估值

一、拆分思想

$$\frac{2(x-1)}{x+1}<\ln x<\frac{1}{2}\left(x-\frac{1}{x}\right)<x-1\quad(x>1)$$

如 $\ln 2=\ln\frac{8}{4}=\ln\frac{5}{4}+\ln\frac{6}{5}+\ln\frac{7}{6}+\ln\frac{8}{7}<\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}=0.759\ldots$

即 $\ln 2=\ln\frac{2n}{n}=\ln\frac{n+1}{n}+\ln\frac{n+2}{n+1}+\cdots+\ln\frac{2n}{2n-1}<\frac{1}{n}+\frac{1}{n+1}+\cdots+\frac{1}{2n-1}.$

$\ln 2<\frac{1}{2}\left(2-\frac{1}{2}\right)=\frac{3}{4}$

$\ln 2=\ln\frac{4}{2}=\ln\frac{3}{2}+\ln\frac{4}{3}<\frac{1}{2}\left(\frac{3}{2}-\frac{2}{3}+\frac{4}{3}-\frac{3}{4}\right)=\frac{17}{24}=0.708\ldots$

$\ln 2=\ln\frac{6}{3}=\ln\frac{4}{3}+\ln\frac{5}{4}+\ln\frac{6}{5}<\frac{1}{2}\left(\frac{4}{3}-\frac{3}{4}+\frac{5}{4}-\frac{4}{5}+\frac{6}{5}-\frac{5}{6}\right)=\frac{7}{10}$

二、麦克劳林展开

$$f(x)=f(0)+\frac{f'(0)}{1!}x+\frac{f''(0)}{2!}x^2+\frac{f'''(0)}{3!}x^3+\frac{f^{(4)}(0)}{4!}x^4+\cdots$$

$$e^x=1+x+\frac{x^2}{2!}+\cdots+\frac{x^n}{n!}+o\left(x^n\right)$$

$$\sin x=x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\cdots+(-1)^n\frac{x^{2n+1}}{(2n+1)!}+o\left(x^{2n+2}\right)$$

$$tanx=x+\frac{x^3}{3}+\cdots$$

$$\cos x=1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\cdots+(-1)^n\frac{x^{2n}}{(2n)!}+o\left(x^{2n+1}\right)$$

$$\ln(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\cdots+(-1)^{n-1}\frac{x^n}{n}+o\left(x^n\right)$$

$$(1+x)^\alpha=1+\alpha x+\frac{\alpha(\alpha-1)}{2!}x^2+\cdots+\frac{\alpha(\alpha-1)\cdots(\alpha-n+1)}{n!}x^n+i2\left(x^n\right)$$

三、Pade逼近

$$e^x\sim\frac{x^2+6x+12}{x^2-6x+12}$$

$$lnx\sim\frac{3x^2-3}{6x^2-11x+11}$$