Modelling ultra high frequency financial data: Model with uncertainty zones and associated statistical procedures for volatility, correlation and lead-lag

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Outline

Outline

Properties of the microstructure model

We want the following properties for our model :

- Model for prices **and** durations $\rightarrow$ no hesitation about the sampling frequency.
- Discrete prices.
- Bid-Ask bounce.
- Stylized facts of returns, durations and volatility. In particular, inverse relation between durations and volatility.
- A diffusive behavior at large sampling scales.

Microstructure modelling (2)

Properties of the microstructure model (2)

- No hesitation about the price.
- Finite quadratic variation for the microstructure noise.
- An interpretation of the model.
- **A useful model**.

A proposed answer :

- The model with uncertainty zones.
- In this model, prices and durations are functionals of some hitting times of an underlying continuous semi-martingale.

Outline

Aversion for price changes

- In an idealistic framework, transactions would occur when the efficient price crosses the tick grid.
- In practice, uncertainty about the efficient price and aversion for price changes of market participants.
- The price changes only when market participants are convinced that the efficient price is far from the last traded price.
- We introduce a parameter η quantifying this aversion for price changes.

Model with uncertainty zones : notation

- Efficient price : X_t.
- α : tick size.
- t_i : time of the i-th transaction with price change.
- P_{t_i} : transaction price at time t_i.
- $L_i = |P_{t_{i+1}} - P_{t_i}|/\alpha$: size of the i-th price jump.
- Explanatory variables process for the size of the jumps : χ_t.
- Uncertainty zones : $U_k = [0, \infty) \times (d_k, u_k)$ with

$$d_k = (k + 1/2 - \eta)\alpha \text{ and } u_k = (k + 1/2 + \eta)\alpha.$$

- τ_i : i-th exit time of an uncertainty zone.

Model with uncertainty zones

Model with uncertainty zones : dynamics

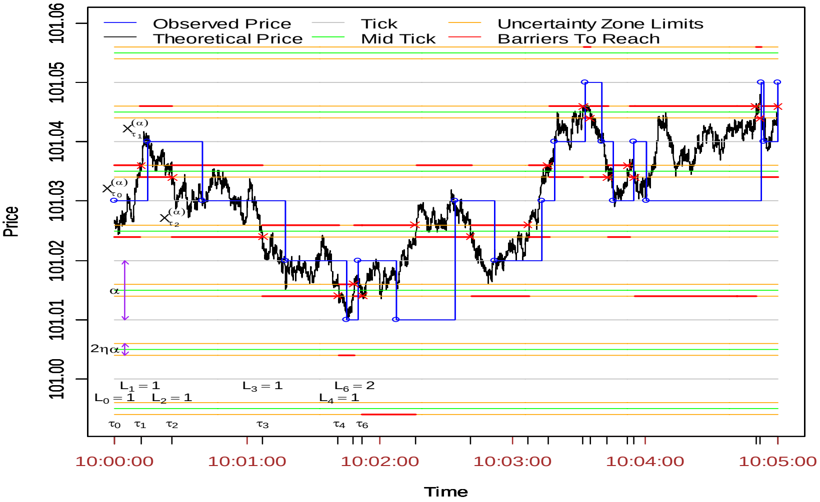
- $$\mathrm{d}\log X_u = a_u \mathrm{d}u + \sigma_{u-} \mathrm{d}W_u.$$

- $$\mathbb{P}[L_i = s | \mathcal{F}_{\tau_i}] = \phi_s(\chi_{\tau_i}).$$

- $$\tau_{i+1} = \inf\left\{t > \tau_i, X_t = X_{\tau_i}^{(\alpha)} \pm \alpha(L_i - \frac{1}{2} + \eta)\right\},$$

  with $X_{\tau_i}^{(\alpha)}$ the value of X_{τ_i} rounded to the nearest multiple of α.

- $$\tau_i \leq t_i < \tau_{i+1} \text{ and } P_{t_i} = X_{\tau_i}^{(\alpha)}.$$


Observed Price
Theoretical Price
Tick
Mid Tick
Uncertainty Zone Limits
Barriers To Reach
Price
Time
101.00
101.01
101.02
101.03
101.04
101.05
101.06
10:00:00
10:01:00
10:02:00
10:03:00
10:04:00
10:05:00
$L_1 = 1$
$L_0 = 1$
$L_2 = 1$
$L_3 = 1$
$L_6 = 2$
$L_4 = 1$
α
$2\eta\alpha$
$X^{(\alpha)}_{\tau_0}$
$X^{(\alpha)}_{\tau_1}$
$X^{(\alpha)}_{\tau_2}$
τ_0 τ_1 τ_2 τ_3 τ_4 τ_6

Comments on the model

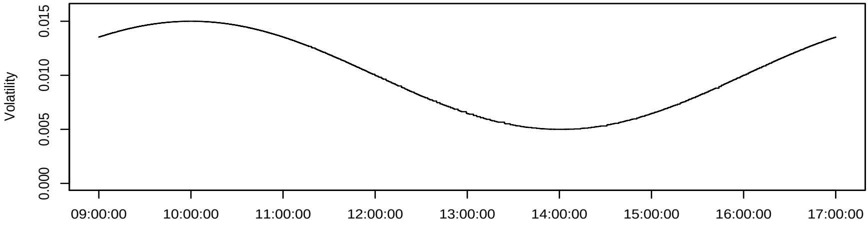
- Price/Durations model for the last traded price.
- Latent semi-martingale efficient price.
- Random times for the transactions. Inverse relation between durations and volatility.
- Bid-Ask bounce.
- Stylized facts of auto and cross correlogramms reproduced, both in tick time and calendar time.
- Jumps of several ticks. Size of the jumps determined by explanatory variables, involving for example the order book.
- Structural model for the microstructure noise.

