Homework Submission

CMDA 3605 Mathematical Modeling: Methods and Tools I

Virginia Tech Honor Code Pledge:

"I have neither given nor received unauthorized assistance on this assignment."

Solution to Exercise 1

(a)

An annual rate of 5% compounded 12 times a year in Klaus' savings account. k = 240 when equation is in terms of months (20 years = 240 months).

$$
\begin{aligned}
x(0) &= 400{,}000 \\
x(t+1) &= x(t) + x(t)\left(\tfrac{0.05}{12}\right) \\
&= x(t)\left(1 + \tfrac{1}{240}\right) \\
x(240) &= \left(\tfrac{241}{240}\right)^{240}(400{,}000) \\
&= \$1{,}085{,}056.12
\end{aligned}
$$

(b)

$t = 1 - 10:$

$$
\begin{aligned}
x(0) &= 400{,}000 \\
x(t+1) &= x(t) + .07x(t) \\
&= 1.07x(t) \\
x(10) &= 1.07^{10}x(0) \\
&= 786860.542916
\end{aligned}
$$

$t = 1 - 5:$

$$
\begin{aligned}
x(0) &= 786860.542916 \\
x(t+1) &= x(t) - .06x(t) \\
x(t+1) &= .94x(t) \\
x(t) &= .94x(t-1) \\
x(5) &= .94^5x(0) \\
&= 577480.117514
\end{aligned}
$$

$t = 1 - 5:$

$$
\begin{aligned}
x(0) &= 577480.117514 \\
x(t+1) &= x(t) + .09x(t) = 1.09x(t) \\
x(t) &= 1.09^t x(0) \\
x(5) &= 1.09^5 x(0) \\
&= \$888{,}524.74
\end{aligned}
$$

The final amount Violet has after 20 years is \$888,524.74.

Solution to Exercise 2

Here is an example on how to use the `itemize` environment in the case of exercises with substructured parts:

(a) Here goes the response to Exercise 2, Part (a);

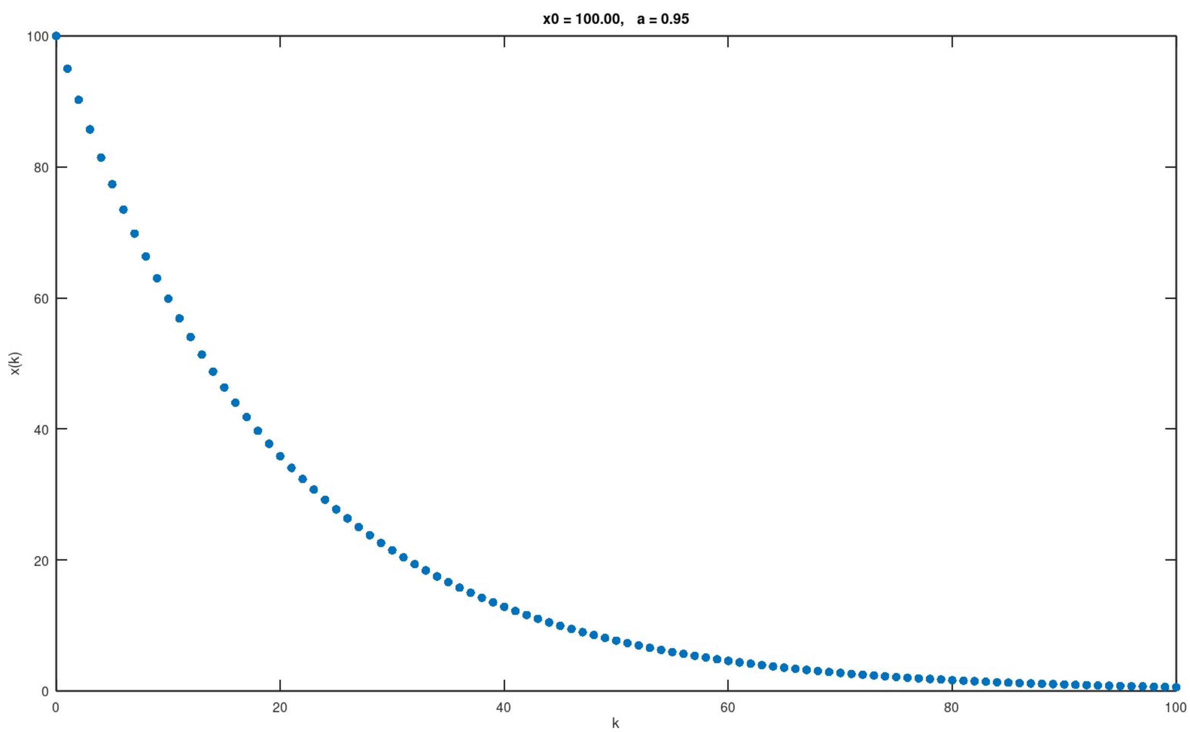
(b) and here to Part (b).

