INSTITUTE OF TECHNOLOGY OF CAMBODIA

Department of Applied Mathematics and Statistics

Stochastic Linear Programming (SLP)

I3-AMS-A

Group 5

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3. Motivation of Method
5. Application of SLP

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6. Lesson learn
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8. Conclusion and Reference

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1. Introduction
2. Background or History

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Content

1 Introduction

- **Stochastic Linear Programming (SLP) extends traditional linear programming by addressing decision-making under uncertainty.**

Scenario:

- A bakery needs to decide how much cake to order tomorrow
- The goal is to minimize costs, but demand is uncertain.
- Sales may vary each day, and the bakery doesn't know exactly how many customers will buy cake.

2 Background or History

Background

- Early Optimization Models (Mid-20th Century): By George Dantzig
- Formation of Stochastic Programming (1950s-1960s)
- Growth and Development in the 1970s-1980s
- Applications of Stochastic Linear Programming
- Modern Era of SLP

3 Motivation of Method

Why Motivation Matters

- Real decisions ≠ perfect data
- Uncertainty is unavoidable

Limits of Traditional Linear Programming

- Fixed parameters
- No uncertainty
- Risky decisions

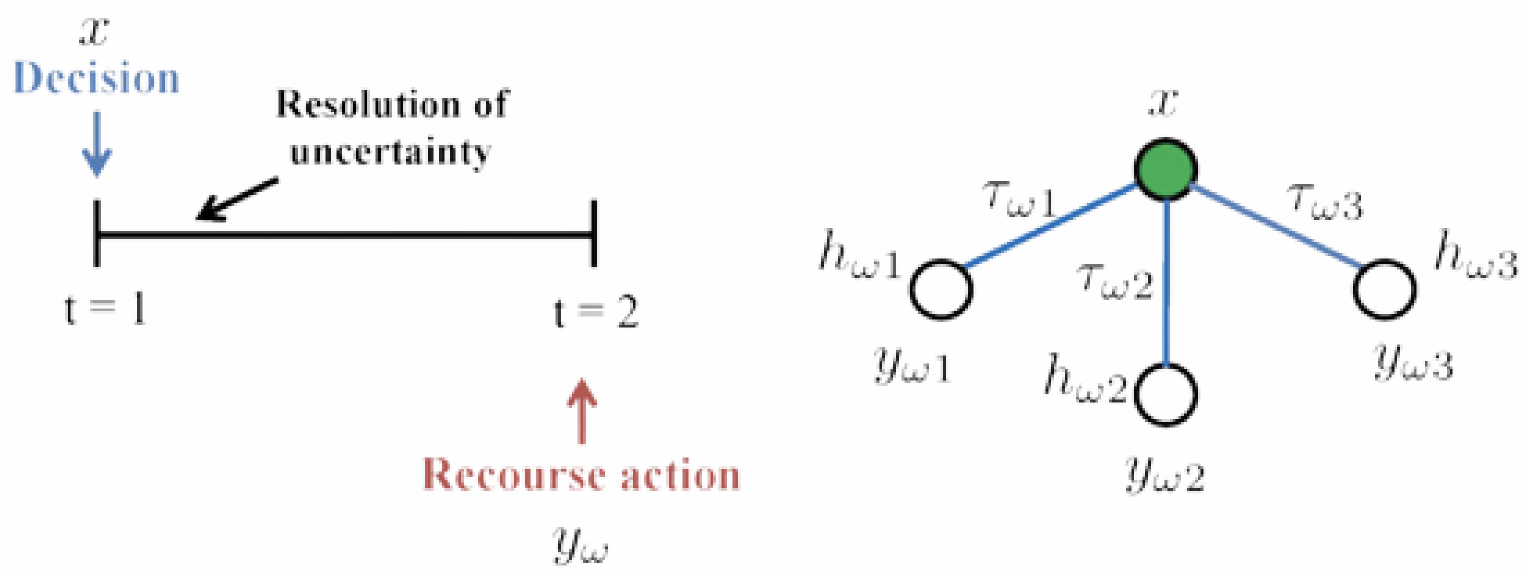
3 Motivation of Method

Why Stochastic Linear Programming

- Scenarios
- Probabilities
- Expected performance

4 Methodology of Stochastic Linear Programming

Stochastic Linear Programming models consist of decision variables, an objective function, and constraints, similar to linear programming. However, some parameters are random variables with known probability distributions.



