

**Directions**

Scan and upload your *handwritten* solutions to eLearning by the end of the day on Monday, April 25. STAT functions are permitted for all problems where applicable but be sure to show organized steps of hypothesis testing. You may use either rejection regions or  $p$ -values (your choice).

**Problem 1 (2 points)**

An election was challenged in court because there was suspicion of fraudulent use of absentee ballots. The judge had to decide whether to overturn the election and remove the winner from office, and would do so if he believed the hypothesis that fraud was involved.

- (a) State the hypotheses of the test.

$H_0$  : No fraud involved vs.  $H_a$  : Fraud involved

- (b) Expert witnesses presented statistical evidence that the  $P$ -value for testing these hypotheses is 0.06. The judge decided that fraud was involved and overturned the results of the election. Did the judge use a level of significance of  $\alpha = 0.05$ ? Explain how you know and then state the level of significance used by the judge as an inequality.

No. We were told that the  $p$ -value was .06, yet the judge rejected the null hypothesis and accepted the alternative hypothesis. We reject  $H_0$  when  $p \leq \alpha$ . So then

$$0.06 \leq \alpha \implies \alpha \text{ must be at least } 0.06.$$

$\alpha \geq 0.06$  or  $\alpha > 0.06$  are both acceptable.

**Problem 2 (2 points)**

The Harris Poll conducted a survey to determine how many tattoos people have on their bodies. Of the 1205 males surveyed, 181 had at least one tattoo. Of the 1097 females surveyed, 143 had at least one tattoo.

- (a) A tattoo artist believes that only 10% of females have at least one tattoo. At  $\alpha = 0.07$ , test their claim using an appropriate significance test. (Note: You do not need data for the males in this part.)

Step	Value
Hypotheses	$H_0 : p = 0.10$ vs. $H_a : p \neq 0.10$
Test Statistic	$z = \frac{0.1304 - 0.10}{\sqrt{\frac{0.1 \times 0.9}{1097}}} \approx 3.4$
$P$ -value	$p \approx 0.0008$ or $0.001$
Decision	Reject $H_0$ since $0.001 < 0.07$

Conclude that the proportion of females with at least one tattoo is not 10% at  $\alpha = 0.07$ .

- (b) Determine whether the proportion of males with at least one tattoo differs from the proportion of females with at least one tattoo using an appropriate significance test at  $\alpha = 0.05$ .

Step	Value
Hypotheses	$H_0 : p_x = p_y$ vs. $H_a : p_x \neq p_y$
Test Statistic	$z \approx 1.37$
$P$ -value	$p \approx 0.171$
Decision	Accept $H_0$ since $0.171 > 0.05$

Conclude there is no difference between the proportions of males and females with at least one tattoo at  $\alpha = 0.05$ .

### Problem 3 (2 points)

Quiz scores for students A and B are given below in the table. At  $\alpha = 0.05$ , determine if the mean grades differ between the two students.

Student	Quiz 1	Quiz 2	Quiz 3	Quiz 4	Quiz 5	Quiz 6
A	85	92	97	65	75	96
B	81	79	76	84	83	77

First, you must determine if you can assume the variances are the same. The sample variances are  $s_A = 12.76$  and  $s_B = 3.22$  so the numerator term should be with respect to student A in the two-sample  $F$  test.

Step	Value
Hypotheses	$H_0 : \sigma_A^2 = \sigma_B^2$ vs. $H_a : \sigma_A^2 > \sigma_B^2$
Test Statistic	$F = \frac{12.76^2}{3.22^2} \approx 15.7$
$p$ -value	$p \approx 0.0045$
Decision	Reject $H_0$ since $p < \alpha$

Conclude there is evidence that the variances are not equal. Next, set Pooled = NO and perform a 2 sample  $t$ -test to determine if their means are different:

Step	Value
Hypotheses	$H_0 : \mu_A = \mu_B$ vs. $H_a : \mu_A \neq \mu_B$
Test Statistic	$t \approx 0.9306$
$p$ -value	$p \approx 0.39$
Decision	Accept $H_0$ since $p > \alpha$

Conclude that their means could be the same. There is no evidence at  $\alpha = 0.05$  that their mean grades differ.

### Problem 4 (2 points)

Zocor is a drug meant to reduce LDL (bad) cholesterol and increase HDL (good) cholesterol. In clinical trials, patients were divided into three groups (Group 1 = Zocor, Group 2 = placebo, and Group 3 = a competing drug). The table contains the number of patients in each group who did or did not experience abdominal pain as a side effect.

	Group A	Group B	Group C	TOTAL
Experienced Pain	51 (59.4)	5 (5.9)	16 (6.7)	72
Experienced No Pain	1532 (1523.6)	152 (151.1)	163 (172.3)	1847
TOTAL	1583	157	179	1919

TABLE 1. All expected values are  $\geq 5$ .

Step	Value
Hypotheses	$H_0$ : Drug group and side effect are independent $H_a$ : otherwise
Test Statistic	$\chi^2_{obs} \approx 14.79$
Degress of freedom	$\nu = (3 - 1)(2 - 1) = 2$
P-value	$p \approx 0.0006$
Decision	Reject $H_0$ since $P < \alpha$

- (a) At  $\alpha = 0.01$ , determine if drug group and side effect are independent. (Show how you computed the expected values.)

The first expected value is computed as  $E = \frac{72 \times 1583}{1919} \approx 59.4$ . The rest are similar.

Conclude that drug group and side effect are not independent at  $\alpha = 0.01$  since we reject the null hypothesis.

- (b) What does your result from (a) indicate about the homogeneity of the three groups? We would reject the null hypothesis that the group proportions are equal. Thus, the groups are not homogeneous. (Note: You did not need to compute anything here. The test is the same as (a) but with different hypotheses.)

## Problem 5 (2 points)

Each day a baker bakes three large cakes. The number of cakes sold per day (0, 1, 2, or 3) over a period of 300 days are summarized in the first two columns of the table below.

Cakes sold	Days	Probability	Expected Value
0	1	$\text{binompdf}(3, 0.85, 0) = 0.003375$	$300 \times 0.003375 = 1.0125$
1	16	$\text{binompdf}(3, 0.85, 1) = 0.057375$	$300 \times 0.057375 = 17.2125$
2	55	$\text{binompdf}(3, 0.85, 2) = 0.325125$	$300 \times 0.325125 = 97.5375$
3	228	$\text{binompdf}(3, 0.85, 3) = 0.614125$	$300 \times 0.614125 = 184.2375$

\*\*\*\*\* Combine the first two rows since the expected value in the first row is less than 5!!! \*\*\*\*\*

Cakes sold	Days	Probability	Expected Value
0-1	17	0.06075	18.225
2	55	0.325125	97.5375
3	228	0.614125	184.2375

Use an appropriate test at  $\alpha = 0.05$  to determine whether the data may be from a binomial distribution with success probability  $p = 0.85$ . Show how you compute the probabilities and expected values.

Conclude that the data are not binomial with success probability  $p = 0.85$  at  $\alpha = 0.05$ .

Step	Value
Hypotheses	$H_0$ : The data are binomial with success probability $p = 0.85$ . $H_a$ : otherwise
Test Statistic	$\chi^2_{obs} \approx 29.03$
Degress of freedom	$\nu = 3 - 1 = 2$
$P$ -value	$p \approx 0.0000005$
Decision	Reject $H_0$ since $P < \alpha$