

Directions

Scan and upload your *handwritten* solutions to eLearning by the end of the day on **Monday, February 27 by 11:59 pm**. Show sufficient work in problems 1 and 2 or no credit may be given. Calculator codes are sufficient for problem 3 (or use formulas/integrals if you prefer). Submit your work in an organized format: solutions should be in question order and please write neatly.

Problem 1 (3 points)

The function $f_X(x)$ below is a probability density function (pdf) for the continuous random variable X . Answer the following.

$$f_X(x) = \begin{cases} x & 0 \leq x \leq 1 \\ 2 - x & 1 < x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

- (a) Neatly sketch $f_X(x)$. Use the plot to guess the value of $\mathbb{E}(X)$. Explain your guess.
- (b) Calculate the standard deviation, σ .
- (c) Set up and solve an integral to calculate $P(1/2 \leq X < 3/2)$.

Problem 2 (4 points)

The time X it takes to complete an exam is uniformly distributed between 50 and 76 minutes. In other words, $X \sim \text{Uniform}(50, 76)$.

- (a) Calculate the mean and variance of X .
- (b) What is the probability that it will take between 65 and 75 minutes to complete the exam?
- (c) Suppose $y = \ln(x - 49)$. Find the probability density function for Y .
- (d) Verify that your answer from (c) is a valid p.d.f.

Problem 3 (3 points)

Chipotle can serve at a rate of two customers every three minutes.

- (a) What is the probability that the wait time for the next customer will be less than one minute?
- (b) What is the probability that the wait time for the next customer will exceed 4 minutes, given that they have been waiting for 2 minutes already?
- (c) There are 4 people in line. What is the probability that it will take at most 6 minutes to serve all of them?

DO NOT WRITE ON THIS FORM