Homework Submission

CMDA 3605 Mathematical Modeling: Methods and Tools I

<u>**Virginia Tech Honor Code Pledge:**</u>

"I have neither given nor received unauthorized assistance on this assignment."

Solution to Exercise 1

(a) $x(k+1) =$

$$\begin{bmatrix} .8 & .1 & .1 \\ .1 & .7 & .2 \\ .1 & .2 & .7 \end{bmatrix} x(k), \text{ with } x(0) = x_0 = \begin{bmatrix} 60,000 \\ 150,000 \\ 90,000 \end{bmatrix}$$

(b) $x(1) = Ax(0) = \begin{bmatrix} .8 & .1 & .1 \\ .1 & .7 & .2 \\ .1 & .2 & .7 \end{bmatrix} \begin{bmatrix} 60000 \\ 150000 \\ 90000 \end{bmatrix} = \begin{bmatrix} 48000 + 15000 + 9000 \\ 6000 + 105000 + 18000 \\ 6000 + 30000 + 63000 \end{bmatrix} = \begin{bmatrix} 72,000 \\ 129,000 \\ 99,000 \end{bmatrix}$

$$x(2) = Ax(1) = \begin{bmatrix} .8 & .1 & .1 \\ .1 & .7 & .2 \\ .1 & .2 & .7 \end{bmatrix} \begin{bmatrix} 72000 \\ 129000 \\ 99000 \end{bmatrix} = \begin{bmatrix} 87600 + 12900 + 9900 \\ 7200 + 90300 + 19800 \\ 7200 + 25800 + 69300 \end{bmatrix} = \begin{bmatrix} 80,400 \\ 117,300 \\ 102,300 \end{bmatrix}$$

$$x(3) = Ax(2) = \begin{bmatrix} .8 & .1 & .1 \\ .1 & .7 & .2 \\ .1 & .2 & .7 \end{bmatrix} \begin{bmatrix} 80400 \\ 117300 \\ 102300 \end{bmatrix} = \begin{bmatrix} 64320 + 11730 + 10230 \\ 8040 + 82110 + 20460 \\ 8040 + 23460 + 71610 \end{bmatrix} = \begin{bmatrix} 86,280 \\ 110,610 \\ 103,110 \end{bmatrix}$$

Solution to Exercise 2

(a) $x(k+1) = \begin{bmatrix} F(k+1) \\ F(k+2) \end{bmatrix} = \begin{bmatrix} F(k+1) \\ F(k+1)+F(k) \end{bmatrix} = A \begin{bmatrix} F(k) \\ F(k+1) \end{bmatrix}$

$x(k+1) = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} x(k)$, with $x(0) = x_0 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

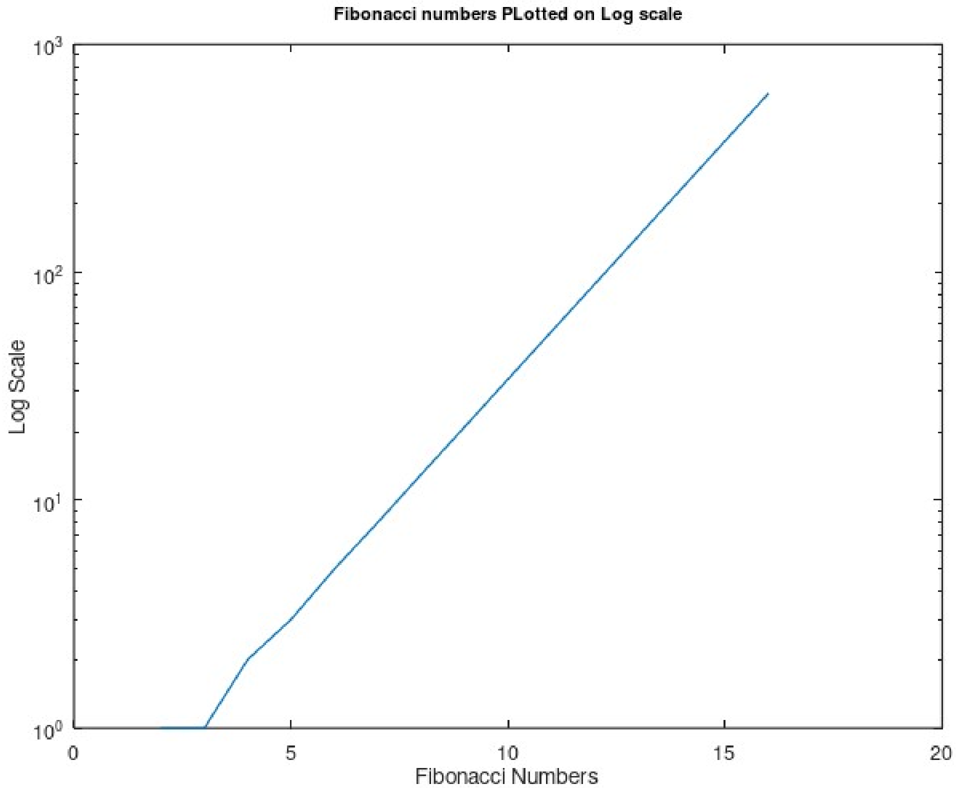
(b) The code for my simulation is below:

```
% Clean screen and workspace.
clc;
clear;
close all;

% Setup system components.
A  = [0,1;1,1];

x0 = [0;1];

% Compute simulation.
kmax    = 15;
X       = zeros(2, kmax+1);
X(:, 1) = x0;
for k = 1:kmax
    X(:, k+1) = A * X(:, k);
end

% Get first 16 numbers of the Fibonacci Sequence
Fib = X(1,1:16)

semilogy(1:16, Fib, '-')
title('Fibonacci numbers PLotted on Log scale');
xlabel('Fibonacci Numbers'); ylabel('Log Scale')

r = Fib(3:end) ./ Fib(2:end-1);        % ratios from F2/F1 onward
k_rat = 2:kmax;                        % aligns with r
figure; plot(k_rat, r, '-'); grid on
xlabel('k'); ylabel('F_{k+1} / F_k');
title('Consecutive Fibonacci ratios');
```

