Homework 1

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1. What will be the possible minimum and maximum cardinality of following sets, if X is a set of n numbers and Y is a set of m numbers and n is greater than m. (12 points)
   (a) $X \cap Y$
   The minimum cardinality: 0
   The maximum cardinality: m

   (b) $X \cup Y$
   The minimum cardinality: m
   The maximum cardinality: n

   (c) $X - Y$
   The minimum cardinality: 0
   The maximum cardinality: n – m

   (d) $Y - X$
   The minimum cardinality: 0
   The maximum cardinality: m

2. Prove or disprove following by giving examples: (15 points)
   (a) If $X \subset Y$ and $X \subset Z$, then $X \subset Y \cap Z$
   Let $X = \{1,2\}$ and $Y = \{1,2,3\}$ and $Z = \{1,2,3,4\}$
   $X \subset Y$ and $X \subset Z$, and $Y \cap Z = \{1,2,3\}$
   Since every element in X also in $Y \cap Z = \{1,2,3\}$, then $X \subset Y \cap Z$.

   (b) If $X \subseteq Y$ and $Y \subseteq Z$, then $X \subseteq Z$
   Let $X=\{1,2\}$, $Y=\{1,2,3,4\}$ and $Z=\{1,2,3,4\}$
   Since every element in X is also in Z, then $X \subseteq Z$

   (c) If $X \in Y$ and $Y \in Z$, then $X \in Z$
   Let $X = \{1,2\}$, $Y=\{ \{1,2\}, 3,4\}$ and $Z=\{\{\{1,2\},3,4\},5,6\}$
   Since $X \in Y$ and $Y \in Z$, then $X \in Z$

3. If $P(X) \subseteq P(Y)$, then what will be the relation between X and Y? (5 points)
   Where $P(X)$ = Power set of set X and $P(Y)$ is Power set of set Y.

   The relation between $P(X)$ and $P(Y)$, the Power set of X is a subset of the Power set of set Y. The relation between X and Y is dependent on a specific sets X and Y and not solely on the fact that $P(X) \subseteq P(Y)$.

4. Suppose U = {1, 2,…, 9}, A = all multiples of 2, B = all multiples of 3, and C = {3, 4, 5, 6, 7}. Find C - (B - A). (5 points)

   A = {2,4,6,8}, B = {3,6,9}, and C = {3, 4, 5, 6, 7}
   B – A = {3,9}
   C – (B – A) = {4, 5, 6, 7}

5. What is the difference between function and relation? Explain by giving example. (5 points)
   - Function is for each input, there will be an output that associate with them.
     Ex: $f(x) = 2x$ → (1,2), (2,4)
   - Relation is a set of ordered pair (a, b) where a is an element of one set, and b is an element of another set.
     Ex: {(1,1), (1,2)}

6. Find the domain and range of these functions. (20 points)
   (a) the function that assigns to each pair of positive integers the first integer of the pair
       The domain: all positive integer pairs (x, y), where x and y are positive integers.
       The range: all positive integers

   (b) the function that assigns to each positive integer its largest decimal digit
       The domain: the set of positive integers
       The range: the set of all single-digit non-negative integers {0,1,2,3,4,5,6,7,8,9}

   (c) the function that assigns to a bit string the number of ones minus the number of zeros in the string
       The domain: the set of all bit strings
       The range: the set of all integers

(d) the function that assigns to each positive integer the largest integer not exceeding the square root of the integer

The domain: the set of positive integers

The range: the set of positive integers less than or equal to the square root of the input integer

(e) the function that assigns to a bit string the longest string of ones in the string

The domain: the set of all bit strings

The range: the set of all positive integers, as the length of the longest string of ones in a bit string can be any positive integer.

7. Determine whether each of these functions is a bijection from R to R. (16 points)

(a) $f(x) = -3x + 4$ → bijection
- $f(x) = f(y) \to -3x + 4 = -3y + 4 \to x=y \to f(x)$ is injective
- $-3x+4 = 0 \to x = \frac{3}{4}$, for every element of $-3x + 4$ there are an element of x. → $f(x)$ is surjective

(b) $f(x) = -3x^2 + 7$ → bijection
- $f(x) = f(y) \to -3x^2 + 7 = -3y^2 + 7 \to x=y \to f(x)$ is injective
- $-3x^2 + 7 = 0 \to x = \pm\frac{\sqrt{21}}{3}$ for every element of $-3x^2 + 7$ there are an element of x. → $f(x)$ is surjective

c) $f(x) = (x + 1)/(x + 2)$ → not bijection
- $f(x) = f(y) \to (x + 1)/(x + 2) = (y + 1)/(y + 2)$.
  $f(x) = f(y)$ if $x = y \neq -2$. → $f(x)$ is not injective

d) $f(x) = x^5 + 1$ → not bijection
- $f(x) = f(y) \to x^5 + 1 = y^5 + 1 \to x=y \to f(x)$ is injective
- $x^5 + 1 = 1 \to x = 0 \to f(x)$ is not subjective

8. If f and f ∘ g are one-to-one, does it follow that g is one-to-one? Justify your answer. (4 points)

It does not follow that g is one-to-one if f and f ∘ g are one-to-one.

For example, let $f(x) = x+1$ and $g(x) = -x^2$.
$f \circ g = f(g(-x^2)) = -x^2 + 1$
f and f ∘ g are one-to-one, but g is not one-to-one because $g(2) = g(-2) = -4$

9. What can you say about the sets A and B if we know the following:-   (12 points)
   a) A ∪ B = A?

   Every element in set A is also an element of set B.

   b) A ∩ B = A?

   Every element in set A is also an element of set B, and every element in set B is also an element of set A.

   c) A − B = A?

   Set B is a subset of set A, and there are no elements in set A that are not also in set B.

   d) A − B = B − A?

   Set A and set B have the same elements, and there are no elements that are in one set and not in the other.

10. Rewrite the following sets using set builder notation (6 points)
    a) $\{-8, -6, -4, -2, 0, 2, 4, 6, 8\} = \{x \mid x = -8, -6, -4, -2, 0, 2, 4, 6, 8\}$

    b) $\{1, 4, 9, 16, 25, 36, 64, 81, 100\} = \{x^2 \mid x = 1,2,3,4,5,6,,8,9,10\}$

    c) $\{1, 3, 5, 7, 9, 11, 13, \ldots \} = \{2x - 1 \mid x \in Z+\}$