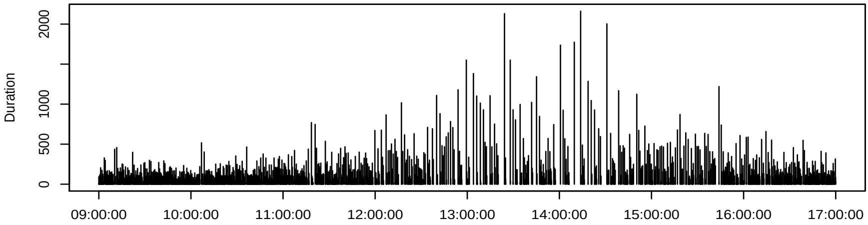
The parameter η

Interpretation of η

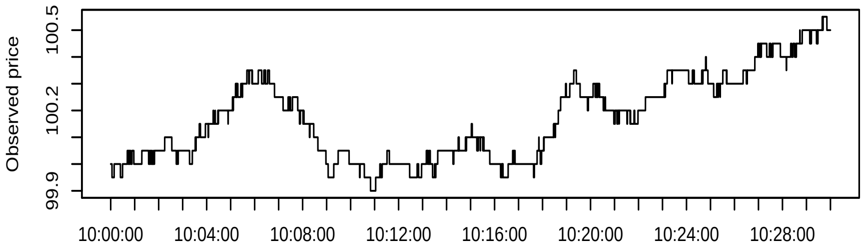
- Quantifies the aversion for price changes (with respect to the tick size) of market participants.
- In the UHF, the order book can not "follow" the efficient price and is reluctant to price changes. Reluctancy measured by η.
- $2\eta\alpha$ represents the **implicit spread** of a large tick asset (see later).
- A small η ($< 1/2$) means that for market participants, the tick size is too big and conversely.