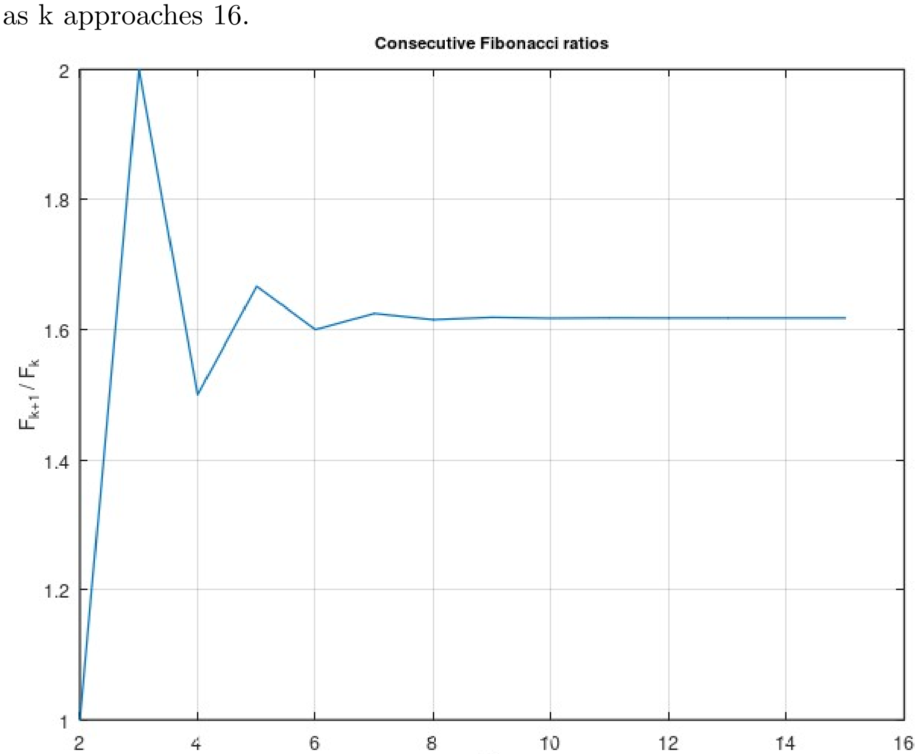
The output for the simulation is below:

```
Fib =

 Columns 1 through 15:

     0     1     1     2     3     5     8    13    21    34    55    89   144   233   377

 Column 16:

   610
```

Plot of the Fibonacci number with logarithmically scaled y-axis:



The image below shows the ratio between Fibonacci numbers Fib^{k+1}/Fib^{k}, or how the growth rate, as k approaches 16.



From the graph, the Fibonacci growth rate is approximately 1.6 in the long run. This also matches the dominant eigenvalue of 1.6180 returned from MATLAB.

if $F_{k+1} \approx 1.6F^k$

Then, $F_k \approx F_0 1.6^k$

Then taking the log gives us:

$\log(F_k) \approx \log(F_0) + k\log(1.6)$

The equation above approximately represents how the Fibonacci numbers are scaled on the why axis as k approaches 16. The equation is linear since the constant k is raised to the first power. This is why the plot of the Fibonacci numbers on a logarithmically scaled y-axis shows a straight line.

Also, $\lambda_d = 1.6180$ which is related to exponential growth.