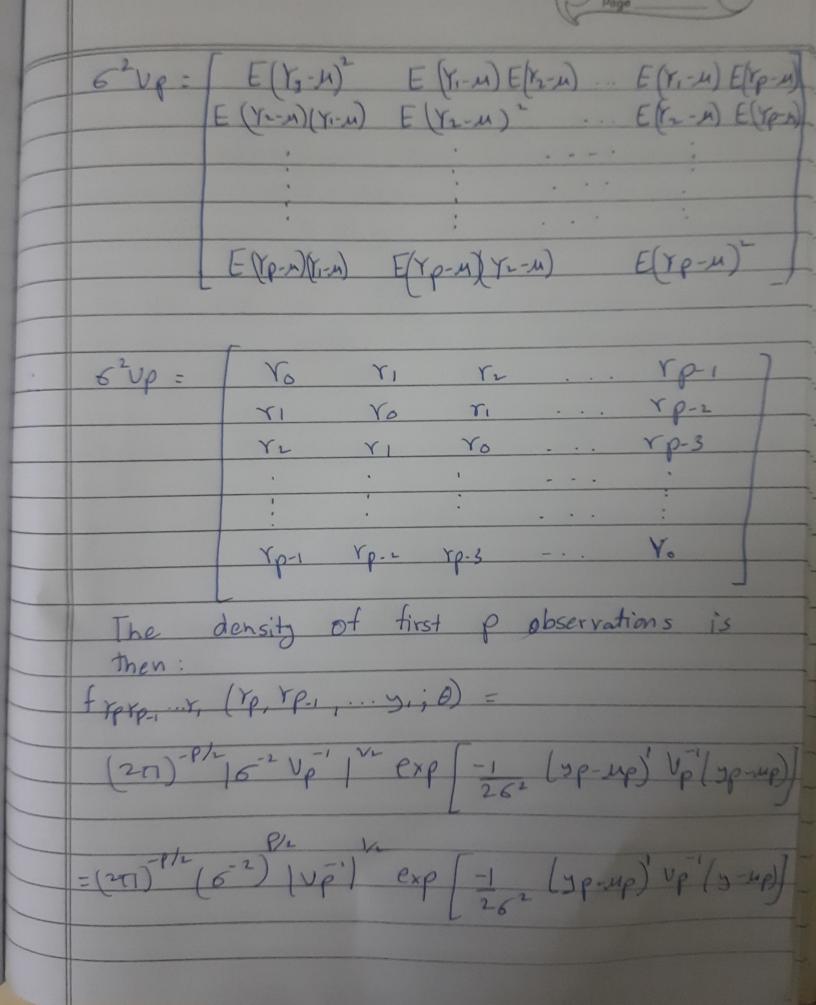


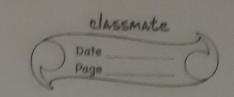
The goal of maximum likelyihood is to find the optimal way to fit a distribution to the dota. Let's say there are nobservations We first collect the first p observations in the sample. (Y, Yz... Yp) in a (px1) vector sp. For a gaussian AR(P) Process Where Et N i.j.d. N(0,62)

 $\frac{1}{y_{\beta}} \text{ has mean}$ M = C $1 - \theta_1 - \theta_2 - \dots - \theta_{\beta}$

Let 6'Up denote the (pxp) variance covariance matrix of (t, tz... tp)



for the remaining observations in the sample (Yp+1, Yp+2 ... YT), Conditional on the first t-1 observations, the the observation is gaussian with mean C+ d, yt-1 + dzytz + Dzyt-z + ... + pp yt-p. b variance 52. Only the p most recent observations matter for this distribution hence for the TP for top f Ye | Yt-1, ... Y, (yt | yt-1, ... y, i 0) = f (Tt | Tt-1 ... Yt-p; 0) = 1 exp/-(3t-c-0,3t-1-...-pp 3t-p)]-J21162 262 The likelighood function for the complete sample is then + YT, YTA ... Y, (yT, yT-, ... Y, ; 0) = f Yp Yp-1 ... Y, (yp, yp-1 ..., y, i0) x 17 f Y1 / Y+-1, ..., Y+-p (y+) y+-p ; 0)



The log likelihood is therefore

L(0) = log fr, r. -- K. (tr, Yr., Y., 0)

= - Plog(211) - Plog(62) + 1log | vpl - 1 (yp-up)

VP (J-10) - (I-P) 103211 - I-P 103(62)

- \(\frac{1}{5t} - C - \partition 13t - \partition 25t - \ldots - \partition - \partition p 5tp)^{\frac{1}{5}} \\
\tag{t=p+1} \]

The value of $\phi_1, \phi_2...\phi_p$ that miximizes the above equation are the same as those that minimize

 $= \frac{1}{1} \left(y_{t-c-\phi_1 y_{t-1}} - \phi_2 y_{t-2} - \cdots - \phi_p y_{t-p} \right)^2$

This is an exactly similar equation like OLS.

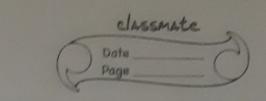
The conditional MLE of These parameters

(an be obtained from an OLS regression

of yt on a constant & p of it's own

lagged terms.

.. The 2 estimates are exactly the same.



[2.2] (onsider simple MA(1) model, Ye = et-Oet-1

The true lag-1 autocorrelation in This model is $l_1 = -\theta$

If we equate e. to r., we get a quadratic equation in B.

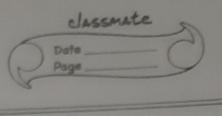
if |r, | < 0.5, then only one of the 2 real solutions satisfies the invertibility condition | 0| < 1

That solution is ê: (-1+ VI-Axi /2xi

But if $|x_1| = 0.5$, no investible solution exists by if $|x_1| \neq 0.5$ then no real solution at all exists by the method of moments fail to give any estimator of θ .

With higher order MA(9) models, the set of equations for estimating B1,... Bp is highly hon-linear & could be only solved humerically

Hence model estimation procedure is a bit harder



Consider now the MA(1) model

This can be written as $Y_t = -\partial Y_{t-1} - \partial^2 Y_{t-2} - \partial^3 Y_{t-3} - \dots + C_t$

So a least square estimator of θ Can be abtained by finding the value

of θ that minimizer $Sc(\theta) = \sum \int \int_{\mathbb{T}} dt + \theta \int_{\mathbb{T}} dt + \theta \int_{\mathbb{T}} dt + \frac{1}{2} \int_{\mathbb{T}} dt$

But this is non-linear in B & the infinite series causes technical problems