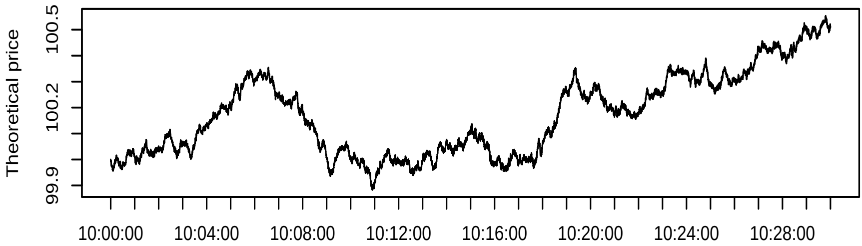
Some properties : Durations



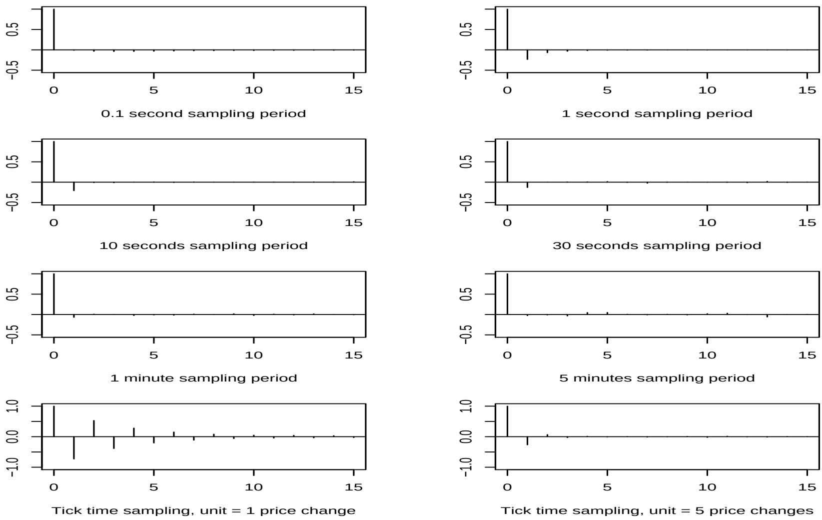


Some properties : The price

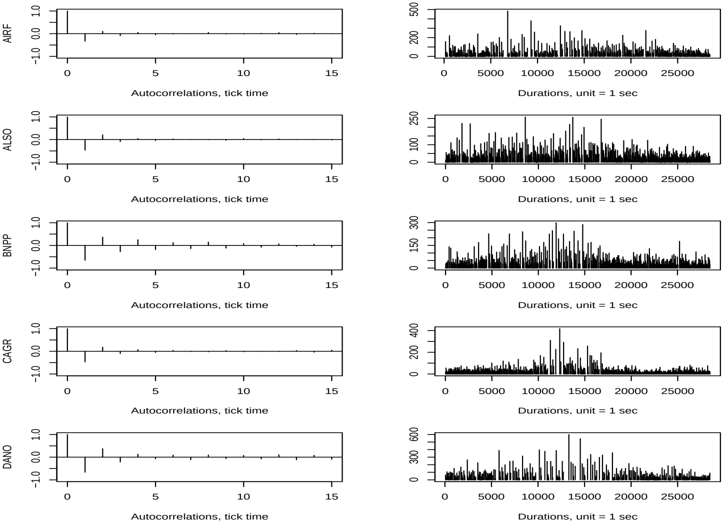




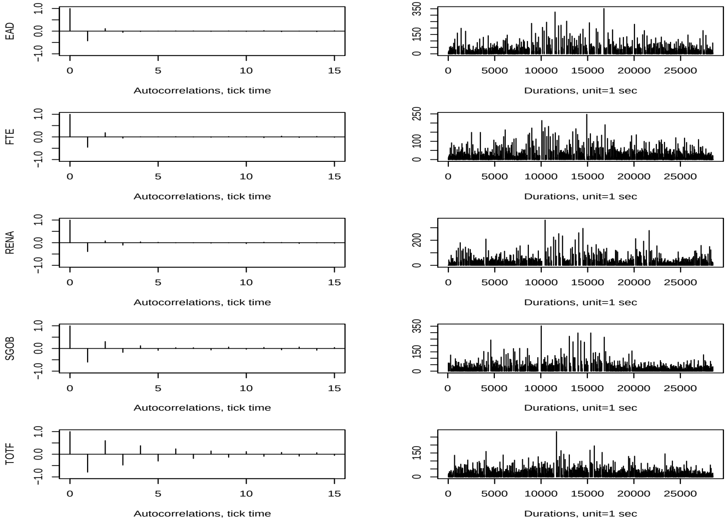
Some properties : ACF log returns ($\eta < 1/2$)



Real data : 2007/01/16



Real data : 2007/01/16



Outline

Retrieving the efficient price

Efficient price

- $$X_{\tau_i} = P_{t_i} - \alpha(\frac{1}{2} - \eta) sign(P_{t_i} - P_{t_{i-1}}).$$
- $$\hat{X}_{\tau_i} = P_{t_i} - \alpha(\frac{1}{2} - \hat{\eta}) sign(P_{t_i} - P_{t_{i-1}}).$$

t_i : observation time, τ_i : exit time,
P_t : observed price, X_t : efficient price.

Estimation of η

Let $N_{\alpha,t} = \text{card}\{t_i, t_i \leq t\}$. For $k = 1, ..., m$,

$$N^{(c)}_{\alpha,t,k} = \sum_{i=1}^{N_{\alpha,t}} \mathbb{I}_{\{|X_{\tau_i} - X_{\tau_{i-1}}| = \alpha k\}},$$

$$N^{(a)}_{\alpha,t,k} = \sum_{i=1}^{N_{\alpha,t}} \mathbb{I}_{\{|X_{\tau_i} - X_{\tau_{i-1}}| = \alpha(k-1+2\eta)\}}.$$

We define

$$\hat{\eta}_t = \sum_{k=1}^{m} \lambda_{\alpha,t,k} u_{\alpha,t,k},$$

with

$$\lambda_{\alpha,t,k} = \frac{N^{(a)}_{\alpha,t,k} + N^{(c)}_{\alpha,t,k}}{\sum_{j=1}^{m} \left[N^{(a)}_{\alpha,t,j} + N^{(c)}_{\alpha,t,j}\right]} \text{ and } u_{\alpha,t,k} = \frac{1}{2}\Big(k\big(\frac{N^{(c)}_{\alpha,t,k}}{N^{(a)}_{\alpha,t,k}} - 1\big) + 1\Big).$$

Theorem

Let

$$\widehat{RV}_t = \sum_{i=1}^{N_{\alpha,t}} \left(\log(\hat{X}_{\tau_i}) - \log(\hat{X}_{\tau_{i-1}}) \right)^2.$$

We have

$$\alpha^{-1}(\widehat{RV}_t - RV_t) \xrightarrow{\mathcal{L}s} \gamma_t \int_0^t v_u \mathrm{d}W_{\theta_u},$$

where W is a Brownian motion independent of B and θ_u, γ_u and v_u depend on X_u, σ_u and explanatory variables, involving for example the order book.

Discussion

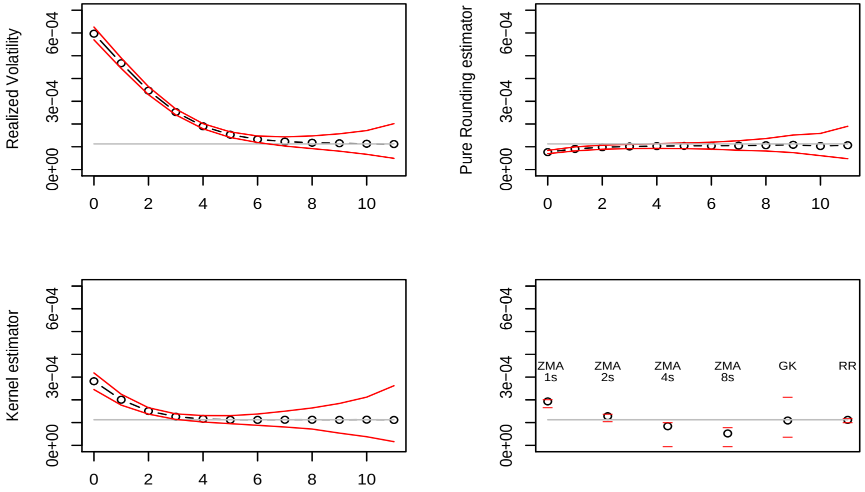
Comments on the theorem

- a_u only needs to be progressively measurable and locally bounded. σ_u only needs to be adapted càdlàg. In particular, σ_u is not necessarily an Ito semi-martingale.
- Rounding happens on the original scale. Thus, estimating the quadratic variation of the log price is more intricate than estimating those of the price.
- The observation times are random, endogenous. So, usual theorems for deterministic or exogenous sampling can not be applied.
- The key ideas are to work in a modified time in which the observation times are equidistant and to use stability properties of the convergence in $\mathbb{D}[0, T]$.

