Drawing Road Networks with Focus Regions

Edited from Jan-Henrik Haunert's slides

If you are here

Providence, RI

Fish-Eye Views (here: Yamamoto et al., ACM GIS 2009)

- enlarge areas of interest
- still give the big picture
- ***but*** introduce large distortions

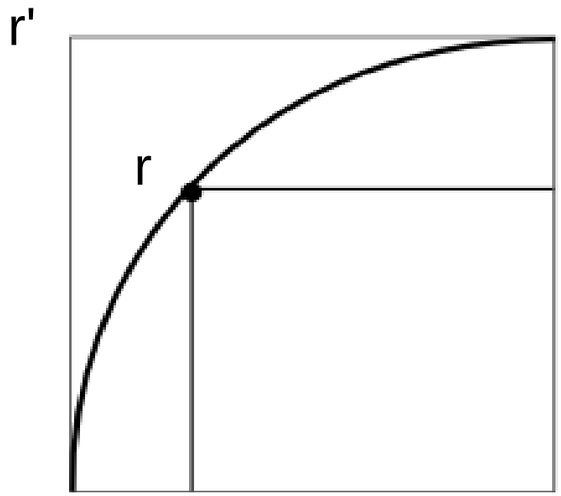
- **Aim:**

  Focus-and-context view with ***minimum distortion***

Providence, RI

Fish-Eye Views (here: Yamamoto et al., ACM GIS 2009)

- *For every pixel (x, y), transform its representation to (θ, r)*
- *Transform r to r' using a spherical function*
  - $r' = r + (1 - sqrt(1 - r^2))$
- *Recalculate (x', y') using*
  - $x' = rcos(\theta)$
  - $y' = rsin(\theta)$



Quadratic Programming

Find $x \in \mathbb{R}^n$ minimizing

- a given ***cost function***

while satisfying

- a given set of ***hard constraints***.

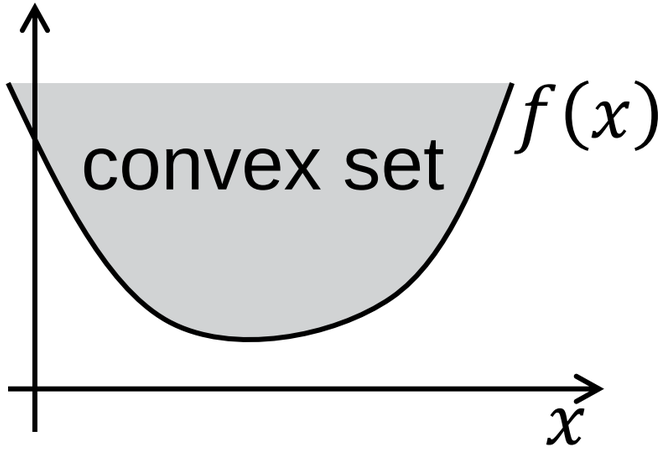
Convex Quadratic Programming

Find $x \in \mathbb{R}^n$ minimizing

- $f(x) = cx + x^T Dx$, where f is a ***convex function***,

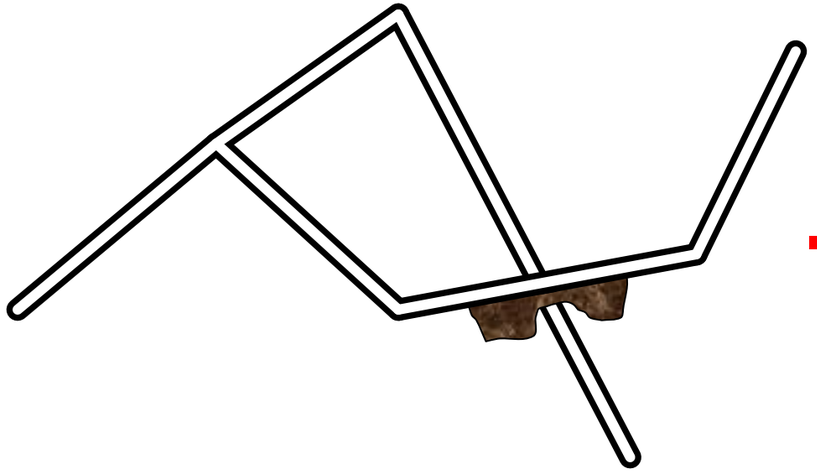
while satisfying

- $Ax \geq b$ and
- $x \geq 0$

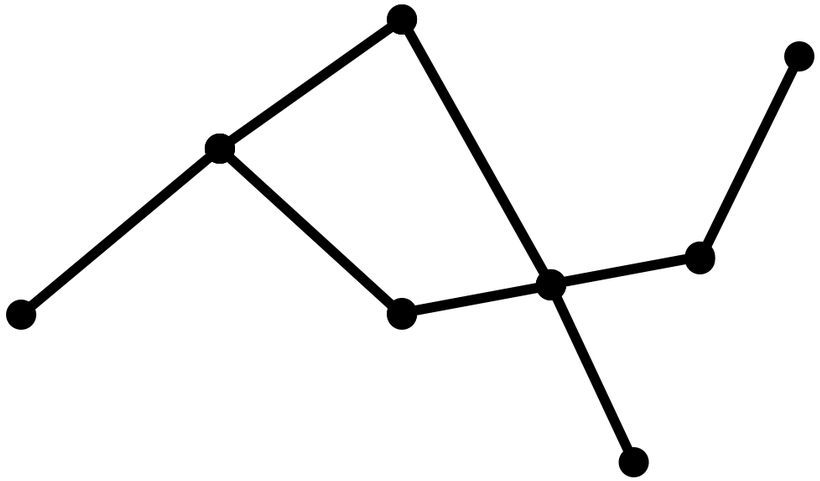


Preliminaries

road network



graph $G = (V, E)$



X_v, Y_v: ***input*** coordinates of node $v \in V$

x_v, y_v: ***output*** coordinates of node $v \in V$

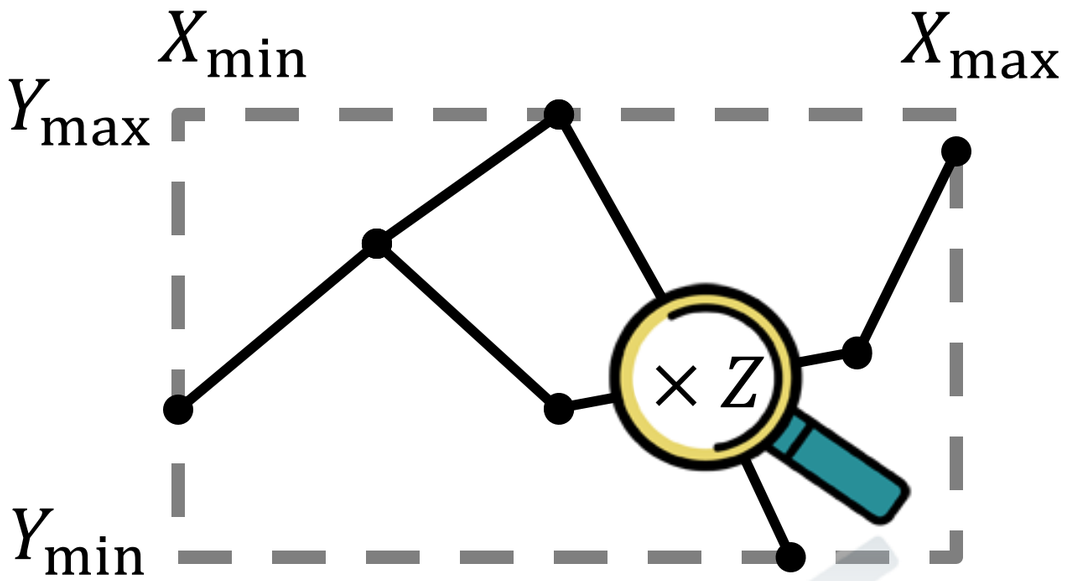
Preliminaries

Output map must fit into given frame!

