Directions

Scan and upload your *handwritten* solutions to eLearning by the end of the day on **Monday, April 3**. Show sufficient work or credit may not be given.

Part 1: Show all work 'by hand'. No STAT functions are permitted.

Problem 1 (3 points @ 1.5 points each)

In a previous class survey, 112 out of 190 students said they do not currently have a job.

(a) Assuming these results are from a random sample, construct a 95% confidence interval to estimate the proportion of students that do not currently have a job. Write a sentence to summarize your findings.

Let $\hat{p} = 112/190 \approx 0.589$, $z_c = 1.96$, and $n = 190$. Then, the 95% confidence interval for p is

$$0.589 \pm 1.96\sqrt{\frac{0.589(1-0.589)}{190}} \approx [0.52, 0.66]$$

We are 95% confident that true proportion of students that do not currently have a job is between 52% and 66%.

(b) What sample size is needed to construct a 99% confidence interval for the proportion of students that do not currently have a job if we want our estimate to be within 4% of the true proportion and we know from a previous survey that this proportion was estimated to be 0.62?

Let $z_c = 2.576, \Delta = 0.04, \hat{p} = 0.62$. Then, the minimum sample size required is

$$n \geq 0.62(1-0.62)\left(\frac{2.576}{0.04}\right)^2$$

$$n \geq 977.1$$

Therefore, we need at least 978 students.

Problem 2 (5 points @ 2.5 points each)

Let $X_1, X_2, \ldots, X_n$ have the pdf

$$f(x|\theta) = \theta x^{\theta-1}$$

for $0 < x \leq 1$ and $\theta > 0$. Let $X_1, X_2, \ldots, X_n$ have the pdf

$$f(x|\theta) = \theta x^{\theta-1}$$

for $0 < x \leq 1$ and $\theta > 0$.

(a) Derive the method of moments estimator (MOME) for θ.

Let $m_1 = \overline{x}$ and $\mu_1 = \int_0^1 x \cdot \theta x^{\theta-1}\, dx = \left[\frac{\theta}{\theta+1} x^{\theta+1}\right]_0^1 = \frac{\theta}{\theta+1}$

Next, equate the empirical and theoretical moments and solve for θ :

$$\begin{aligned} m_1 &= \mu_1 \\ \overline{x} &= \frac{\theta}{\theta+1} \\ \theta &= \frac{\overline{x}}{1-\overline{x}} \end{aligned}$$

$$\therefore \boxed{\hat{\theta}_{MOME} = \frac{\overline{x}}{1-\overline{x}}}$$

(b) Derive the maximum likelihood estimator (MLE) for θ.

Step 1: Likelihood Function $L(\theta) = \prod_{i=1}^{n} \theta x_i^{\theta-1} = \theta^n \left(\prod_{i=1}^{n} x_i\right)^{\theta-1}$

Step 2: Log-likelihood: $\log L(\theta) = n \log \theta + (\theta - 1)\left(\sum_{i=1}^{n} \log x_i\right)$

Step 3: Partial-Derivative wrt θ : $\frac{\partial}{\partial \theta} \log L(\theta) = \frac{n}{\theta} + \sum_{i=1}^{n} \log x_i = 0$

$\Longrightarrow$ Critical point: $\theta = -\frac{n}{\sum_{i=1}^{n} \log x_i}$

Step 4: Second derivative test $(\log L(\theta))'' = -\frac{n}{\theta^2} < 0$ for all values of n and θ. Thus, the critical point is a maximum.

$$\therefore \boxed{\hat{\theta}_{MLE} = -\frac{n}{\sum_{i=1}^{n} \log x_i}}$$

Part 2: STAT functions are permitted. No work is necessary.

Problem 3 (2 points)

In a previous class survey, 51 of 140 males play a musical instrument and 19 out of 49 females also play a musical instrument. Construct a 97% confidence interval for the difference of proportions of male and female students who play a musical instrument. Using the confidence interval, determine if there evidence that one sex is more musically inclined than the other. Explain.

The 97% confidence interval for $p_1 - p_2$ is computed using the `2-PropZInt` STAT function which gives

$$[-0.1984, 0.15148]$$

Since zero is in the interval, then $p_1 - p_2 = 0 \implies p_1 = p_2$. Thus, there is no evidence that one sex is more musically inclined than the other since the proportions are about the same.