Bund and DAX, estimation of η, October 2010

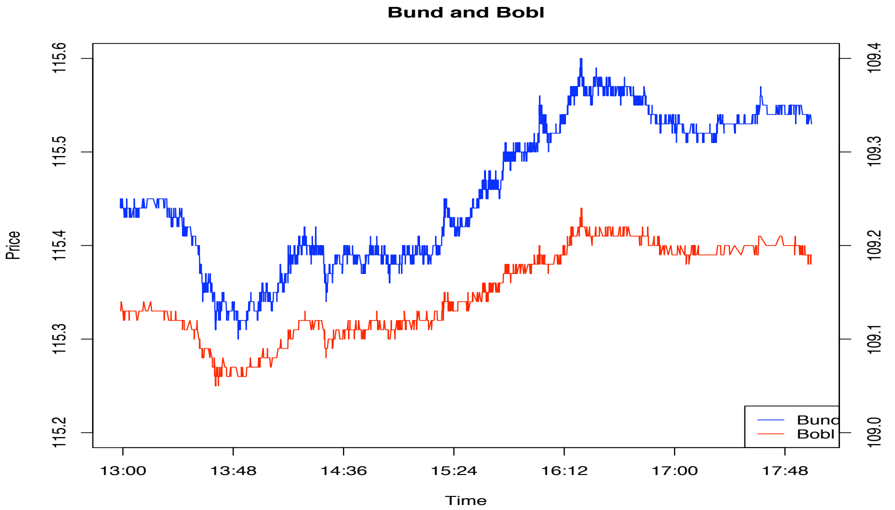
Day	η (Bund)	η (FDAX)	Day	η (Bund)	η (FDAX)
1 Oct.	0.18	0.41	18 Oct.	0.16	0.33
5 Oct.	0.15	0.37	19 Oct.	0.13	0.37
6 Oct.	0.15	0.37	20 Oct.	0.13	0.33
7 Oct.	0.15	0.38	21 Oct.	0.15	0.33
8 Oct.	0.15	0.41	22 Oct.	0.11	0.33
11 Oct.	0.14	0.36	25 Oct.	0.12	0.31
12 Oct.	0.14	0.36	26 Oct.	0.14	0.31
13 Oct.	0.14	0.32	27 Oct.	0.14	0.32
14 Oct.	0.16	0.35	28 Oct.	0.14	0.32
15 Oct.	0.16	0.35	29 Oct.	0.14	0.34

1000 MC simulations, 90% confidence interval



Outline

Correlated assets : Bund-Bobl



Dynamics of the efficient log prices

We now consider two assets.

$$d\log X_t = \mu_t^X dt + \sigma_{t-}^X dW_t,$$

$$d\log Y_t = \mu_t^Y dt + \sigma_{t-}^Y dB_t,$$

with

$$d\langle W, B\rangle_t = \rho_t dt.$$

Quantity of interest

Integrated covariance

We want to estimate

$$\int_0^1 \rho_t \sigma_t^X \sigma_t^Y \mathrm{d}t.$$

Difficulties

Practical problems

We will face two main difficulties :

- Asynchronicity of the data,
- Microstructure effects.

Usual estimator of the covariation

Realized covariation

We start with the usual case without asynchronicity or microstructure effects. We observe $(X_{i/n}, Y_{i/n})$, $i = 0, \dots, n$. Let

$$\Delta_i^n X = \log X_{i/n} - \log X_{(i-1)/n}.$$

An estimator of

$$\int_0^1 \rho_t \sigma_t^X \sigma_t^Y \mathrm{d}t$$

with accuracy $n^{-1/2}$ is given by

$$\widehat{c}_n = \sum_{i=1}^{n} \Delta_i^n X \Delta_i^n Y.$$

Case of a constant correlation

Constant volatilities

Very often, traders like to think in term of correlation. When the correlation and volatility parameters are supposed to be constant :

$$\rho_t = \rho, \; \sigma_t^X = \sigma^X, \; \sigma_t^Y = \sigma^Y,$$

an estimator of ρ with accuracy $n^{-1/2}$ is given by

$$\frac{\widehat{c}_n}{\sqrt{\sum_{i=1}^n (\Delta_i^n X)^2 \sum_{i=1}^n (\Delta_i^n Y)^2}}.$$

Non constant volatilities

In the case where the volatility parameters are no longer constant, one can consider

$$\widehat{\rho}_n = \frac{2}{\pi} \frac{\widehat{c}_n}{\widehat{a}_n},$$

with

$$\widehat{a}_n = \sum_{i=1}^{n-1} |\Delta_{i+1}^n X \Delta_i^n Y|.$$

Indeed, $\widehat{a}_n$ is an estimator of

$$\frac{2}{\pi} \int_0^1 \sigma_s^X \sigma_s^Y ds.$$

Case of a constant correlation

Non constant volatilities : convergence of $\widehat{a}_n$

- Step 1 : we note that

$$\widehat{a}_n \approx \sum_{i=1}^{n-1} \sigma^X_{\frac{i-1}{n}} \sigma^Y_{\frac{i-1}{n}} |W_{\frac{i+1}{n}} - W_{\frac{i}{n}}||B_{\frac{i}{n}} - B_{\frac{i-1}{n}}|,$$

  with the Brownian increments in the preceding sum being independent.

