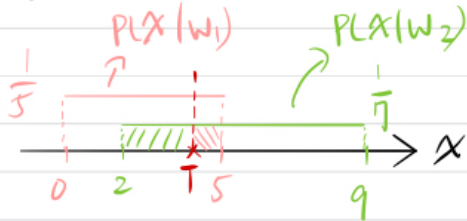


Problem 1: Bayes Decision Rule [1]

single feature $x \in [0, 9]$ (time of spent time per week)



Bayes: What we want!

$$P(w_j, x) = P(w_j | x) \cdot P(x)$$

$$= P(x | w_j) \cdot P(w_j)$$

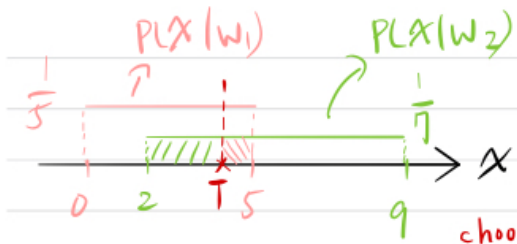
$$\Rightarrow P(w_1 | x) = \frac{P(x | w_1) \cdot P(w_1)}{P(x)} \Rightarrow \begin{cases} \text{if } P(w_1 | x) > P(w_2 | x) \\ \text{decide } w_1 \\ \text{else} \\ \text{decide } w_2 \end{cases}$$

\therefore compare $P(w_1 | x)$ with $P(w_2 | x)$

\Rightarrow compare $P(x | w_1) P(w_1)$ with $P(x | w_2) P(w_2)$ $2 < T < 5$

$$P_e = \min \{ P(w_1 | x), P(w_2 | x) \} \Rightarrow \int_2^T \frac{1}{7} \cdot \frac{1}{9} dx + \int_T^5 \frac{1}{5} \cdot \frac{2}{9} dx = \frac{1}{9} (T-2) + \frac{2}{45} (5-T)$$

$$= \int_2^T P(x | w_2) P(w_2) dx + \int_T^5 P(x | w_1) P(w_1) dx = \frac{1}{45} (5T - 2T - 10 + 10) = \frac{3}{45} T = \frac{1}{15} T$$



$$P_e = \min \{ P(w_1 | x), P(w_2 | x) \}$$

$$= \frac{T}{15}$$

$$, 2 < T < 5$$

choose $T = 2 \Rightarrow P_e \downarrow$

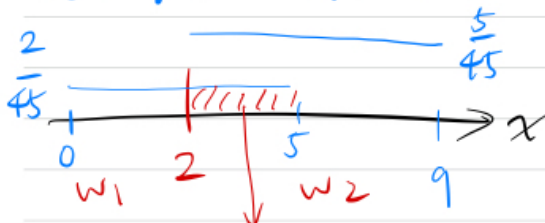
$$\Rightarrow P_e = \frac{2}{15}$$

$0 < x \leq 2$, choose w_1

$R_1 = \{ 0 < x \leq 2 \}$, choose w_1

$R_2 = \{ 2 < x < 9 \}$, choose w_2

resulting probability of error

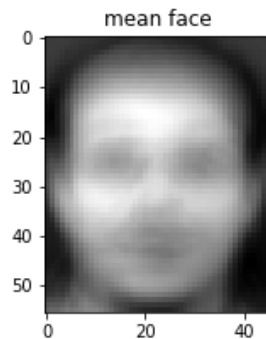


$$P_e = \int_2^5 \frac{2}{45} dx = \frac{2}{45} \cdot 3 = \frac{2}{15}$$

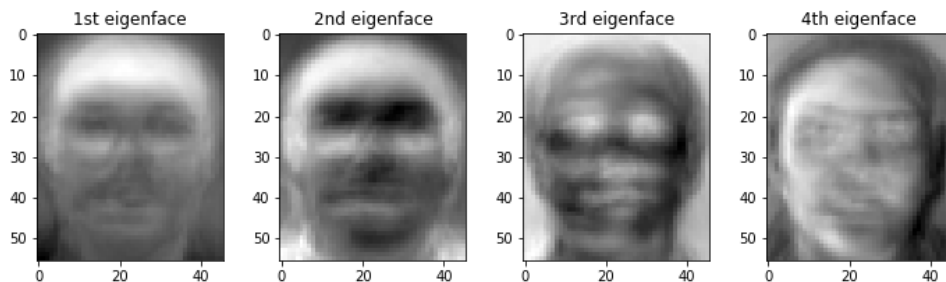
Problem 2: Principal Component Analysis [2]