Constraints:

$$X_{\min} \leq x_v \leq X_{\max}$$
$$Y_{\min} \leq y_v \leq Y_{\max}$$

focus region = subset of node set V

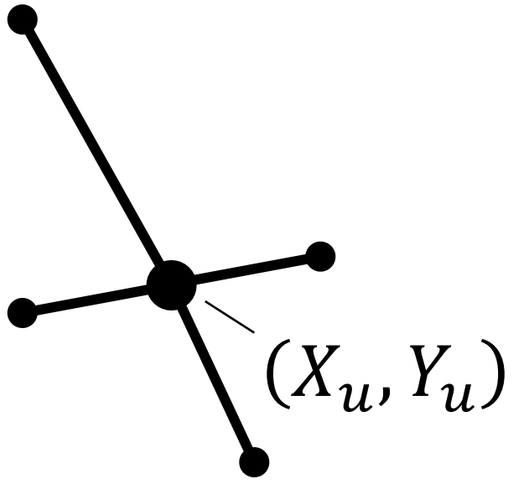


X_v, Y_v: ***input*** coordinates of node $v \in V$

x_v, y_v: ***output*** coordinates of node $v \in V$

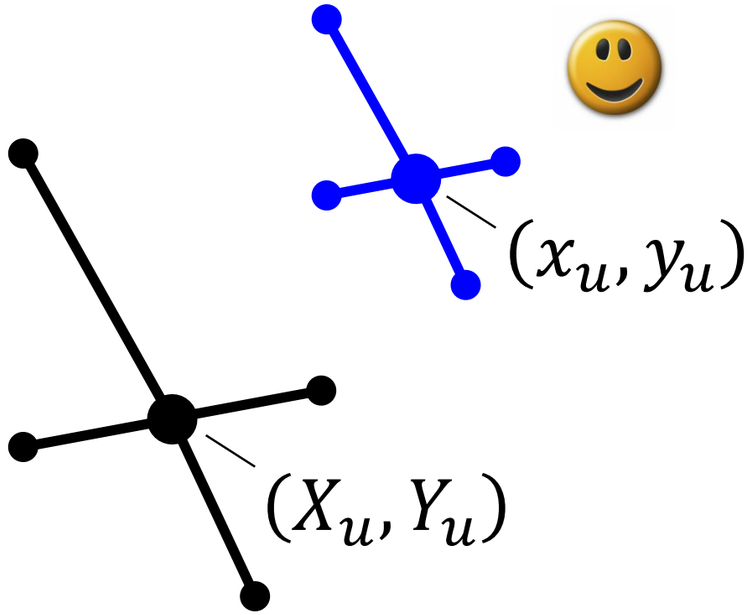
Measuring Distortion

- ***locally***, i.e., for each node $u \in V$ with its neighbors



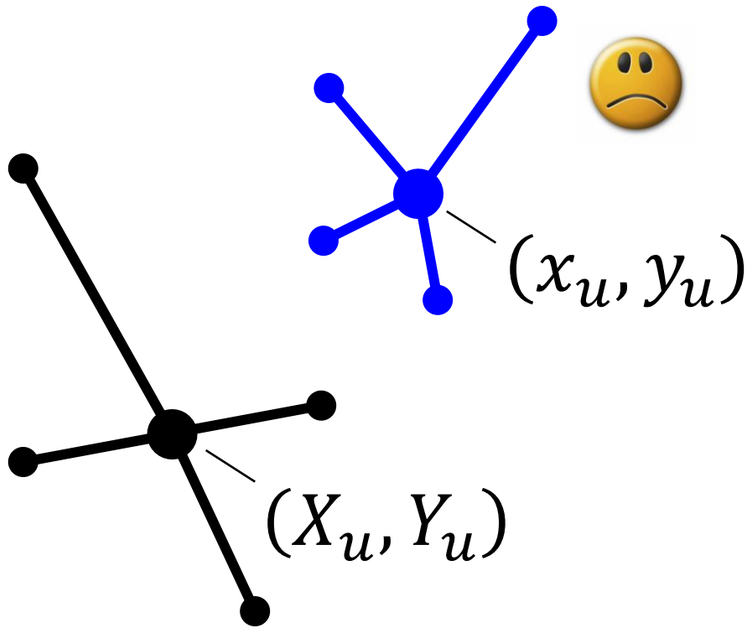
Measuring Distortion

- ***Scaling*** and ***translating*** is not distorting



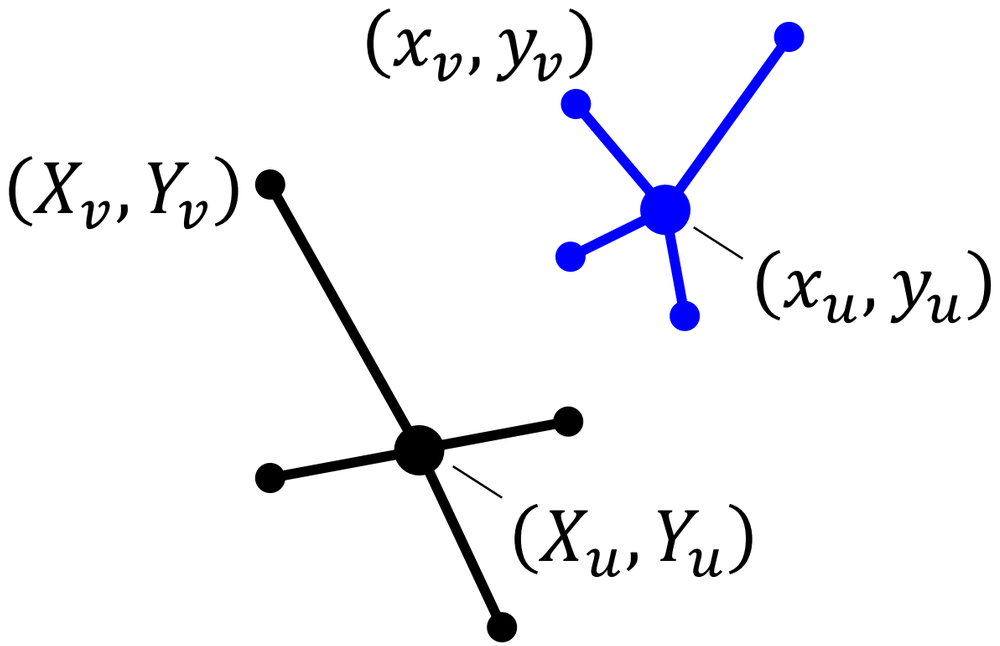
Measuring Distortion

- ***Scaling*** and ***translating*** is not distorting,
- **but every other change is.**



