Directions

Scan and upload your *handwritten* solutions to eLearning by the end of the day on **Monday, April 17**. Calculator functions are permitted. Show all steps of hypothesis testing for problems 3 and 4: Include hypotheses, rejection region, test statistic, p-value, and conclusion.

Problem 1 (2 points)

Consider the following hypotheses from the classical court of law example.

$$H_0: \text{ not guilty}$$
$$H_a: \text{ guilty}$$

Two errors that can be made by the jury are (1) letting a truly guilty person go free and (2) letting a truly innocent person go to jail. Which is a Type I error? Which is a Type II error? Justify your answers by applying the error definitions.

(1) Letting a truly guilty person go free is a type II error because the judge wrongly accepts "not guilty".
(2) Letting a truly innocent person go to jail is a type I error because the judge wrongly rejects "not guilty".

Problem 2 (2 points)

Suppose the null hypothesis for a population mean is $H_0: \mu = 14$. Suppose data is collected from a sample of size $n = 25$ and σ is unknown ($\Longrightarrow$ use the t-distribution!).

(a) A researcher believes the mean is higher. The computed test statistic is 3.024. Compute the P-value of the test. At $\alpha = 0.05$, what do you conclude?

Right tailed test: $p = P(t > 3.024) =$ `tcdf(3.024,1E99,25-1)` $\approx \boxed{0.0029}$. Since $p < \alpha$, we decide to reject H_0 at $\alpha = 0.05$

(b) A researcher believes the mean is not 14. The computed test statistic is -1.275. Compute the P-value of the test. At $\alpha = 0.10$, what do you conclude?

Two-tailed test: $p = 2 \cdot P(t < -1.275) = 2\cdot$`tcdf(-1E99,-1.275,25-1)` $\approx \boxed{0.2145}$. Since $p > \alpha$, we decide to accept H_0 at $\alpha = 0.10$.

Problem 3 (3 points)

According to the American Time Use Survey, the typical American spends 154.8 minutes per day watching television. A survey of 50 internet users results in a mean time watching television per day of 148.7 minutes with a sample standard deviation of 46.5 minutes. The researcher wants to know if the true mean differs from 154.8 minutes. At $\alpha = 0.04$, conduct an appropriate hypothesis test to answer the researcher's inquiry. Show all steps including hypotheses, rejection region, test statistic, p-value, and conclusion.

The critical value for the two-tailed rejection region is $t_c =$ `InvT(1-0.04/2, 50-1)` $= 2.110$

Step	Value (may use `T-Test`)
Hypotheses	$H_0 : \mu = 154.8$ vs $H_a : \mu \neq 154.8$
Rejection Region	$(-\infty, -2.110] \cup [2.110, \infty)$
Test Statistic	$t = \frac{148.7-154.8}{46.5/\sqrt{50}} \approx -0.9276$ (not in RR)
p-value	0.3582
Decision	Accept H_0 since $p > \alpha$

Thus, we can accept that the mean might be 154.8 minutes. So, there is no evidence at $\alpha = 0.04$ to support that the mean is different than 154.8 minutes.

Problem 4 (3 points)

An account on server A is more expensive than an account on server B. However, server A is faster. To see if it is optimal to go with the faster but more expensive server, a manager needs to know if server A is significantly faster than server B. A certain computer algorithm is executed $n_A = 30$ times on server A and $n_B = 20$ times on server B with sample mean and standard deviation of the performance speed of each server (in minutes):

Server A	Server B
$\bar{x}_A = 6.7$ minutes	$\bar{x}_B = 7.5$ minutes
$s_A = 0.6$ minutes	$s_B = 1.2$ minutes

At $\alpha = 0.05$, determine if server A is faster than server B using an appropriate significance test. Assume $\sigma_A^2 \neq \sigma_B^2$. Show all steps including hypotheses, rejection region, test statistic, p-value, and conclusion.

If server A is faster, then A should take less time than server B $\implies$ $\mu_A < \mu_B$. For the rejection region: $t_c = \texttt{InvT}(0.05, 25.399) = -1.71$ (The degrees of freedom are estimated by Satterthwaite's approximation.)

Step	Value (may use `2-SampTTest`)	Other
Hypotheses	$H_0 : \mu_A = \mu_B$ vs $H_a : \mu_A < \mu_B$	$H_a : \mu_A \neq \mu_B$
Rejection Region	$(-\infty, -1.71]$	$(-\infty, -2.06] \cup [2.06, \infty)$
Test Statistic	$t = -2.76$ (is in the RR)	Same
p-value	0.0053	0.0106
Decision	Reject H_0 since $p < \alpha$	Same

We can support $\mu_A < \mu_B$. So, there is evidence that A is significantly faster at $\alpha = 0.05$.