No microstructure effects $\Rightarrow$ Flat signature plot and ACV of tick returns $\Rightarrow$ Uncertainty zone $= 1$ tick $\Rightarrow$ the tick size is, in some sense, **optimal**

The market order areas

- Simplification: $S = \alpha$, constant, 1 tick.
- For given bid-ask quotes, we have:
  - Ask zone with only buy market orders.
  - Bid zone with only sell market orders.
  - Uncertainty or Buy/Sell zone.

Ask Zone, Bid Zone and Uncertainty Zone



Intuitions about η

- Distance between Ask Zone and Bid Zone is $2\eta\alpha$.
- $2\eta\alpha$ represents an **implicit unobservable spread**.
- M: **Total number of trades** (null returns and not).
- Can we extend

$$\frac{S}{2} \sim \frac{\sigma}{\sqrt{M}} \text{ to } \eta\alpha \sim \frac{\sigma}{\sqrt{M}}?$$

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Outline

The assets

- We want to investigate the relationship

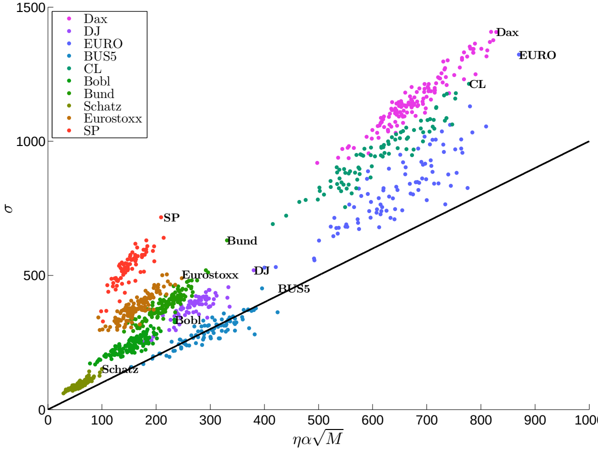
$$\eta\alpha \sim \frac{\sigma}{\sqrt{M}} + \phi$$

  for large tick assets.

- We consider Futures on: the DAX index (DAX), the Euro-Stoxx 50 index (ESX), the Dow Jones index (DJ), SP500 index (SP), 10-years Euro-Bund (Bund), 5-years Euro-Bobl (Bobl), 2-years Euro-Schatz (Schatz), 5-Year U.S. Treasury Note Futures (BUS5), EUR/USD futures (EURO), Light Sweet Crude Oil Futures (CL).

Empirical results

- Cloud $(\eta\alpha\sqrt{M}, \sigma)$, for each day, for each asset.
- Linear relationship, same slope, different intercepts.



Linear regression

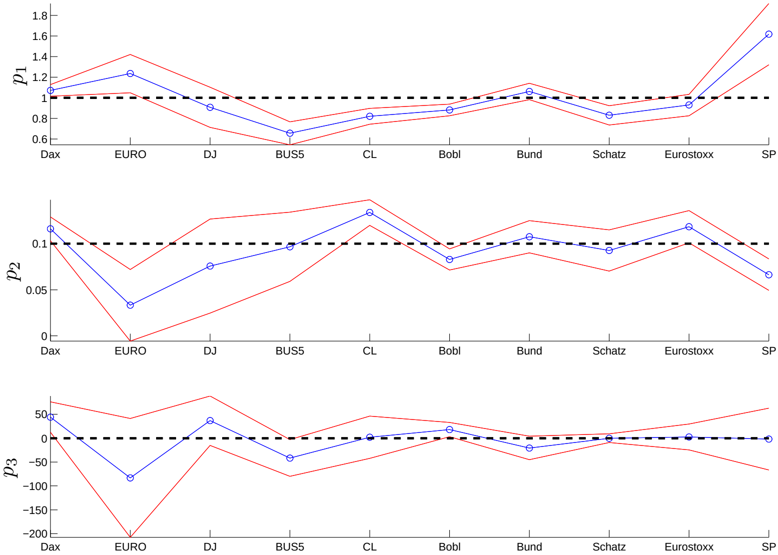
- We consider the relationship

$$\eta\alpha \sim \frac{\sigma}{\sqrt{M}} + \phi$$

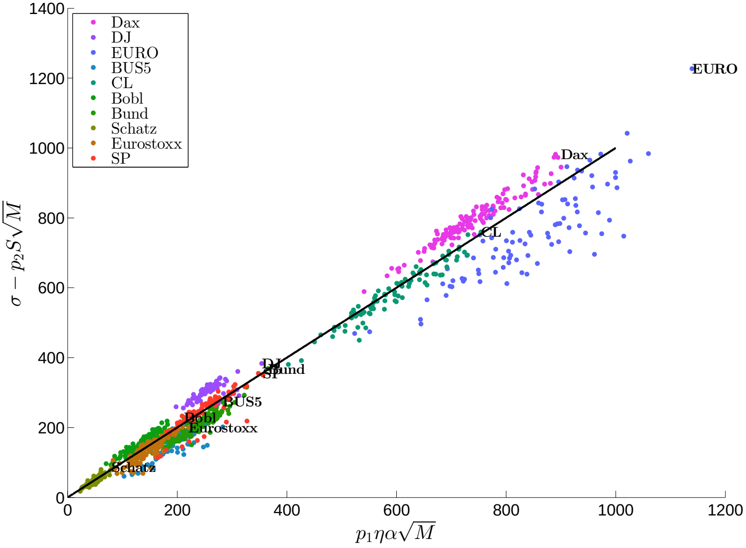
  for large tick assets.
- ϕ includes operational costs and **inventory control** $\Rightarrow$ $\phi = k * S$.
- Daily regression:

$$\sigma = p_1 \eta\alpha\sqrt{M} + p_2 S\sqrt{M} + p_3.$$

Daily regression



The constant is equal to zero



Market orders cost

- Average ex post cost of a market order (relative to X_t):

$$\alpha/2 - \eta\alpha.$$

- Average P&L per trade of the market makers = average cost of a market order ⇒

$$\eta\alpha = c\frac{\sigma}{\sqrt{M}} + \phi.$$

- $\eta < 1/2$: Limit orders are profitable whereas market orders are costly.
- Note: $\eta < 1/2$ is a natural state. Otherwise market makers would increase the spread ⇒ decreasing signature plot.

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Outline

Consequences of a change

- α too small encourages free-riding (directional HFT) and traditional market makers cannot fix their quotes.
- α too large implies price sloppiness. Moreover it favors speed (race to the top of book) $\Rightarrow$ High investments in infrastructure.
- What happens to η if one changes the tick value ?
- How to obtain the following **optimal** situation:
  - $S \sim 1$
  - η close to 1/2
  - Cost of market orders = cost of limit orders = 0.

Changing α

We assume that when changing the tick value:

- σ constant $\Rightarrow$

$$\eta_0\alpha_0\sqrt{M_0} + 0.1\alpha_0\sqrt{M_0} = \eta\alpha\sqrt{M} + 0.1\alpha\sqrt{M}.$$

- Constant volume.
- Linear shape for the cumulative latent liquidity.

Forecasting formula and optimal tick value (simplified version)

$$\eta \sim (\eta_0 + 0.1)(\frac{\alpha_0}{\alpha})^{1/2} - 0.1.$$

$$\alpha^* \sim (\frac{\eta_0 + 0.1}{0.6})^2 \alpha_0.$$

- Ex-ante assessment of the consequences of a tick value change !

Outline

Description of the pilot

- 55 Japanese stocks of the TOPIX 100 index involved.
- Phase 0 (before the pilot program): from June 3, 2013 to January 13, 2014.
- Phase 1 (between the first and second implementations of the tick value reduction program): from January 14, 2014 to July 21, 2014.
- Phase 2 (after the second implementation of the tick value reduction program): from July 22, 2014 to December 30, 2014.

Tick value reduction table

Price below (¥)	Phase 0 tick(¥)	Phase 1 tick(¥)	Phase 2 tick (¥)
1,000	1	1	0.1
3,000	1	1	0.5
5,000	5	1	0.5
10,000	10	1	1
30,000	10	5	5
50,000	50	5	5
100,000	100	10	10
300,000	100	50	50
500,000	500	50	50
1,000,000	1000	100	100
3,000,000	1000	500	500
5,000,000	5000	500	500
10,000,000	10000	1000	1000
30,000,000	10000	5000	5000
50,000,000	50000	5000	5000
Higher prices	100000	10000	10000

Classification of the stocks in Phase 0

Tick size

- Small tick stocks: $S > 1.6$.
- Large tick stocks: $S \leq 1.5$.

For large tick stocks

- Balanced stocks: $\eta \geq 0.4$.
- Market makers favorable stocks: $\eta < 0.4$.

Phase 0-Phase 1 analysis

Selection of stocks

- Large tick assets during Phase 0.
- Stocks involved in the tick value reduction program during Phase 1.
- Stable tick value during the period.

Forecasting device

- If $\eta_1^p \geq 0.55$, the asset is predicted to become a small tick asset after the tick value change.
- If $\eta_1^p < 0.5$, the asset is predicted to remain a large tick asset after the tick value change, with the forecast value for the new η being meaningful and given by η_1^p.

Phase 0-Phase 1 analysis

Company name	S_0	η_0	S_1	η_1	η_1^p
**Astellas Pharma Inc	1.05	0.14	1.72		0.66 [0.50,0.71]
**Canon Inc	1.04	0.06	1.13	0.23	0.26 [0.19,0.27]
**Honda Motor Co Ltd	1.04	0.10	1.23	0.32	0.34 [0.26,0.37]
**Japan Tobacco Inc	1.04	0.12	1.23	0.32	0.39 [0.26,0.41]
**Mitsui Fudosan Co Ltd	1.07	0.23	1.95		0.63 [0.52,0.69]
*Nippon Telegraph Corp	1.04	0.08	2.00		0.46 [0.35,0.51]
**Seven I Holdings Co Ltd	1.06	0.16	1.55	0.51	0.49 [0.38,0.55]
*Softbank Corp	1.05	0.06	1.85		0.40 [0.32,0.40]
*Sumitomo Mitsui Fin. Gr.	1.15	0.08	1.33	0.34	0.47 [0.27,0.47]
**Takeda Pharm. Co Ltd	1.06	0.13	1.46	0.43	0.42 [0.28,0.45]
**Tokio Marine Holdings Inc	1.05	0.18	1.39	0.46	0.53 [0.41,0.57]
**Toyota Motor Corp	1.03	0.04	1.36	0.32	0.35 [0.27,0.33]

*The number of stars * represents the number of good predictions (one for being large tick or not, one for being balanced or not).*