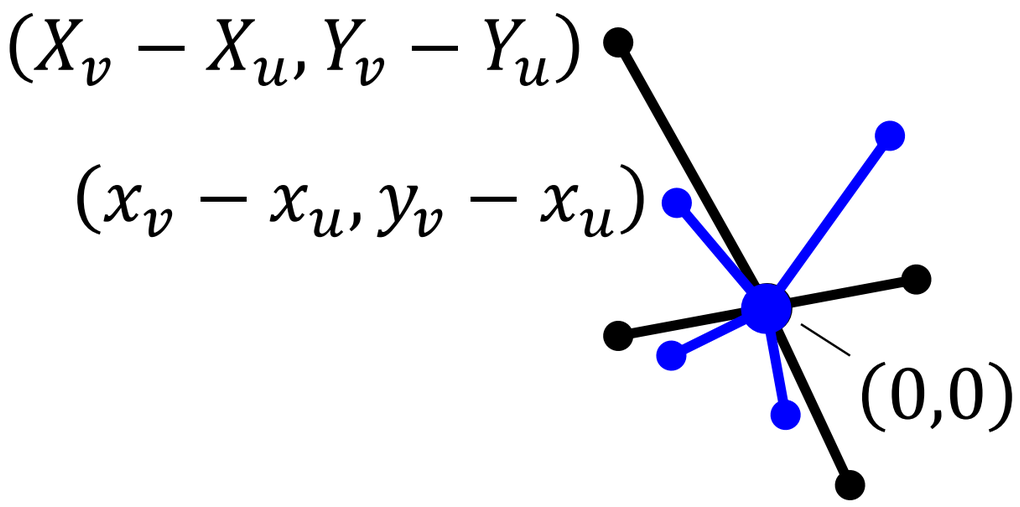
Measuring Distortion

- compare both (sub-)graphs in ***local reference frame***



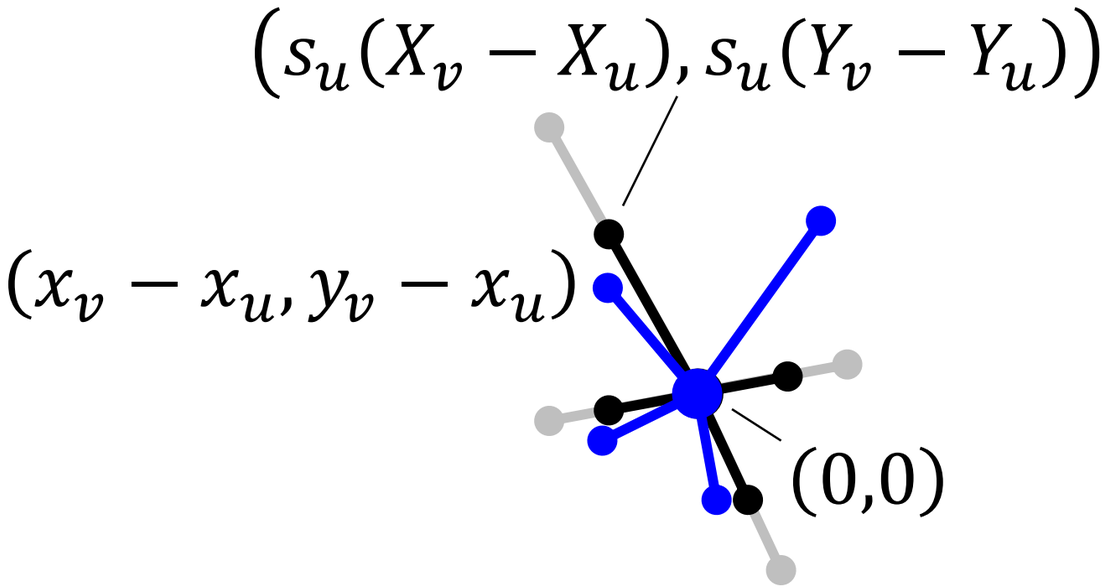
Measuring Distortion

- compare both (sub-)graphs in ***local reference frame***



Measuring Distortion

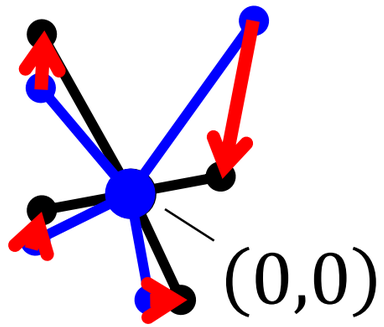
- compare both (sub-)graphs in ***local reference frame***
- apply (unknown) scale factor $s_u \in \mathbb{R}^+$ to compensate scaling



Measuring Distortion

- compare both (sub-)graphs in ***local reference frame***
- apply (unknown) scale factor $s_u \in \mathbb{R}^+$ to compensate scaling
- ***now*** measure ***squared residuals***

$$\begin{pmatrix} \underline{s_u(X_v - X_u) - (x_v - x_u)} \\ \underline{s_u(Y_v - Y_u) - (y_v - x_u)} \end{pmatrix}$$

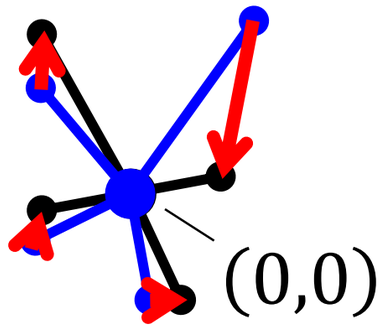


$$[\underline{s_u(X_v - X_u) - (x_v - x_u)}]^2 + [\underline{s_u(Y_v - Y_u) - (y_v - x_u)}]^2$$

Measuring Distortion

- compare both (sub-)graphs in ***local reference frame***
- apply (unknown) scale factor $s_u \in \mathbb{R}^+$ to compensate scaling
- ***now*** measure ***squared residuals***

$$\begin{pmatrix} \underline{s_u(X_v - X_u) - (x_v - x_u)} \\ \underline{s_u(Y_v - Y_u) - (y_v - x_u)} \end{pmatrix}$$



$$f(x) = \sum_{u \in V} \sum_{v \in \mathrm{Adj}(u)} [\underline{s_u(X_v - X_u) - (x_v - x_u)}]^2 + [\underline{s_u(Y_v - Y_u) - (y_v - x_u)}]^2$$

Measuring Distortion

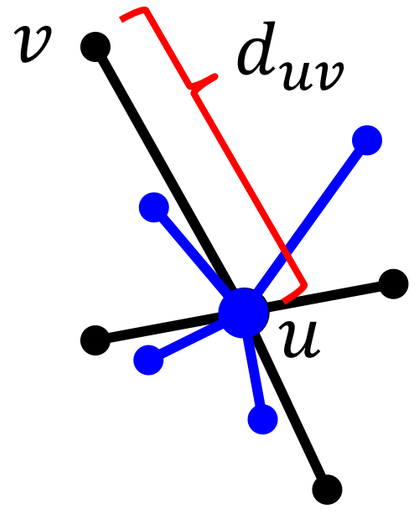
- compare both (sub-)graphs in ***local reference frame***
- apply (unknown) scale factor $s_u \in \mathbb{R}^+$ to compensate scaling
- ***now*** measure ***squared residuals***

$$f(x) = \sum_{u \in V} \sum_{v \in \mathrm{Adj}(u)} \Delta X_{uv}{}^2 + \Delta Y_{uv}{}^2$$

$$\Delta X_{uv} = s_u(X_v - X_u) - (x_v - x_u)$$
$$\Delta Y_{uv} = s_u(Y_v - Y_u) - (y_v - y_u)$$

Measuring Distortion

- compare both (sub-)graphs in ***local reference frame***
- apply (unknown) scale factor $s_u \in \mathbb{R}^+$ to compensate scaling
- ***now*** measure ***squared residuals***



$$f(x) = \sum_{u \in V} \sum_{v \in \mathrm{Adj}(u)} \frac{1}{{d_{uv}}^2} \left({\Delta X_{uv}}^2 + {\Delta Y_{uv}}^2 \right)$$

weighted by squared edge length.

$$\Delta X_{uv} = s_u (X_v - X_u) - (x_v - x_u)$$
$$\Delta Y_{uv} = s_u (Y_v - Y_u) - (y_v - y_u)$$

Measuring Distortion

$$f(x) = \sum_{u \in V} \sum_{v \in \mathrm{Adj}(u)} \frac{1}{{d_{uv}}^2} \left({\Delta X_{uv}}^2 + {\Delta Y_{uv}}^2 \right)$$

$$\Delta X_{uv} = s_u (X_v - X_u) - (x_v - x_u)$$

$$\Delta Y_{uv} = s_u (Y_v - Y_u) - (y_v - y_u)$$

variables

constants

Basic Quadratic Program

Minimize

$$f(x) = \sum_{u \in V} \sum_{v \in \mathrm{Adj}(u)} \frac{1}{{d_{uv}}^2} \left({\Delta X_{uv}}^2 + {\Delta Y_{uv}}^2 \right)$$

subject to

$$\Delta X_{uv} = s_u (X_v - X_u) - (x_v - x_u)$$
$$\Delta Y_{uv} = s_u (Y_v - Y_u) - (y_v - y_u)$$