Solution to Exercise 3

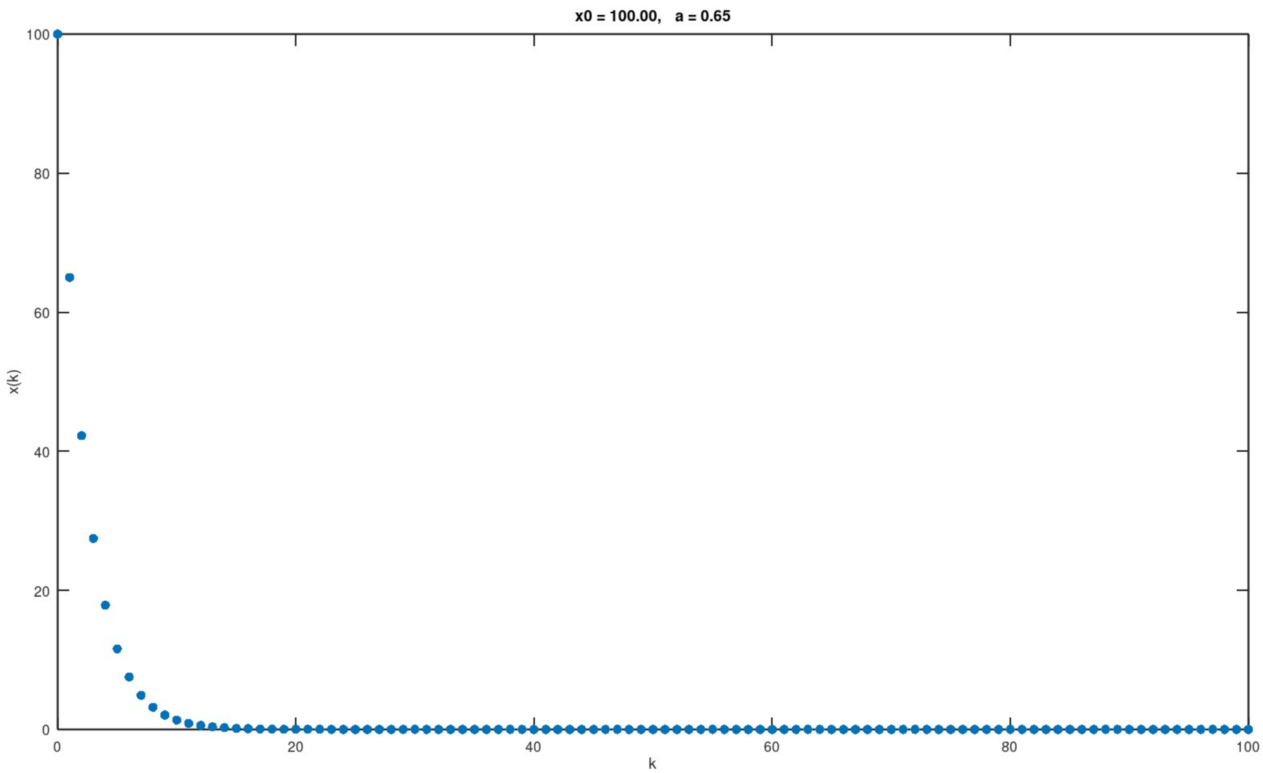
(a) According to the manual, the function eig *computes the eigenvalues (LAMBDA) and optionally the right eigenvectors (V) and the left eigenvectors (W) of a matrix or pair of matrices.*

Using the command 'help' opens up the manual for the function (eig) in the command prompt. Using the command 'doc' searches the manual index, which opens the documentation tab which has the same information. The docs view gives you access to the whole manual so that you can search for other functions or scroll to see related functions.

(b) This is my output simulating the difference equation $x(k+1) = 0.95x(k)$, with $x(0) = 1.00$, for $k = 0, 1, ..., 100$.



This is my output for my output simulating the difference equation $x(k+1) = 0.65x(k)$, with, $x(0) = 100.0$, for $k = 0, 1, ..., 100$.



The difference in behavior amoungst the two cases that I observed is that the second simulation converges to zero in much fewer steps. The model converges around $k = 10$, while the first model converges to zero closer to $k = 100$. The rate of change before converging is higher in the second model.