# Homework 2

**Name:** Chhay Lay Heng

**Net ID:** cxh220032

1. $R^* = R \cup (a, a)$

2. Determine relations on the set of all people are equivalence relations.

    a) {(a, b) | a and b are the same age} → equivalence relation

    Reflexive: Yes

    Symmetric: Yes

    Transitive: Yes

    b) {(a, b) | a and b have the same parents} → not equivalence relation

    Reflexive: No

    c) {(a, b) | a and b share a common parent} → not equivalence relation

    Reflexive: Yes

    Symmetric: Yes

    Transitive: No

    d) {(a, b) | a and b have met} → not equivalence relation

    Reflexive: Yes

    Symmetric: Yes

    Transitive: No

    e) {(a, b) | a and b speak a common language} → equivalence relation

    Reflexive: Yes

    Symmetric: Yes

    Transitive: Yes

3. Find equivalence classes of following relations if they exist.

    a) {(0, 0), (1, 1), (2, 2), (3, 3)}

       [0] = {0}

       [1] = {1}

       [2] = {2}

       [3] = {3}

    b) {(0, 0), (0, 2), (2, 0), (2, 2), (2, 3), (3, 2), (3, 3)}

       [0] = {0, 2}

       [2] = {0, 2, 3}

       [3] = {2, 3}

    c) {(0, 0), (1, 1), (1, 2), (2, 1), (2, 2), (3, 3)}

       [0] = {0}

       [1] = {1, 2}

       [2] = {1, 2}

       [3] = {3}

    d) {(0, 0), (1, 1), (1, 3), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)}

       [0] = {0}

       [1] = {1, 3}

       [2] = {2, 3}

       [3] = {1, 2, 3}

    e) {(0, 0), (0, 1), (0, 2), (1, 0), (1, 1), (1, 2), (2, 0), (2, 2), (3, 3)}
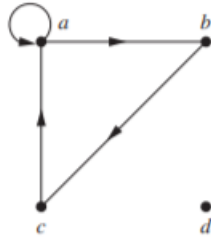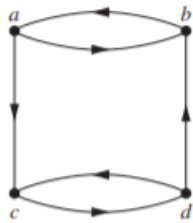
       [0] = {0, 1, 2}

       [1] = {0, 1, 2}

       [2] = {0, 2}

       [3] = {3}

4. Find reflexive closure graph, symmetric closure graph and transitive closure graph for following graphs



- Reflexive closure: $R *= R \cup \{(b,b)\ (c,c)\ (d,d)\}$
- Symmetric closure: $R *= R \cup \{(b,a)\ (a,c)\ (c,b)\}$
- Transitive closure: $R *= R \cup \{(a,c)\ (c,b)\ (b,a)\}$

(a,b) (b,c) → (a,c)

(c,a) (a,b) → (c,b)

(b,c) (c,a) → (b,a)



- Reflexive closure: $R *= R \cup \{(a,a)\ (b,b)\ (c,c)\ (d,d)\}$
- Symmetric closure: $R *= R \cup \{(c,a)\ (b,d)\}$
- Transitive closure: $R *= R \cup \{(a,d)\ (c,b)\ (b,c)\}$

(a,c) (c,d) → (a,d)

(c,d) (d,b) → (c,b)

(b,a) (a,c) → (b,c)

5. Find the matrices

$R_1 = \{(1,2)\ (2,1)\ (2,2)\ (2,3)\ (3,1)\}$

$R_2 = \{(1,2)\ (2,2)\ (2,3)\ (3,1)\ (3,2)\ (3,3)\}$

a) $R_1 \cup R_2 = \{(1,2)\ (2,1)\ (2,2)\ (2,3)\ (3,1)\ (3,2)\ (3,3)\}$

b) $R_1 \cap R_2 = \{(1,2)\ (2,2)\ (2,3)\ (3,1)\}$

c) $R_2 \circ R_1 = \{(1,2)\ (1,3)\ (2,1)\ (2,2)\ (2,3)\ (3,2)\}$

d) $R_1 \circ R_1 = \{(1,1)\ (1,2)\ (1,3)\ (2,1)\ (2,2)\ (2,3)\ (3,2)\}$

e) $R_1 \oplus R_2 = \{(2,1)\ (3,2)\ (3,3)\}$

6. Find

$R = \{(a,b) \mid a < b\}$

a) $R^{-1} = \{(b,a) \mid a < b\}$

b) $\bar{R} = \{(a,b) \mid a \geq b\}$