1. Perform PCA on the training set. Plot the mean face and the first four eigenfaces.

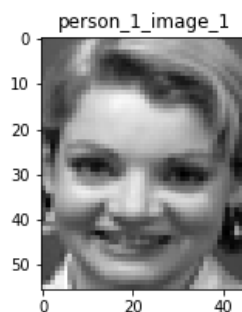
use
from sklearn.preprocessing import StandardScaler
to compute the mean face



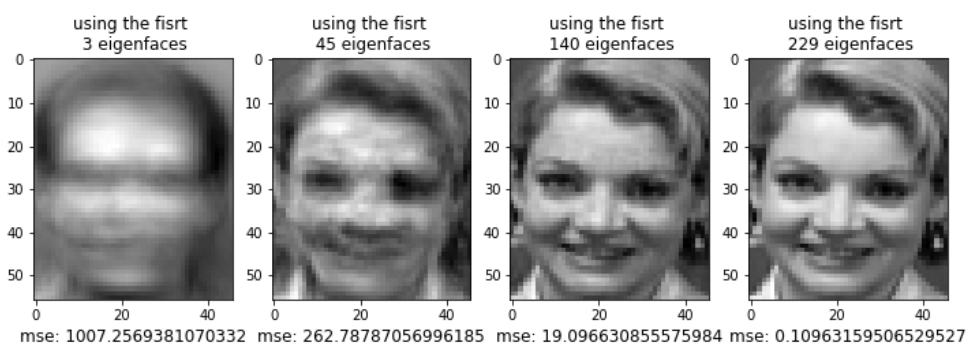
use
from sklearn.decomposition import PCA
to form eigenfaces



2. and 3.
Original image:



Reconstructed images:



Using the first n eigenfaces	3	45	140	229
MSE	1007.25	262.78	19.09	0.10

Table 1: Using the first n eigenfaces to reconstruct the original images and the corresponding mean square error (MSE)

As we using more eigenfaces to reconstruct the original image, less mean square error is obtained.

4. apply the k-nearest neighbors algorithm to classify the testing set images

I use the sklearn package to do 3-fold cross-validation for me.

By setting the cv parameter to 3 in

```
cross_val_score(knn, train_knn, train_class, cv=3, scoring='accuracy')
```

```
from sklearn.cross_validation import cross_val_score
```

k \ n	3	45	140
1	0.7041	0.9291	0.9291
3	0.6166	0.8583	0.8583
5	0.5208	0.7916	0.7541

Table 2: Accuracy of different k and n pairs.

(k: the k of k-nearest neighbors, n: using the first n eigenfaces)

Although the highest accuracy obtained by k-nn is the same when (n,k) = (45,1) and (140,1), I choose (n,k) = (45,1) for reducing to a lower dimension.

5. Choose (n,k) = (45,1)

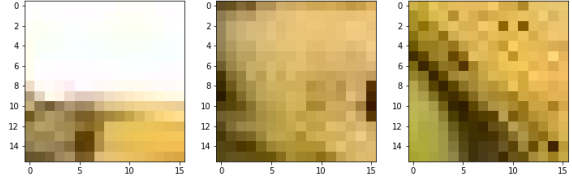
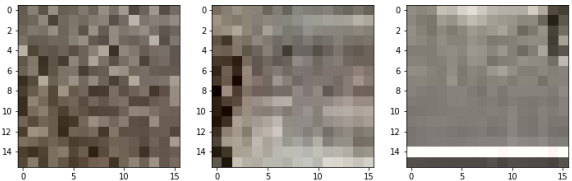
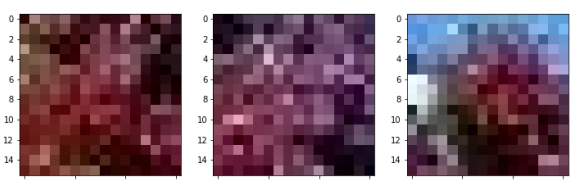
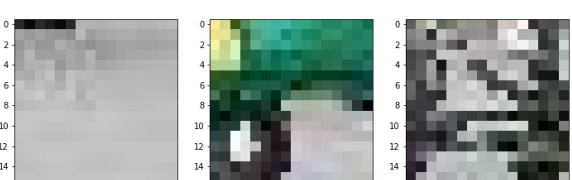
	precision	recall	f1-score	support
1	1.00	1.00	1.00	4
2	1.00	1.00	1.00	4
3	1.00	1.00	1.00	4
4	1.00	1.00	1.00	4
5	1.00	0.75	0.86	4
6	1.00	1.00	1.00	4
7	1.00	1.00	1.00	4
8	1.00	1.00	1.00	4
9	1.00	1.00	1.00	4
10	1.00	0.75	0.86	4
11	1.00	0.75	0.86	4
12	1.00	1.00	1.00	4
13	1.00	1.00	1.00	4
14	1.00	1.00	1.00	4
15	0.67	1.00	0.80	4
16	0.80	1.00	0.89	4
17	1.00	1.00	1.00	4
18	1.00	1.00	1.00	4
19	1.00	0.75	0.86	4
20	1.00	0.75	0.86	4
21	1.00	1.00	1.00	4
22	1.00	1.00	1.00	4
23	1.00	1.00	1.00	4
24	1.00	1.00	1.00	4
25	1.00	1.00	1.00	4
26	1.00	1.00	1.00	4
27	1.00	1.00	1.00	4
28	1.00	0.75	0.86	4
29	1.00	1.00	1.00	4
30	0.80	1.00	0.89	4
31	1.00	1.00	1.00	4
32	1.00	1.00	1.00	4
33	1.00	1.00	1.00	4
34	1.00	1.00	1.00	4
35	1.00	0.75	0.86	4
36	1.00	1.00	1.00	4
37	0.80	1.00	0.89	4
38	0.80	1.00	0.89	4
39	1.00	1.00	1.00	4
40	0.80	1.00	0.89	4
avg / total	0.97	0.96	0.96	160