Step1: Problem Formulation

Identify:

- Decision variables x1,x2,…,xn
- Objective function (maximize profit or minimize cost)
- Constraints
- Which parameters are random

Step2: Model the Uncertainty

Represent uncertain parameters as:

- Discrete scenarios with probabilities
  - Example: demand = {100, 150, 200} with probabilities {0.3, 0.4, 0.3}
- Or continuous distributions
  - Example: demand ~ Normal(150, 20^2)

4 Methodology of Stochastic Linear Programming

Step 3: Choose a Stochastic Programming Structure

Most common: Two-Stage Stochastic Linear Programming

Stage 1: "Here-and-now" decisions

Decide before knowing the future

$$x = \text{production today}$$

Stage 2: "Recourse" decisions

Adjust after uncertainty is revealed

$$y(\omega) = \text{extra production / shortage handling}$$

Step4: Convert to Deterministic Equivalent

Transform the stochastic problem into a large deterministic LP by:

- Listing all scenarios
- Weighting them by probabilities
- Adding recourse variables

$$\max \ c^T x + \sum_{s=1}^{S} p_s q_s^T y_s$$

subject to:

$$Ax \leq b$$

$$T_s x + W_s y_s = h_s \quad \forall s$$

4 Methodology of Stochastic Linear Programming

Step5: Solve Using Linear Programming

Solve using:

- Simplex Method (as Linear Programming Problem)
- Algebraic Method
- L-Shape Method
- Solver (Excel, Python, Gurobi, CPLEX, PuLP)

Step 6: Evaluate the Solution

The Solution:

- Expected profit / cost
- Risk (variance, CVaR)
- Sensitivity analysis

Example: Stochastic Linear Programming

Problem

A bakery must decide how many cakes to bake before knowing tomorrow's demand.

- Cost to bake a cake: $4
- Selling price: $8
- If demand is higher than production → lost sales
- If production is higher than demand → leftover sold at $2

Scenario	Demand	Probability
Low	80	0.3
Medium	120	0.5
High	160	0.2

Step 1: Decision Variable

x=number of cakes baked (Stage 1)

Step 2: Recourse Variables

For each scenario :

- y_s^+ = shortage
- y_s^- = leftover

Step 3: Stochastic Model

$$\max \; 8x - 4x + \sum_s p_s(2y_s^- - 8y_s^+)$$

subject to:

$$x + y_s^- - y_s^+ = d_s \quad \forall s$$

$$x, y_s^+, y_s^- \geq 0$$

Step 4: Deterministic Equivalent

$$\max \; 4x + 0.3(2y_L^- - 8y_L^+) + 0.5(2y_M^- - 8y_M^+) + 0.2(2y_H^- - 8y_H^+)$$

subject to:

$$x + y_L^- - y_L^+ = 80$$

$$x + y_M^- - y_M^+ = 120$$

$$x + y_H^- - y_H^+ = 160$$

$$x, y_s^+, y_s^- \geq 0$$

- Cost to bake a cake: $4
- Selling price: $8
- If demand is higher than production → lost sales
- If production is higher than demand → leftover sold at $2

Scenario	Demand	Probability
Low	80	0.3
Medium	120	0.5
High	160	0.2

Step 5: Solve the Stochastic Model (Algerbraic Method)

Expected Profit: $$E[Profit] = 4x + 0.3(2y_L^- - 8y_L^+) + 0.5(2y_M^- - 8y_M^+) + 0.2(2y_H^- - 8y_H^+)$$