- Step 2 : $\widehat{a}_n$ has the same limit as

$$\sum_{i=1}^{n-1} \mathbb{E}_{\mathcal{F}_{\frac{i}{n}}} \Big[\sigma^X_{\frac{i-1}{n}} \sigma^Y_{\frac{i-1}{n}} |W_{\frac{i+1}{n}} - W_{\frac{i}{n}}||B_{\frac{i}{n}} - B_{\frac{i-1}{n}}| \Big].$$

Case of a constant correlation

Non constant volatilities : convergence of $\widehat{a}_n$

We have

$$\sum_{i=1}^{n-1} \mathbb{E}_{\mathcal{F}_{\frac{i}{n}}}\left[\sigma^X_{\frac{i-1}{n}} \sigma^Y_{\frac{i-1}{n}} |W_{\frac{i+1}{n}} - W_{\frac{i}{n}}||B_{\frac{i}{n}} - B_{\frac{i-1}{n}}|\right]$$

$$= \sum_{i=1}^{n-1} \sigma^X_{\frac{i-1}{n}} \sigma^Y_{\frac{i-1}{n}} |B_{\frac{i}{n}} - B_{\frac{i-1}{n}}| \mathbb{E}[|W_{\frac{i+1}{n}} - W_{\frac{i}{n}}|]$$

$$= \sqrt{\frac{2}{\pi}} \frac{1}{\sqrt{n}} \sum_{i=1}^{n-1} \sigma^X_{\frac{i-1}{n}} \sigma^Y_{\frac{i-1}{n}} |B_{\frac{i}{n}} - B_{\frac{i-1}{n}}|.$$

Case of a constant correlation

Non constant volatilities : convergence of $\widehat{a}_n$

The term

$$\sqrt{\frac{2}{\pi}}\frac{1}{\sqrt{n}}\sum_{i=1}^{n-1}\sigma^X_{\frac{i-1}{n}}\sigma^Y_{\frac{i-1}{n}}|B_{\frac{i}{n}}-B_{\frac{i-1}{n}}|$$

has the same limit as

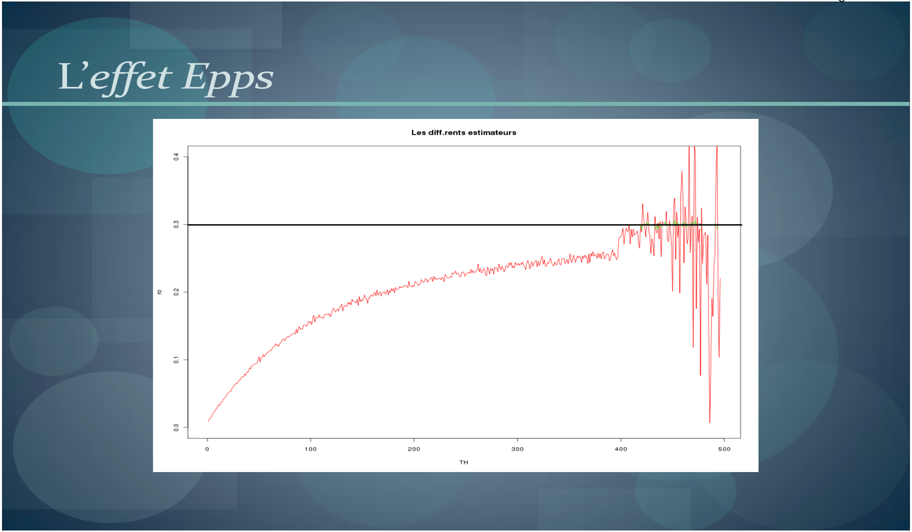
$$\sqrt{\frac{2}{\pi}}\frac{1}{\sqrt{n}}\sum_{i=1}^{n-1}\sigma^X_{\frac{i-1}{n}}\sigma^Y_{\frac{i-1}{n}}\mathbb{E}[|B_{\frac{i}{n}}-B_{\frac{i-1}{n}}|]=\frac{2}{\pi}\frac{1}{n}\sum_{i=1}^{n-1}\sigma^X_{\frac{i-1}{n}}\sigma^Y_{\frac{i-1}{n}},$$

which tends to (Riemann sum)

$$\frac{2}{\pi}\int_0^1\sigma^X_s\sigma^Y_s\,ds.$$

Previous tick scheme

- Assume now we observe X at times $(T^{X,i}), i = 1, \dots$ and Y at times $(T^{Y,i}), i = 1, \dots$
- We build
$$\overline{X}_t = X_{T^{X,i}} \text{ for } t \in [T^{X,i}, T^{X,i+1})$$
and
$$\overline{Y}_t = Y_{T^{Y,i}} \text{ for } t \in [T^{Y,i}, T^{Y,i+1}).$$
- For given h, the previous tick covariation estimator is
$$V_h = \sum_{i=1}^{m} \left(\log \overline{X}_{ih} - \log \overline{X}_{(i-1)h}\right) \left(\log \overline{Y}_{ih} - \log \overline{Y}_{(i-1)h}\right).$$


L'effet Epps
Les diff.rents estimateurs
0.0
0.1
0.2
0.3
0.4
0
100
200
300
400
500
TH

Drawback of this estimator

Epps effect

- Systematic bias for this estimator.
- Example : Assume that $\log X$ and $\log Y$ are two Brownian motions with correlation ρ and that the trade times are arrival times of two independent Poisson processes, then one can show that

$$\mathbb{E}[V_h] \to 0, \text{ as } h \to 0.$$

A convergent estimator under asynchronicity

Hayashi-Yoshida Estimator

- Let $I_i^X = (T^{X,i}, T^{X,i+1}]$ and $I_i^Y = (T^{Y,i}, T^{Y,i+1}]$
- The Hayashi-Yoshida estimator is

$$U_n = \sum_{i,j} \Delta X(I_i^X)\Delta Y(I_j^Y)\mathbf{1}_{\{I_i^X \cap I_j^Y \neq \varnothing\}}.$$

- This estimator does not need any selection of h and is convergent if the arrival times are independent from the price.
- Nevertheless, it is not robust to microstructure effects.