$$X_{\min} \leq x_v \leq X_{\max}$$
$$Y_{\min} \leq y_v \leq Y_{\max}$$

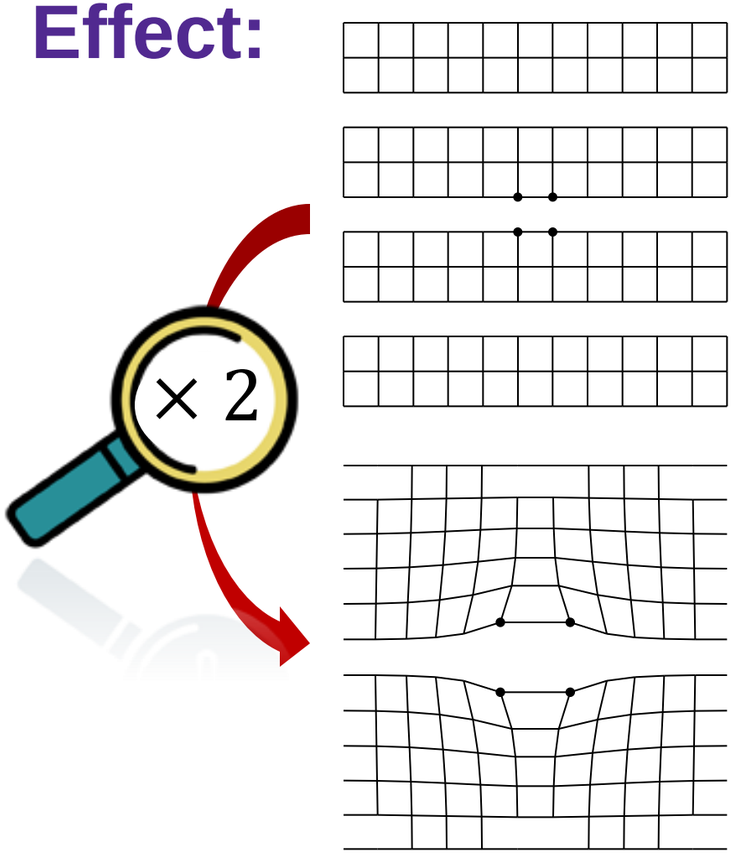
$$s_u = Z$$

$$\Delta X_{uv} = \Delta Y_{uv} = 0$$

for each node u in focus region
($Z =$ ***zoom factor***, e.g., $Z = 3$)

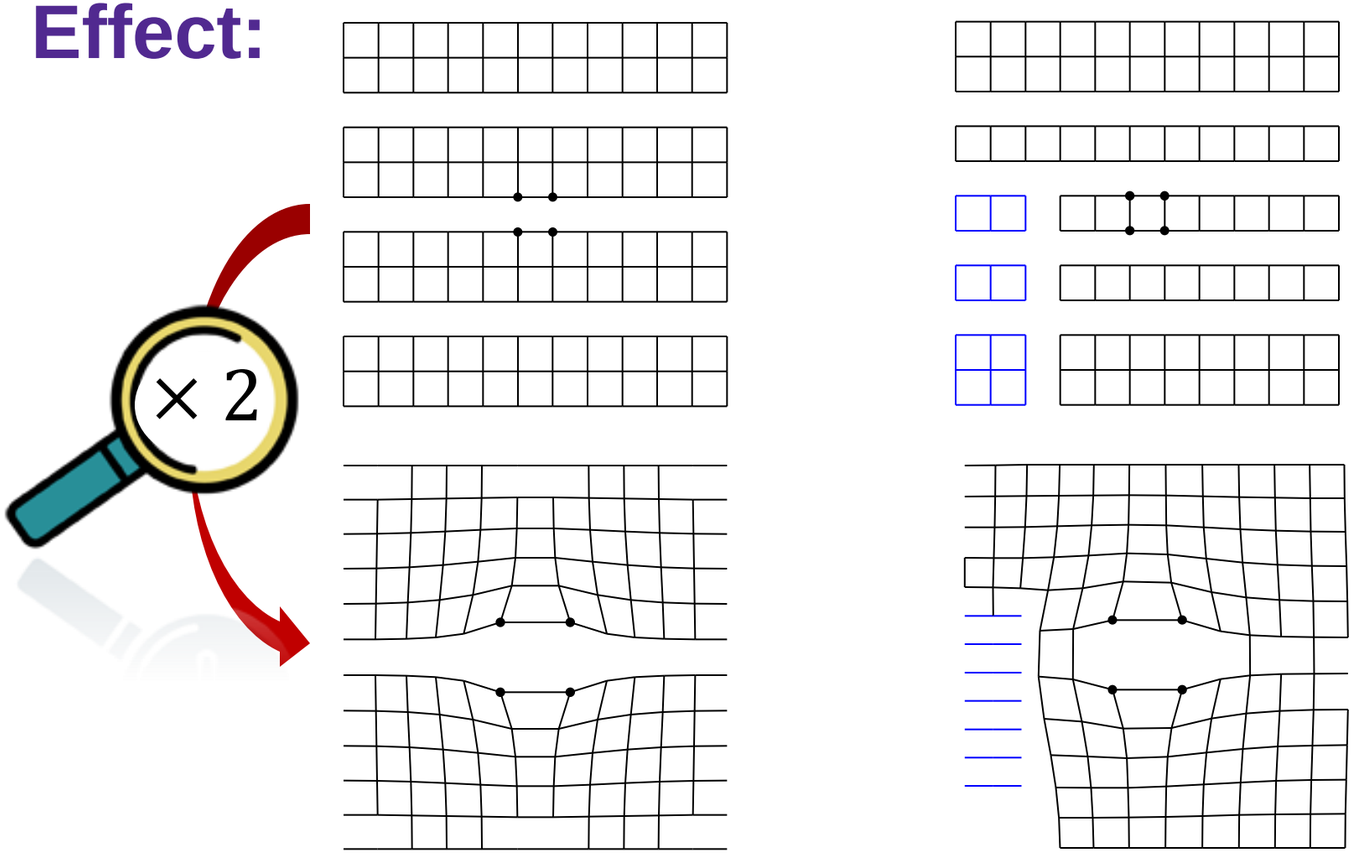
Basic Quadratic Program

Effect:



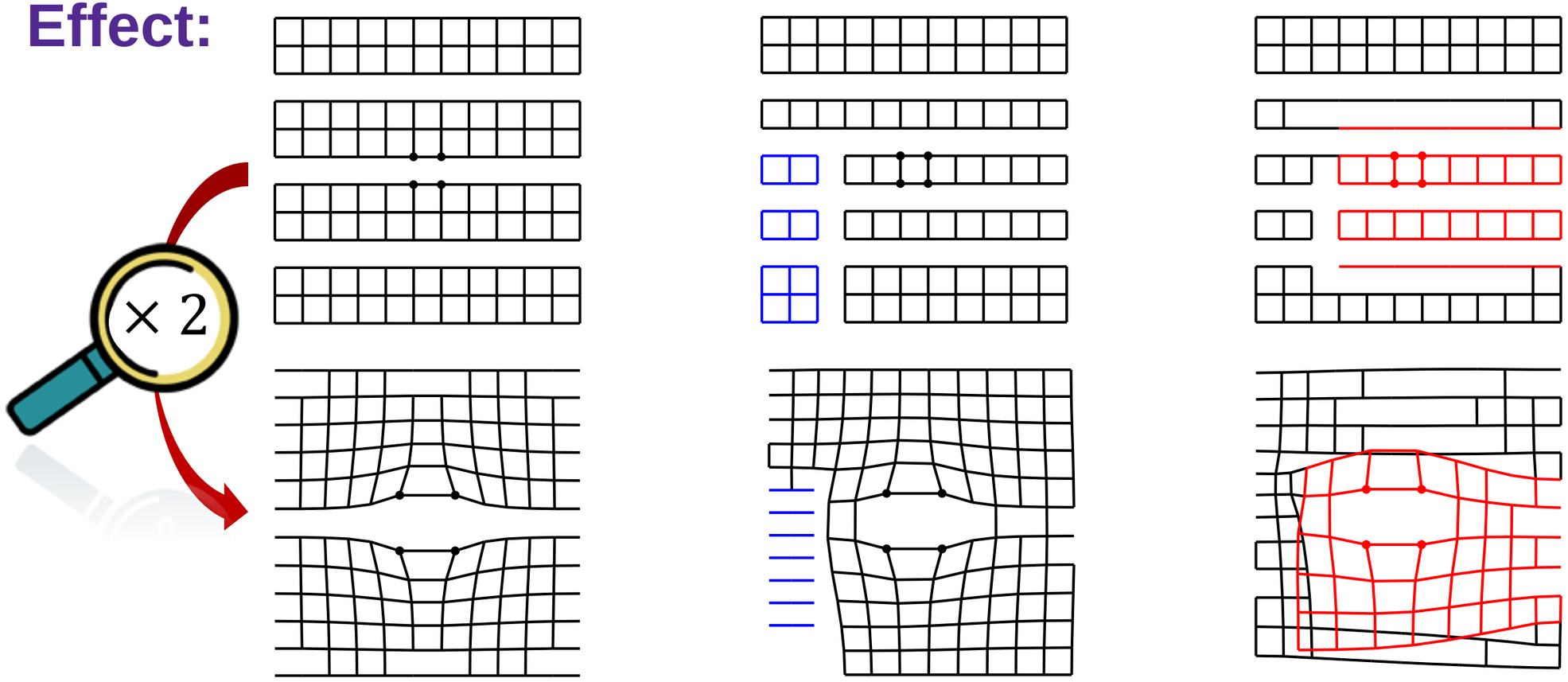
Basic Quadratic Program

Effect:



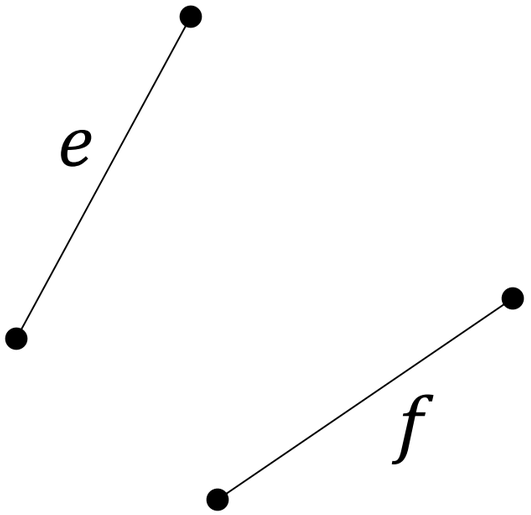
Basic Quadratic Program

Effect:



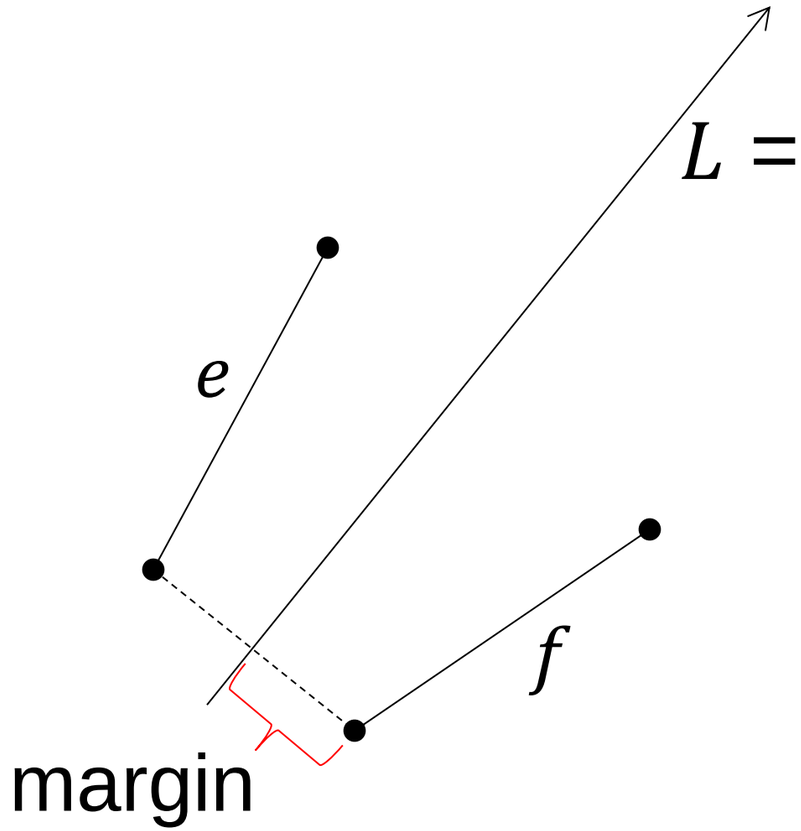
to do: avoid edge crossings

Avoiding Edge Crossings



e and f do not cross

Avoiding Edge Crossings



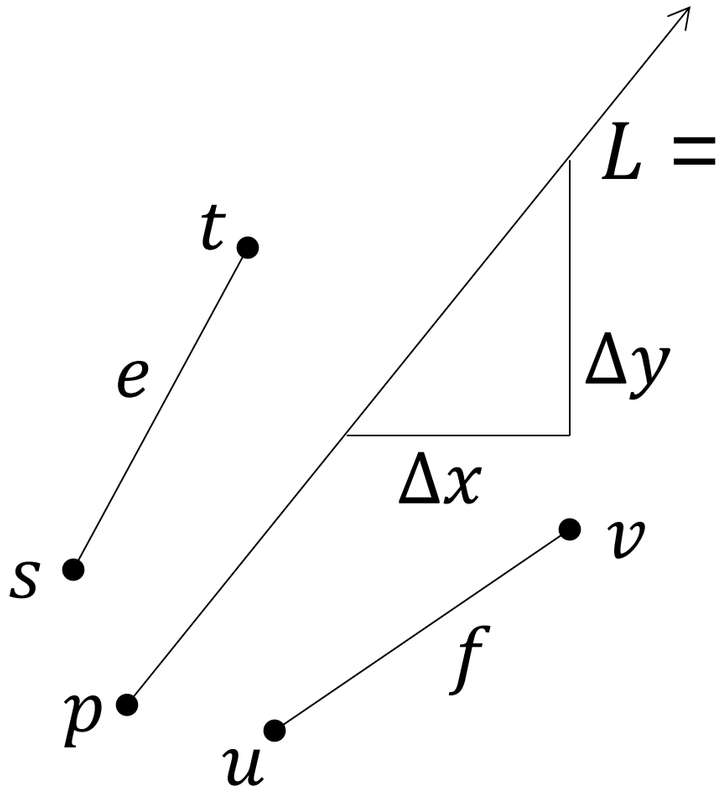
L = maximum-margin line for e and f in input

e and f do not cross

$\Leftarrow$ there exists a line ℓ with

- e to its ***left*** and
- f to its ***right***
- and same slope as L

Avoiding Edge Crossings



L = maximum-margin line for e and f in input

$$(y_s - y_p)/(x_s - x_p) \geq \Delta y/\Delta x$$

$$(y_t - y_p)/(x_t - x_p) \geq \Delta y/\Delta x$$

$$(y_u - y_p)/(x_u - x_p) \leq \Delta y/\Delta x$$

$$(y_v - y_p)/(x_v - x_p) \leq \Delta y/\Delta x$$

Avoiding Edge Crossings

- ***Algorithm:***
  - Solve QP allowing edge crossings
  - Check solution for edge crossings
  - Only add those constraints that forbid crossings found
  - Resolve QP.

$$(y_s - y_p)\Delta x - (x_s - x_p)\Delta y \geq 0$$

$$(y_t - y_p)\Delta x - (x_t - x_p)\Delta y \geq 0$$

$$(y_u - y_p)\Delta x - (x_u - x_p)\Delta y \leq 0$$

$$(y_v - y_p)\Delta x - (x_v - x_p)\Delta y \leq 0$$

These are ***linear*** constraints. 🙂

But their number is $\Theta(|E|^2)$.