Where: $$y^- = \max(x - d, 0), \quad y^+ = \max(d - x, 0)$$

Case 1: $x \leq 80$

Shortage in all scenarios:

$$E[Profit] = 4x - 8(0.3(80 - x) + 0.5(120 - x) + 0.2(160 - x))$$

$$= 4x - 8(116 - x) = 12x - 928$$

Max at $x = 80$:

$$Profit = 12(80) - 928 = 32$$

Case 2: $80 \leq x \leq 120$

$$E[Profit] = 4x + 0.3(2(x - 80)) - 0.5(8(120 - x)) - 0.2(8(160 - x))$$

$$= 4x + 0.6(x - 80) - 4(120 - x) - 1.6(160 - x)$$

$$= 10.2x - 784$$

Max at $x = 120$:

$$Profit = 10.2(120) - 784 = 440$$

Case 3: $120 \leq x \leq 160$

$$E[Profit] = 4x + 0.3(2(x - 80)) + 0.5(2(x - 120)) - 0.2(8(160 - x))$$

$$= 4x + 0.6(x - 80) + 1(x - 120) - 1.6(160 - x)$$

$$= 7.2x - 424$$

Max at $x = 160$:

$$Profit = 7.2(160) - 424 = 728$$

Case 4: $x \geq 160$

$$E[Profit] = 4x + 0.3(2(x - 80)) + 0.5(2(x - 120)) + 0.2(2(x - 160))$$

$$= 6x - 184$$

This increases → but producing more than 160 only creates waste, so **best is x = 160**

Step 6: Final Answer

Optimal Production: $x^* = 160 \text{ cakes}$

Maximum Expected Profit: $E[Profit] = \$728$

Interpretation

The bakery should bake 160 cakes before demand is known.

Why?

Because:

- High demand (120 & 160) is more likely (70%)
- Shortage is very expensive
- Overproduction still gives salvage value

So the stochastic model pushes production higher than the average demand.

Simplex Method

Objective Function

$$\max Z = 4x + 0.3(2y_L^- - 8y_L^+) + 0.5(2y_M^- - 8y_M^+) + 0.2(2y_H^- - 8y_H^+)$$

Constraints

$$x + y_L^- - y_L^+ = 80$$

$$x + y_M^- - y_M^+ = 120$$

$$x + y_H^- - y_H^+ = 160$$

$$x, y_s^-, y_s^+ \geq 0$$

This is a linear program with 7 variables and 3 equality constraints.

L-Shape Method

Step Problem Structure

First-stage problem:

$$\max 4x + \theta$$

where θ represents **expected recourse profit**.

Second-stage (scenario subproblems):

For each scenario s:

$$Q(x, \omega_s) = \max 2y_s^- - 8y_s^+$$

subject to:

$$x + y_s^- - y_s^+ = d_s$$

Step 2: Initial Master Problem

Start with **no information about recourse**:

$$\max 4x + \theta$$

$$\theta \leq 0$$

$$x \geq 0$$

Additional: Machine Learning Models

The L-shaped method is used when the ML problem involves uncertainty or expected loss.

1. Stochastic Linear Regression

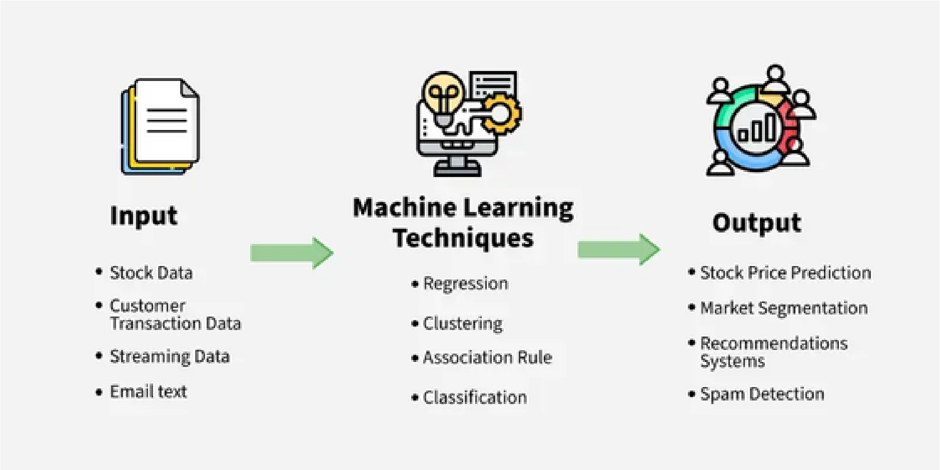
2. Risk-Averse Machine Learning (CVaR Models)