Theorem

In the model with uncertainty zones, the Hayashi-Yoshida estimator is a consistent estimator of the covariation provided one uses the estimated values of the efficient prices.

Monte Carlo

- 1000 simulations,
- Black-Scholes model with U-shaped volatility,
- Constant covariation$= 4.5 \times 10^{-5}$ ($\rho = 0.4$).
-

	Hayashi-Yoshida	New Estimator
Mean	8.55×10^{-5}	4.48×10^{-5}
Standard Deviation	7.8×10^{-6}	5.1×10^{-6}

Outline

Motivation

Observation from practitioners in finance

- Some assets are leading some other assets.
- This means that a "lagger" asset may partially reproduce the behavior of a "leader" asset.
- This common behavior is unlikely to be instantaneous. It is subject to some time delay called "lead-lag".

A toy model for Lead-Lag

Bachelier model

- For $t \in [0,1]$, and $(B^{(1)}, B^{(2)})$ such that $\langle B^{(1)}, B^{(2)} \rangle_t = \rho t$, set

$$X_t := x_0 + \sigma_1 B_t^{(1)}, \;\; \widetilde{Y}_t := y_0 + \sigma_2 B_t^{(2)},$$

- Define $Y_t := \widetilde{Y}_{t-\theta}$, $t \in [\theta, 1]$. Our lead-lag model is given by the bidimensional process (X_t, Y_t).
- We have

$$\begin{cases} X_t &= x_0 + \sigma_1 B_t^{(1)} \\ Y_t &= y_0 + \rho\,\sigma_2 B_{t-\theta}^{(1)} + \sigma_2 (1-\rho^2)^{1/2}\, W_{t-\theta} \end{cases} .$$

Estimation idea (1)

- Assume the data arrive at regular and synchronous time stamps in the Bachelier model, *i.e.* we have data

$$(X_0, Y_0), (X_{\Delta_n}, Y_{\Delta_n}), (X_{2\Delta_n}, Y_{2\Delta_n}), \dots, (X_1, Y_1),$$

  and suppose $\theta = k_0 \Delta_n$, $k_0 \in \mathbb{Z}$.
- Let

$$\mathcal{C}_n(k) := \sum_i \big(X_{i\Delta_n} - X_{(i-1)\Delta_n}\big)\big(Y_{(i+k)\Delta_n} - Y_{(i+k-1)\Delta_n}\big).$$

Intuitive estimator in the Bachelier model (2)

Estimation idea (2)

- Heuristically, we have

$$\mathcal{C}_n(k) \approx \Delta_n^{-1}\mathbb{E}\big[(X_{\cdot} - X_{\cdot-\Delta_n})(Y_{\cdot+k\Delta_n} - Y_{\cdot+(k-1)\Delta_n})\big] + \Delta_n^{1/2}\xi^n.$$

- Moreover,

$$\Delta_n^{-1}\mathbb{E}\big[(X_{\cdot} - X_{\cdot-\Delta_n})(Y_{\cdot+k\Delta_n} - Y_{\cdot+(k-1)\Delta_n})\big] = \begin{cases} 0 & \text{if} \quad k \neq k_0 \\ \rho\,\sigma_1\sigma_2 & \text{if} \quad k = k_0. \end{cases}$$

- Thus we can (asymptotically) detect the value k_0 that defines θ in the very special case $\theta = k_0\Delta_n$ by maximizing in k the contrast sequence

$$k \rightsquigarrow \big|\mathcal{C}_n(k)\big|.$$

The lead-lag model

Let $\theta > 0$ (for simplicity, extensions are quite straightforward) and set $\mathbb{F}^\theta = (\mathcal{F}_t^\theta)_{t\geq 0}$, with $\mathcal{F}_t^\theta = \mathcal{F}_{t-\theta}$.

Assumptions

- We have
$$X = X^c + A, \;\; Y = Y^c + B.$$
- $(X_t^c)_{t\geq 0}$ is a continuous $\mathbb{F}$-local martingale, and $(Y_t^c)_{t\geq 0}$ is a continuous $\mathbb{F}^\theta$-local martingale.
- $\exists v_n \to 0,\; v_n^{-1} \max\big\{\sup\{|I_i^X|\}, \sup\{|I_i^Y|\}\big\} \to 0.$
- The $T^{X,i}$ are $\mathbb{F}^{v_n}$-stopping times and the $T^{Y,i}$ are $\mathbb{F}^{\theta+v_n}$-stopping times.

Estimation strategy

Estimator

- We set

$$U_n(\theta) = \sum_{i,j} \Delta X(I_i^X)\Delta Y(I_j^Y)1_{\{I_i^X \cap (I_j^Y)_{-\theta} \neq \varnothing\}},$$

  with $(I_j^Y)_{-\theta} = (T^{Y,j} - \theta, T^{Y,j+1} - \theta]$.

- Eventually, $\widehat{\theta}_n$ is defined as a solution of

$$\left| U_n(\widehat{\theta}_n) \right| = \max_{\theta \in \mathcal{G}^n} \left| U_n(\theta) \right|,$$

  where $\mathcal{G}^n$ is a sufficiently fine grid.