Avoiding Edge Crossings

- ***Algorithm:***
  - Solve QP allowing edge crossings
  - Check solution for edge crossings
  - Only add those constraints that forbid crossings found
  - Resolve QP.

$$(y_s - y_p)\Delta x - (x_s - x_p)\Delta y \geq 0$$

$$(y_t - y_p)\Delta x - (x_t - x_p)\Delta y \geq 0$$

$$(y_u - y_p)\Delta x - (x_u - x_p)\Delta y \leq 0$$

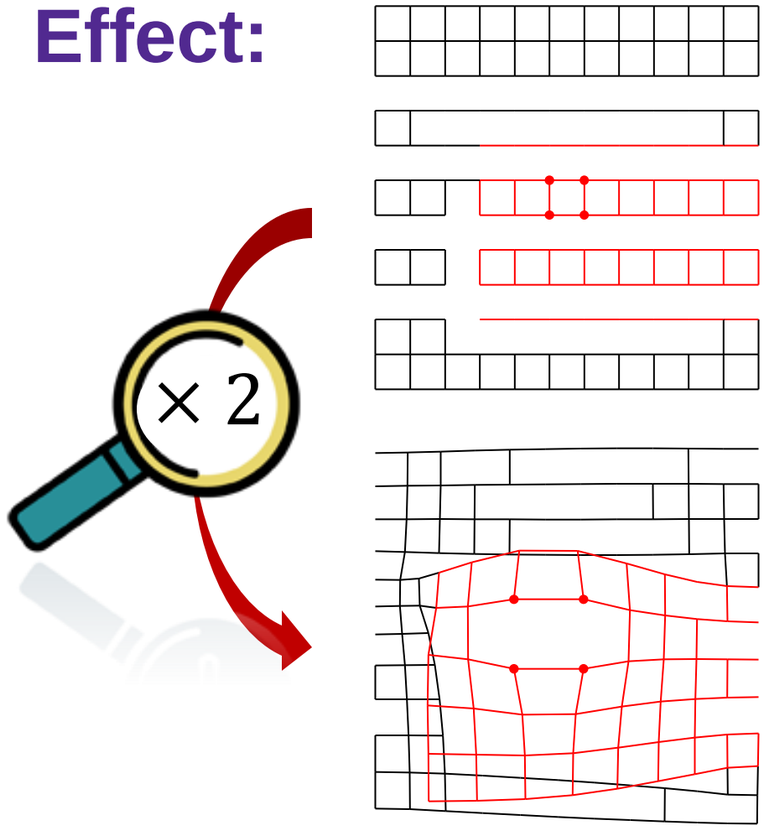
$$(y_v - y_p)\Delta x - (x_v - x_p)\Delta y \leq 0$$

These are ***linear*** constraints. 🙂

But their number is $\Theta(|E|^2)$.

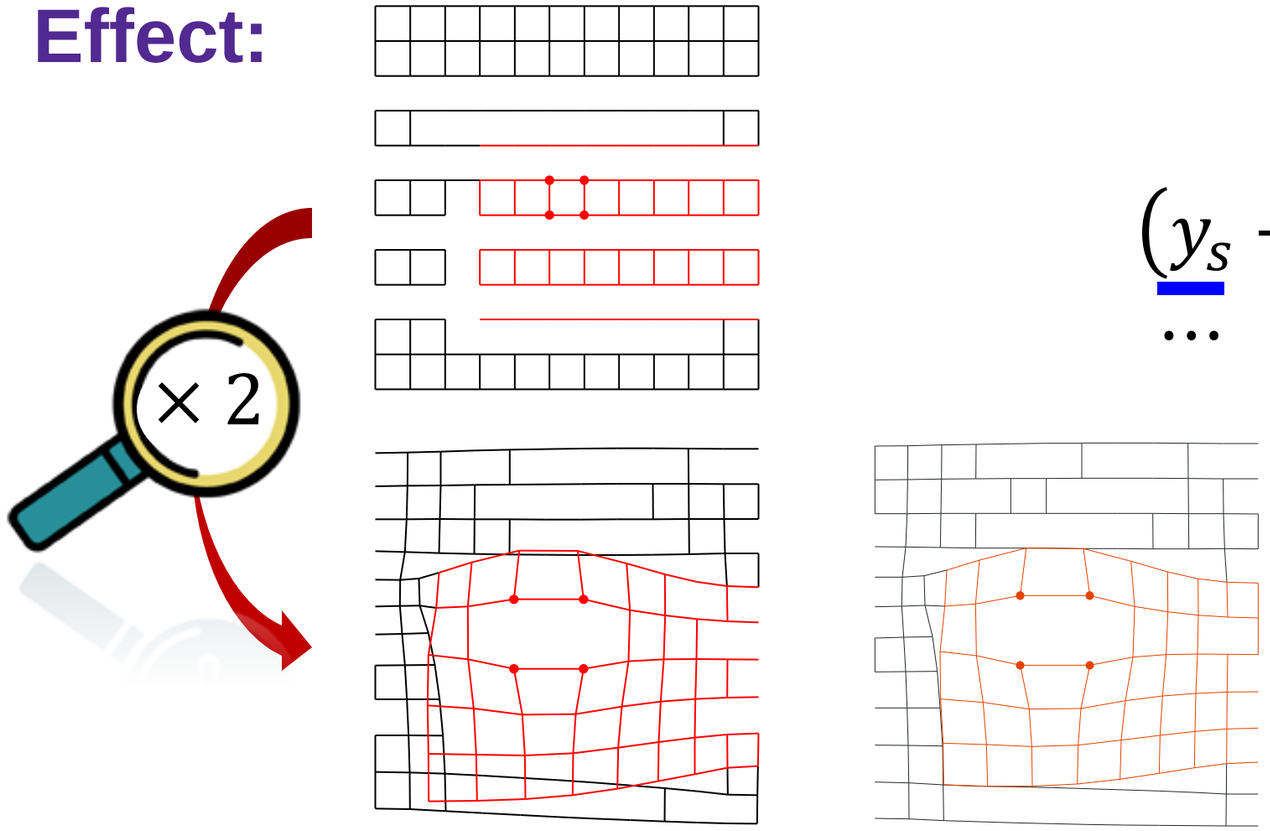
Basic Quadratic Program

Effect:



Basic Quadratic Program

Effect:

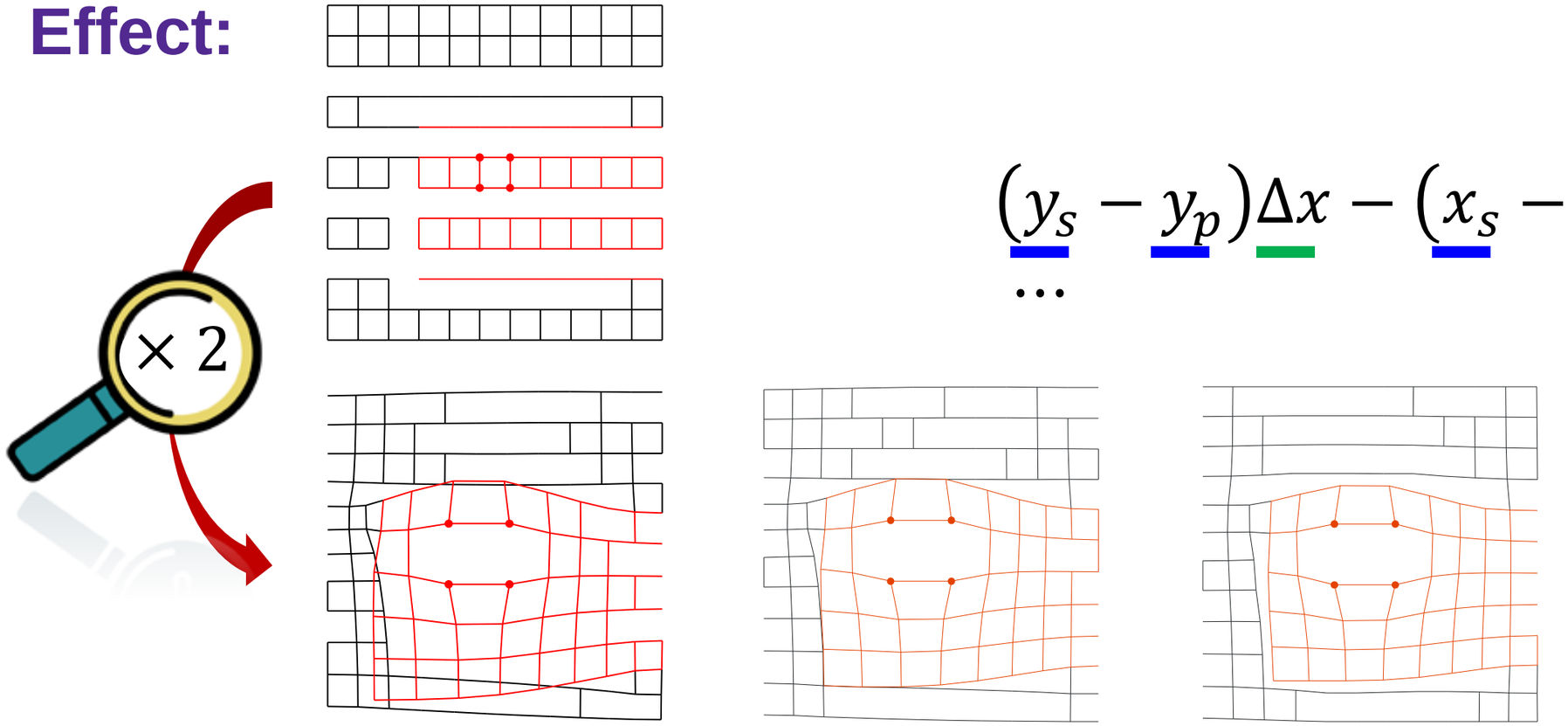


$$(y_s - y_p)\Delta x - (x_s - x_p)\Delta y \geq 0$$

$$\ldots$$

Basic Quadratic Program

Effect:

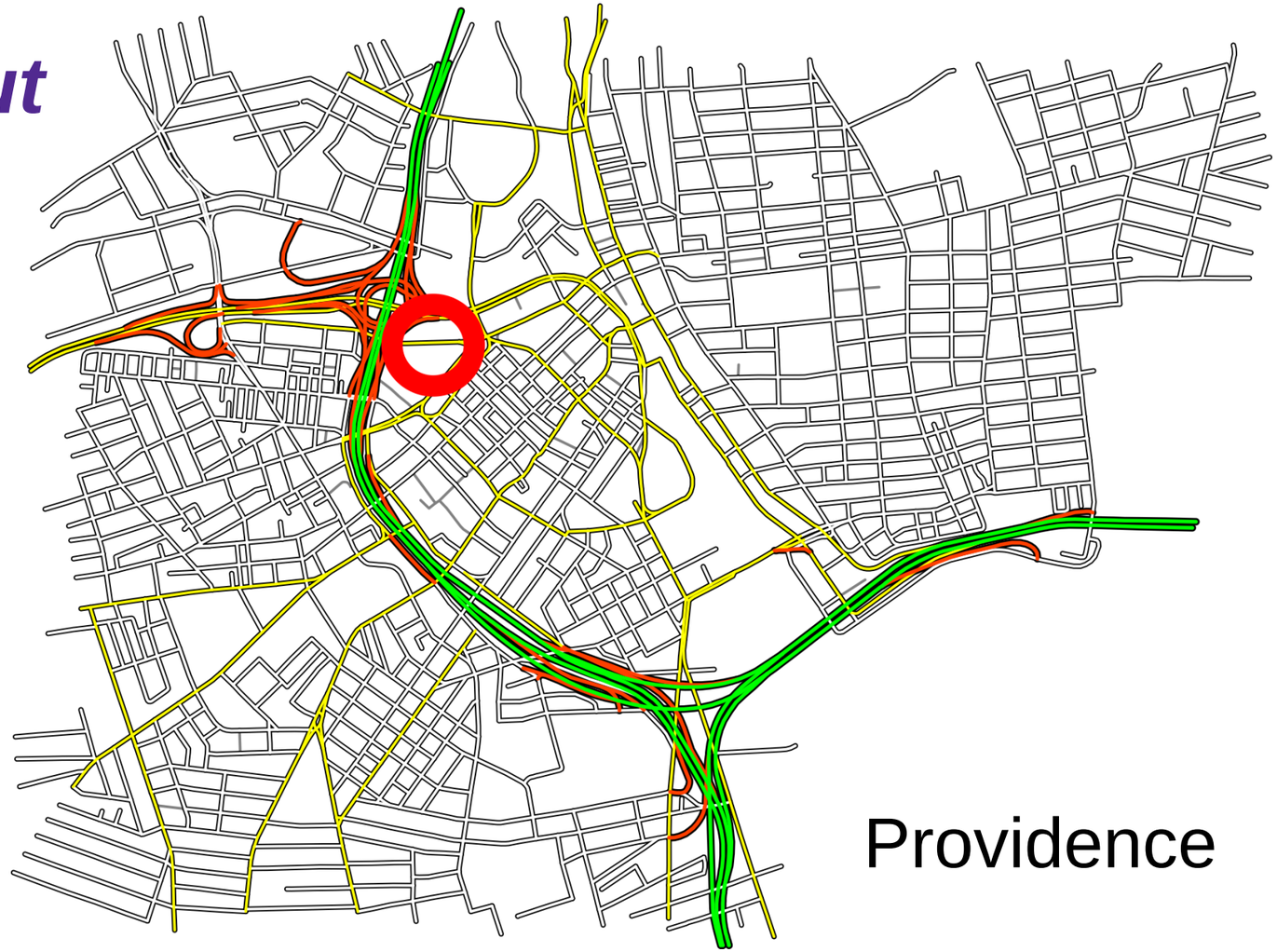


$$(y_s - y_p)\Delta x - (x_s - x_p)\Delta y \geq \varepsilon$$

…

Experiments

Input



Providence

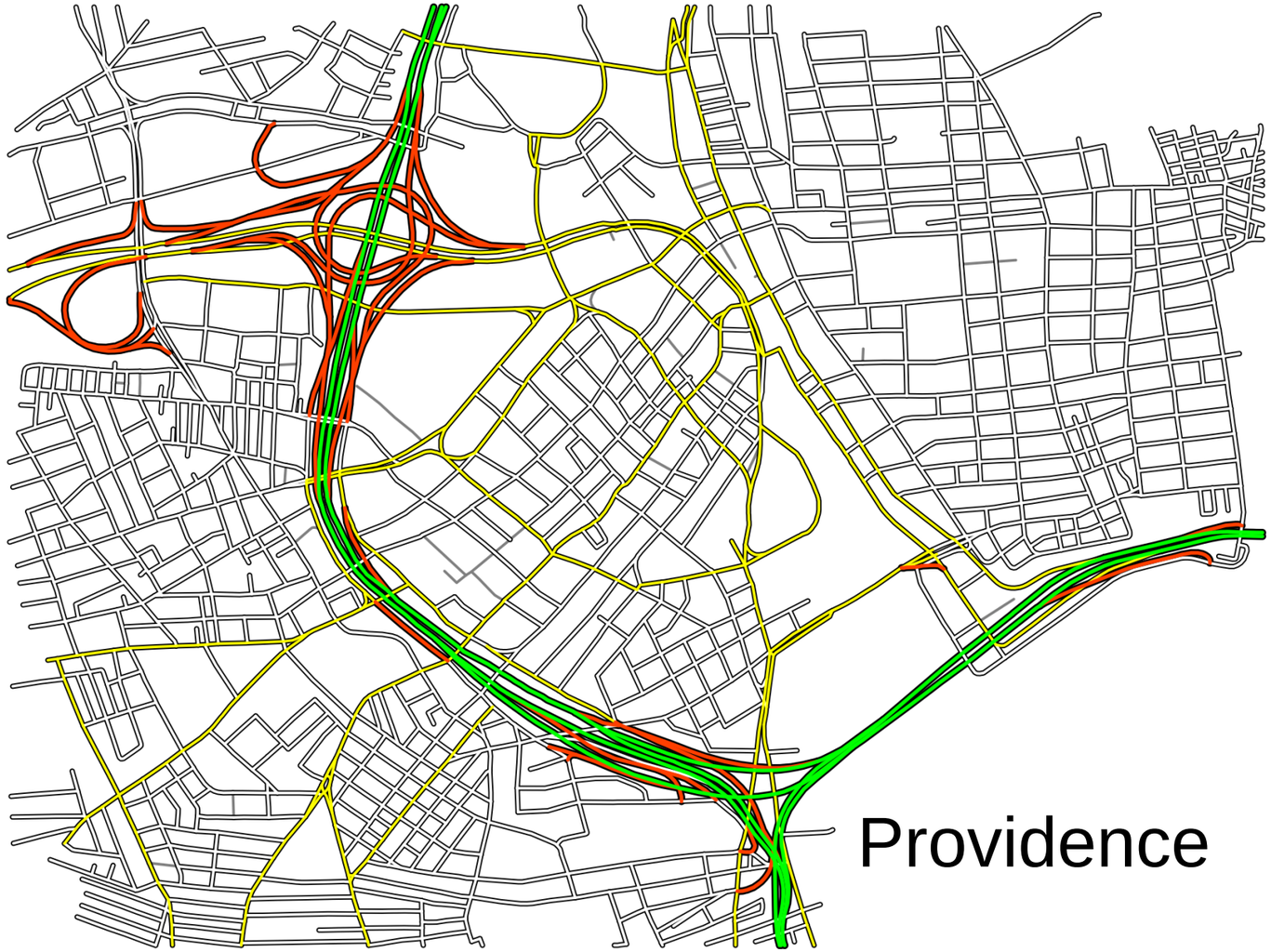
Experiments