Idea

- Inputs or outputs are random
- Minimize expected error

Used in:

- Finance ML
- Fairness-aware ML
- Robust prediction



➡ Converted to two-stage stochastic LP

➡ Solved using L-shaped method

➡ CVaR reformulated as stochastic LP

➡ Solved by L-shaped / Benders decomposition

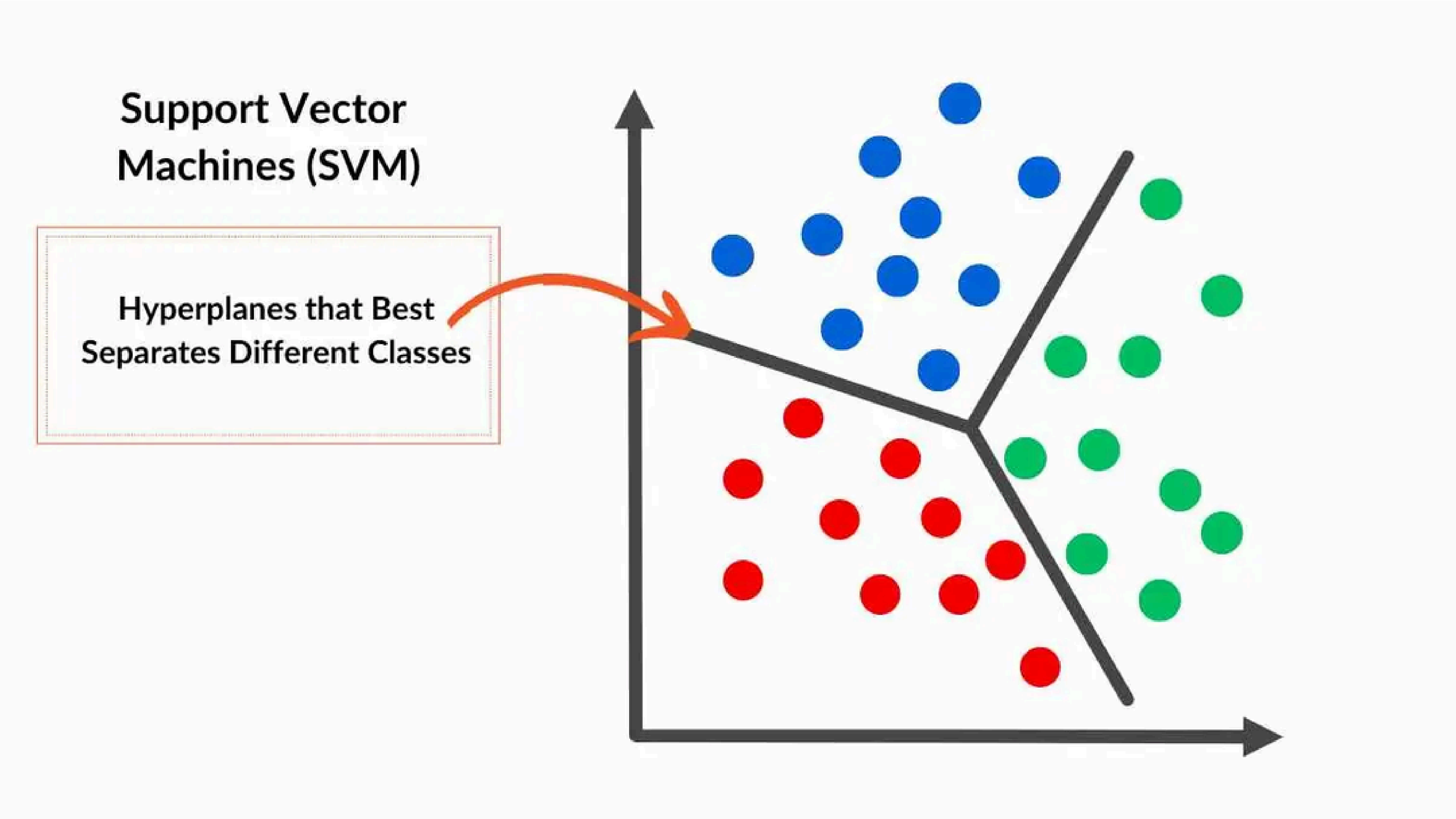
Additional: Machine Learning Models

The L-shaped method is used when the ML problem involves uncertainty or expected loss.