Theorem

As $n \to \infty$,

$$v_n^{-1}\left(\widehat{\theta}_n - \theta\right) \to 0,$$

in probability, on the event $\left\{\langle X^c, \widetilde{Y}^c\rangle_T \neq 0\right\}$.

Idea of proof

We show that

- If we compute the contrast function over points θ_n of the grid $\mathcal{G}^n$ such that $|\theta_n - \theta|$ is bigger than v_n, then the contrast function goes to zero.
- If we compute the contrast function over points θ_n of the grid $\mathcal{G}^n$ such that $|\theta_n - \theta|$ is sufficiently small then the contrast function goes to the covariation between X and $\widetilde{Y}$.

Synchronous case

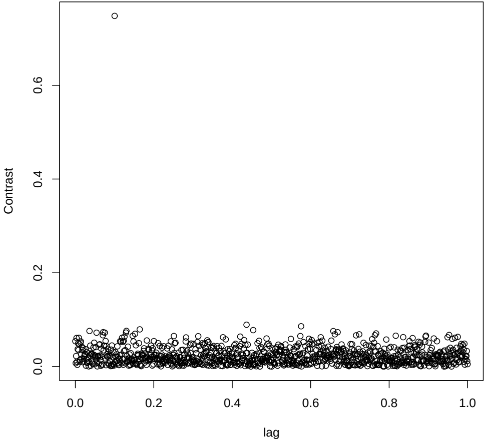
Setup

- We consider 300 simulations of the Bachelier model with synchronous equispaced data with period Δ_n.
- $t \in [0,1],\ \theta = 0.1,\ x_0 = \widetilde{y}_0 = 0,\ \sigma_1 = \sigma_2 = 1.$
- The mesh size of the grid h_n satisfies $h_n = \Delta_n$.
- We consider the following variations :
  - Mesh size : $h_n \in \{10^{-3}(FG), 3.\,10^{-3}(MG), 6.\,10^{-3}(CG)\}$.
  - Correlation value : $\rho \in \{0.25, 0.5, 0.75\}$.

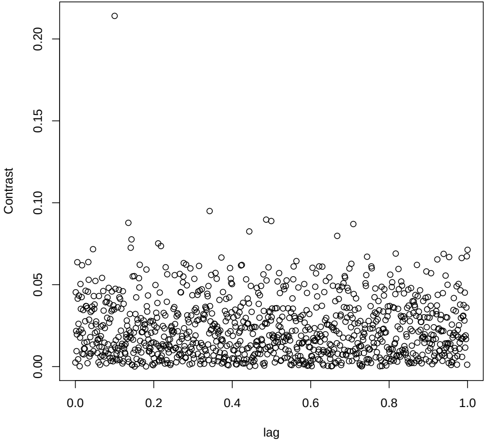
$\widehat{\theta}_n$	0.096	0.099	0.1	0.102	Other
FG, $\rho = 0.75$	0	0	300	0	0
MG, $\rho = 0.75$	0	300	0	0	0
CG, $\rho = 0.75$	1	0	0	299	0
FG, $\rho = 0.50$	0	0	300	0	0
MG, $\rho = 0.50$	0	299	0	1	0
CG, $\rho = 0.50$	13	0	0	280	7
FG, $\rho = 0.25$	0	0	300	0	0
MG, $\rho = 0.25$	0	152	0	11	137
CG, $\rho = 0.25$	10	0	0	66	124

Table 1 : *Estimation of* $\theta = 0.1$ *on 300 simulated samples for* $\rho \in \{0.25, 0.5, 0.75\}$.