Figure 1: The recognition rate of the testing set

The average recognition rate achieved 97% showed in Figure 1.

Problem 3: Visual Bag-of-Words [3]

1.

class	Three random patches
Banana	
Fountain	
Reef	
Tractor	

Two of the sets of patches can be classified to the true class by seeing three patches of each set (Banana and Reef). I can't classify the other two sets of patches (Fountain and Tractor).

2.

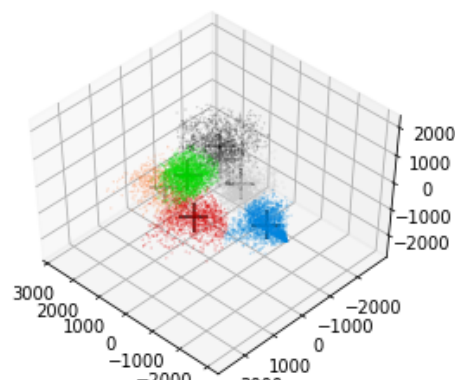
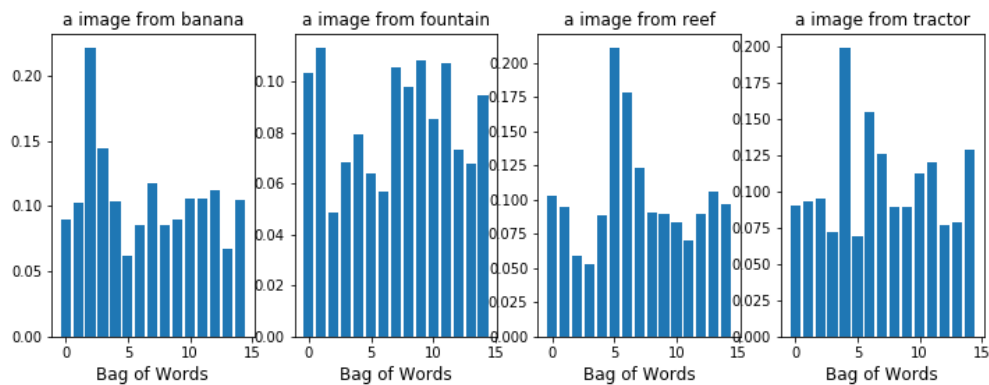


Figure 2: Bag of words (i.e., centroids) and their associated features (i.e., patches) in this PCA subspace.

3. Choose one image from each category and visualize its BoW using histogram plot.



4. Adopt the k-nearest neighbors classifier (k-NN) to perform classification on X_{test} using the above BoW features.

	Classification Accuracy
Banana	0.67
Fountain	0.35
Reef	0.56
Tractor	0.54
average	0.53

Table 3: The classification accuracy for each category and an average accuracy

Reference:

- [1] http://vllab.ee.ntu.edu.tw/uploads/1/1/1/6/111696467/dlcv_w1.pdf
- [2] <https://scikit-learn.org/stable/modules/generated/sklearn.decomposition.PCA.html>
- [3] <http://scikit-image.org/docs/dev/api/skimimage.util.html>