3. Robust Support Vector Machines

Why L-shaped?

- Training data uncertainty
- Worst-case or expected-case optimization



➡ Two-stage stochastic optimization

➡ L-shaped method applies

5 Application of SPL

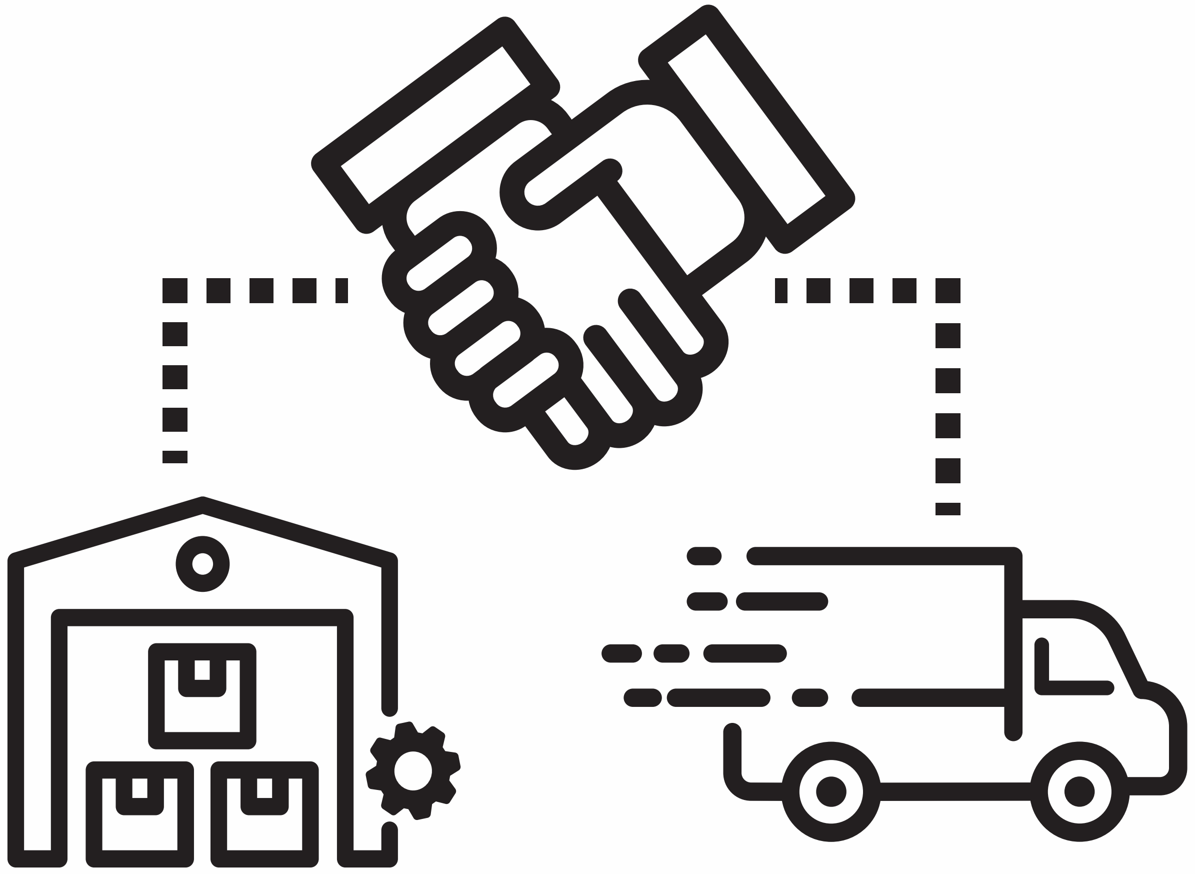
Where Is SLP Used?