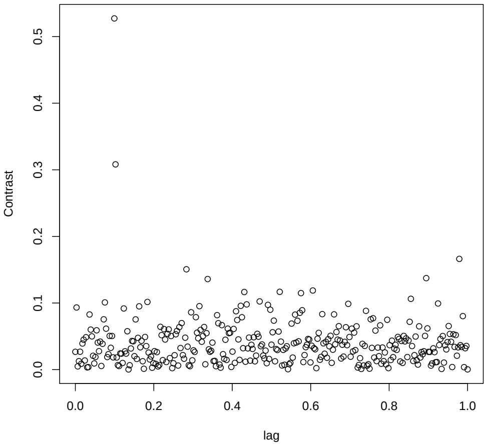
One sample path, FG, $\rho = 0.75$



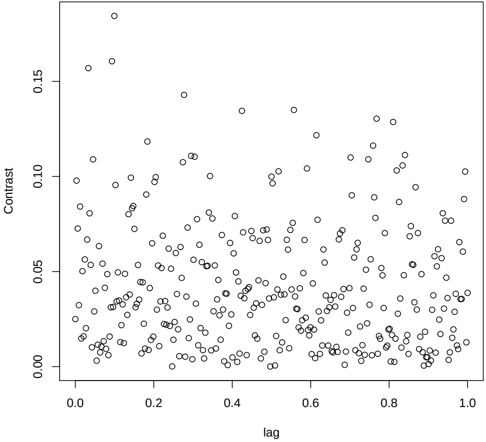
One sample path, FG, $\rho = 0.25$



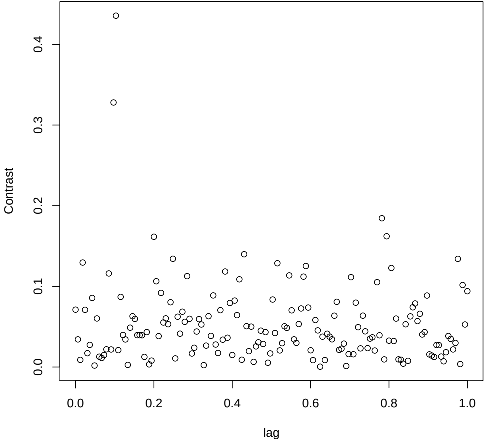
One sample path, MG, $\rho = 0.75$



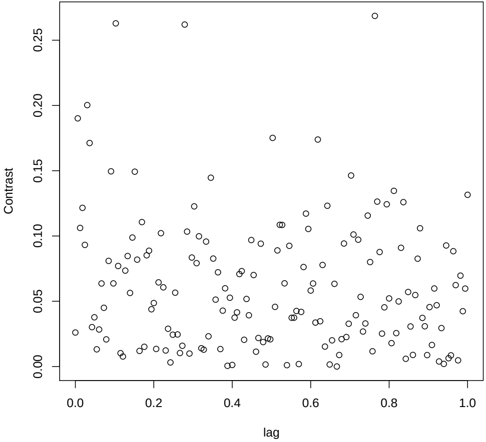
One sample path, MG, $\rho = 0.25$



One sample path, CG, $\rho = 0.75$



One sample path, CG, $\rho = 0.25$



Non synchronous case

Setup

- We randomly pick 300 sampling times for X over $[0, 1]$, uniformly over a grid of mesh size 10^{-3}.
- We randomly pick 300 sampling times for Y likewise, and independently of the sampling for X.
- Fine grid case, with $\theta = 0.1$ and $\rho = 0.75$.

$\widehat{\theta}$	0.099	0.1	0.101	0.102	0.103	0.104	0.105
FG, $\rho = 0.75$	16	106	107	46	19	4	2

Table 2 : *Estimation of* $\theta = 0.1$ *on 300 simulated samples for* $\rho = 0.75$ *and non-synchronous data.*

Financial data

Dataset

We study here the lead-lag relationship between the two following assets :

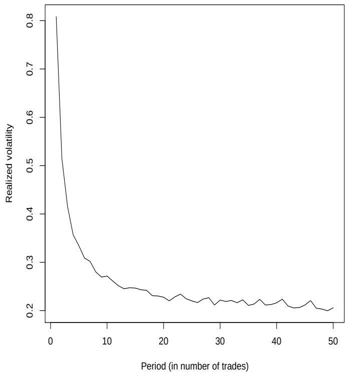
- The future contract on the DAX index, with maturity December 2010,
- The Euro-Bund future contract, with maturity December 2010.

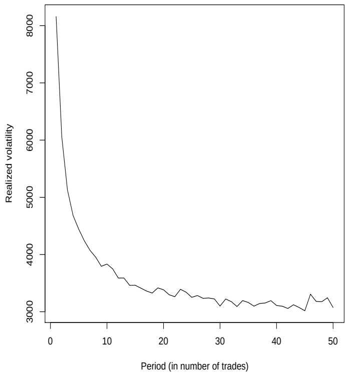
Dealing with microstructure effects

Methodology

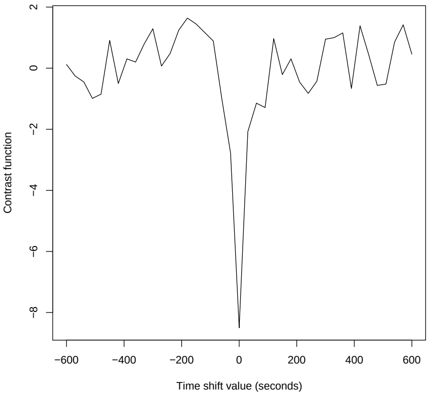
- We want to use high frequency data.
- First approach : use of the model with uncertainty zones.
- Here we just use signature plots in trading times. This enables to take advantage of non synchronous data.
- We keep one trade out of twenty.
- We then compute the function $U_n(\theta)$ over these trades.

Signature plots, October 13, for Bund (left) and FDAX (right).

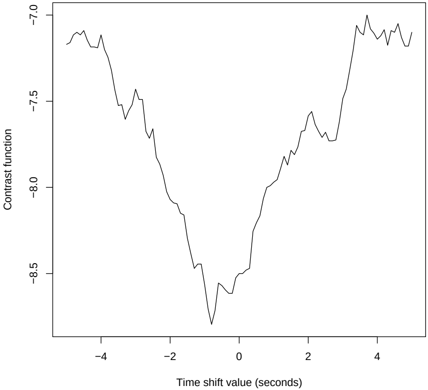




Function U^n, October 13, between -10 and 10 minutes, mesh=30 seconds



Function U^n, October 13, between -5 and 5 seconds, mesh=0.1 second



Jour	Vol.(Bund)	Vol.(FDAX)	LL.	J.	Vol.B.	Vol.F.	LL
1 Oct.	2847	4215	-0.2	18 Oct.	1727	2326	-2.1
5 Oct.	2213	3302	-1.1	19 Oct.	2527	3162	-1.6
6 Oct.	2244	2678	-0.1	20 Oct.	2328	2554	-0.5
7 Oct.	1897	3121	-0.5	21 Oct.	2263	3128	-0.1
8 Oct.	2545	2852	-0.6	22 Oct.	1894	1784	-1.2
11 Oct.	1050	1497	-1.4	25 Oct.	1501	2065	-0.4
12 Oct.	2265	3018	-0.8	26 Oct.	2049	2462	-0.1
13 Oct.	2018	3037	-0.8	27 Oct.	2606	2864	-0.6
14 Oct.	2057	2625	0.0	28 Oct.	1980	2632	-1.3
15 Oct.	2571	3269	-0.7	29 Oct.	2262	2346	-1.6