Problem 1

a) Construct a 95% confidence interval to estimate the proportion of students that do not currently have a job

- $n = 190$
- $z_c = 1.96$
- $\hat{p} = \frac{112}{190} = 0.5895$

$$\hat{p} \pm z_c \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.5895 \pm 1.96 \sqrt{\frac{0.5895(1-0.5895)}{190}} = [0.519, 0.659]$$

We are 95% confident that the true effectiveness rate is between 51.9% and 65.9%.

b) Find the sample size to construct a 99% confidence interval

- $\hat{p} = 0.62$
- $z_c = 2.576$
- $\Delta = 0.04$

$$n \geq \hat{p}(1-\hat{p})\left(\frac{z_c}{\Delta}\right)^2$$

$$n \geq (0.62)(1-0.62)\left(\frac{2.576}{0.04}\right)^2$$

$$n \geq 977.18$$

$n = 978$ or more

Problem 2

a) Derive the method of moment estimator (MOME) for θ.

Population Moment

$$E(x) = \int_0^\infty x\theta x^{\theta-1}\,dx$$

$$= \theta\int_0^1 x^\theta\,dx$$

$$= \theta\frac{x^{\theta+1}}{\theta+1}\Big|_0^1$$

$$= \theta\left[\frac{1^{\theta+1}}{\theta+1} - 0\right] = \frac{\theta}{\theta+1}$$

Sample Moment

$$\bar{X}$$

$$\frac{\theta}{\theta+1} = \bar{X}$$

$$\theta = (\theta+1)\bar{X}$$

$$\theta = \bar{X}\theta + \bar{X}$$

$$\theta - \bar{X}\theta = \bar{X}$$

$$\theta(1-\bar{X}) = \bar{X}$$

$$\boxed{\hat{\theta}_{MOME} = \frac{\bar{X}}{1-\bar{X}}}$$

b) Derive the maximum likelihood estimator (MLE) for θ

Step 1: $L(\theta) = \prod_{i=1}^{n} \theta x_i^{\theta-1} = \theta^n \sum_{i=1}^{n} x_i^{\theta-1}$

Step 2: $\ln L(\theta) = n \ln \theta + (\theta - 1) \ln \sum_{i=1}^{n} (x_i)$

Step 3: $\frac{\partial}{\partial \theta} \ln L(\theta) = \frac{n}{\theta} + \ln \sum_{i=1}^{n} (x_i) = 0$

$$\frac{n}{\theta} = -\ln(\bar{X})$$

$$\theta = -\frac{n}{\ln(\bar{X})}$$

$\therefore$ $\boxed{\hat{\theta}_{MOME} = -\frac{n}{\ln(\bar{X})}}$

Step 4: Confirm $\hat{\theta}$ is the maximum

$$\frac{\partial^2}{\partial \theta^2} \ln L(\theta) = -\frac{n}{\theta^2} < 0$$

$\therefore$ $\hat{\theta}$ is the maximum by 2nd derivative.

Problem 3

Construct a 97% confidence interval for different proportions

- $x_1 = 51$
- $n_1 = 140$
- $x_2 = 19$
- $n_2 = 49$
- C-level = 0.95

$\therefore$ $\boxed{(-0.1984, 0.1515)}$

$P_1 - P_2 = 0$

$P_1 = P_2$

$\therefore$ There are no evidence that one sex is more musically inclined than other.