- Supply Chain
- Energy Planning
- Finance
- Agriculture etc..

5 Application of SPL

Supply Chain Example

- Uncertain demand
- Production decision
- Scenario-based planning



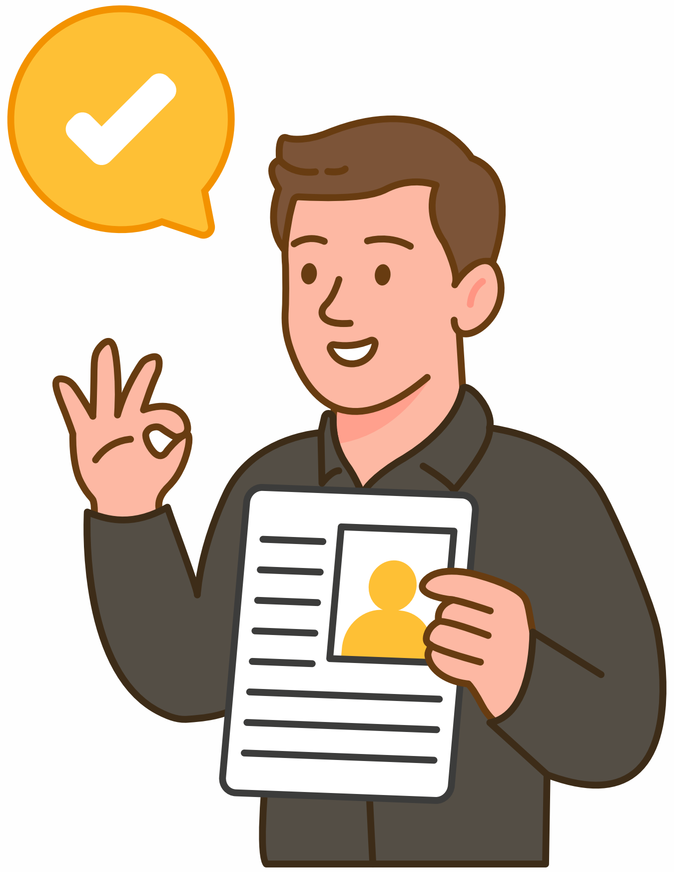
5 Application of SPL

Energy Planning Example

- Demand uncertainty
- Renewable energy variability
- Cost minimization

Finance Application (SLP)

- Uncertain market returns
- Bad market / Good market
- Scenario-based investment



6 Lesson learn

- **The Importance of Modeling Uncertainty Properly**
  - Accurate Decision-Making
  - Risk Management
  - Flexibility in Recourse Action
  - Optimizing Resource Allocation
- **Computational Complexity and Solution Methods**

  Problem
  - Number of Scenario
  - Recourse Decision
  - Multi-Stage Decision

  Solution Methods
  - Decomposition methods
  - Sample Average Approximate (SAA)
  - Stochastic Programming via Approximation
  - Parallel Computing

7 Challenges in SLP

- **High Dimensionality of the Problem**

As the number of scenarios increases, the number of decision variables and constraints can grow exponentially. This leads to curse of dimensionality, where the problem becomes intractable for large-scale instances.

- **Scenario Generation and Representation**
  - A poorly structure of scenario tree can lead to misleading results.
  - Overfitting and underfitting the uncertainty can distort the optimal solution.

- **Real-Time Decision Making**
  - SLP model solves the problem in a batch mode, but real world applications often require real-time decision-making.
  - Real-time decision-making relies on immediate, accurate and complete data.
  - Require fast optimization, challenge for large-scale SLP model.
- **Integration with Existing System**
  - Older system may not support SLP model.
  - Disconnected data sources can cause delay or inaccuracies.
  - Systems may struggle to scale with growing complexity.

8 Conclusion and Reference

- Uncertainty is unavoidable in real problems
- SLP models uncertainty using scenarios
- Supports better and more robust decisions
- Widely applied in practice

Reference

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Thank You