Classical Fish-Eye View

Providence

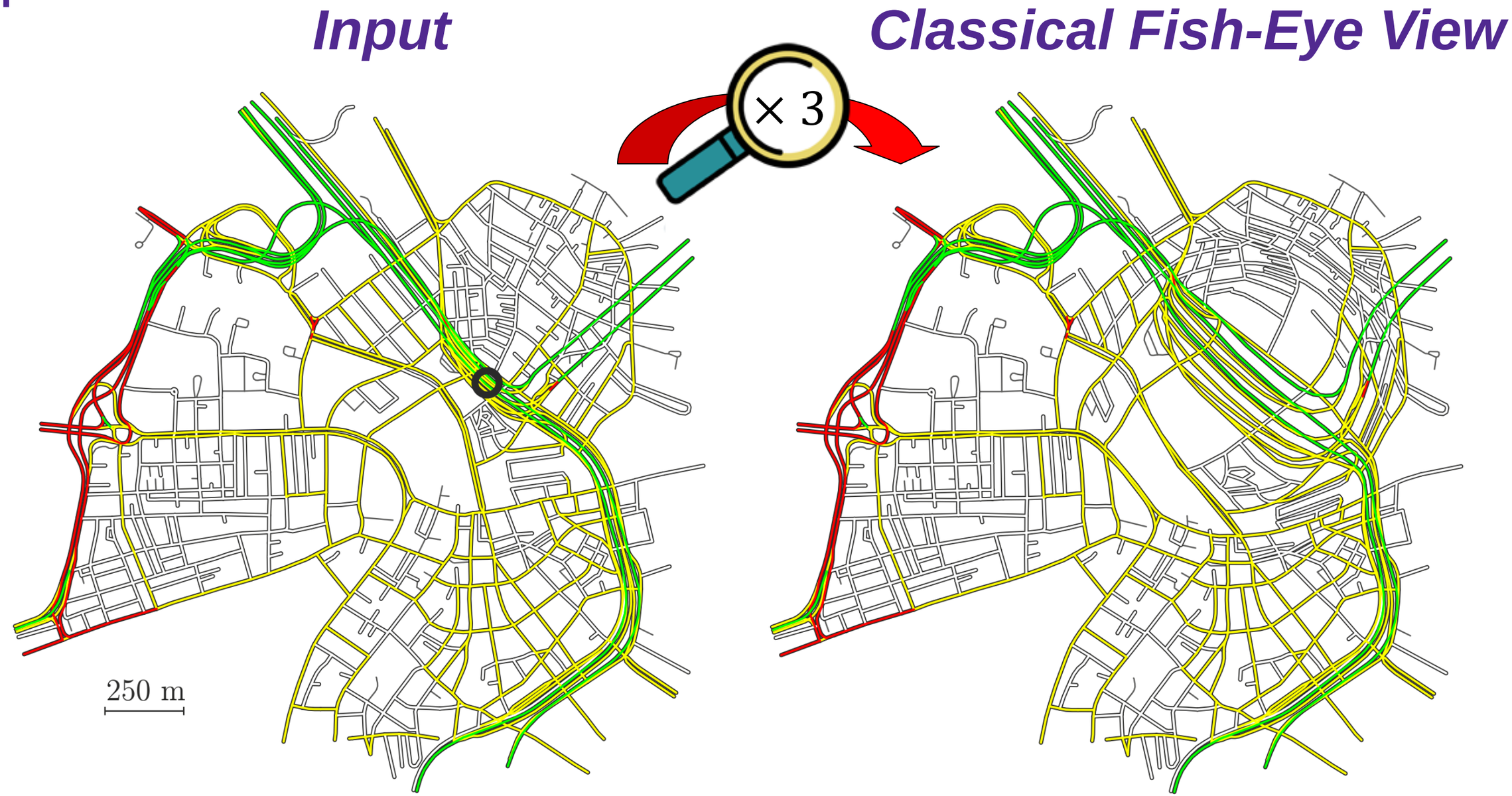
Experiments

Our Result

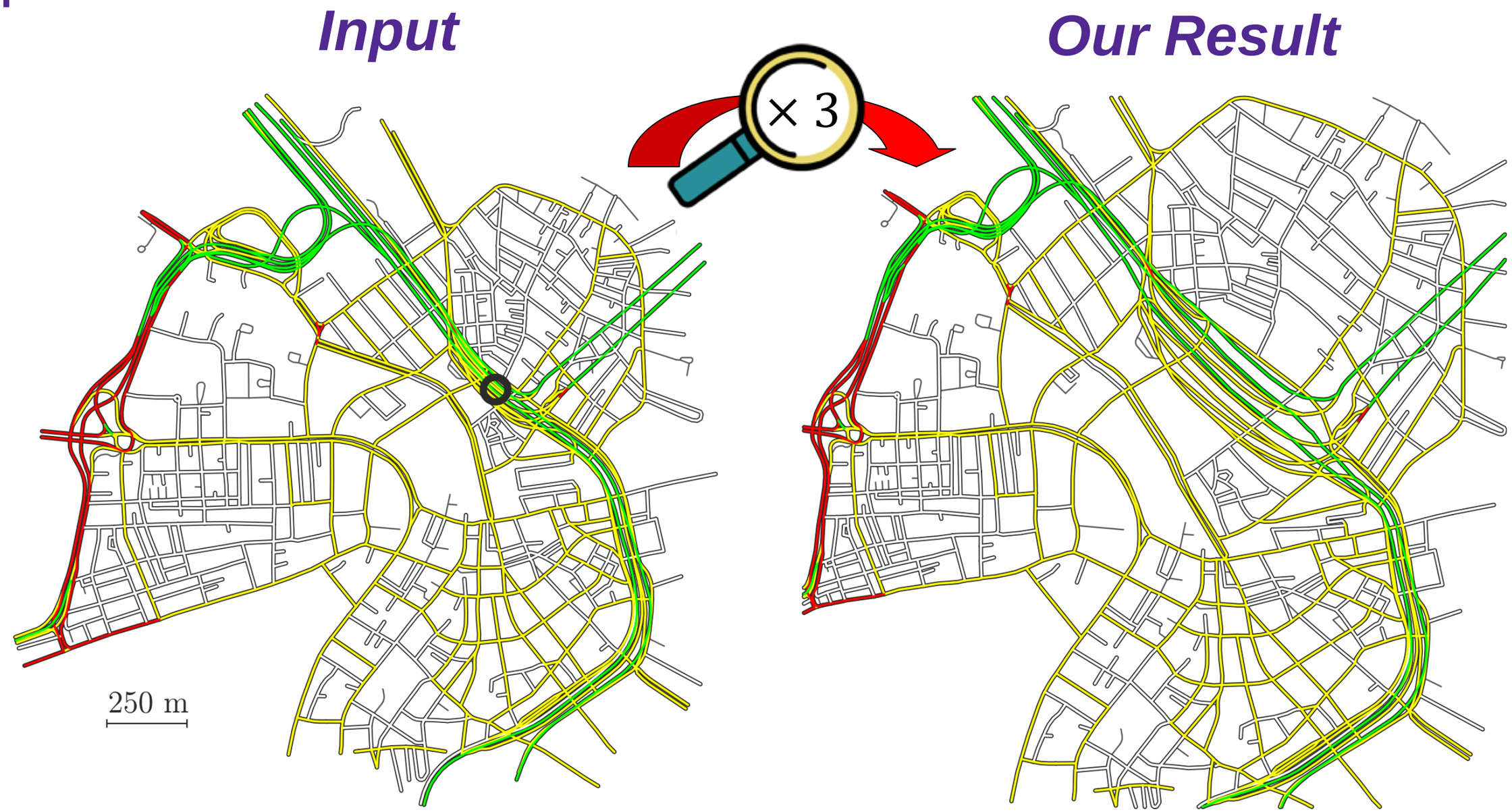


Experiments

Boston



Experiments



Experiments

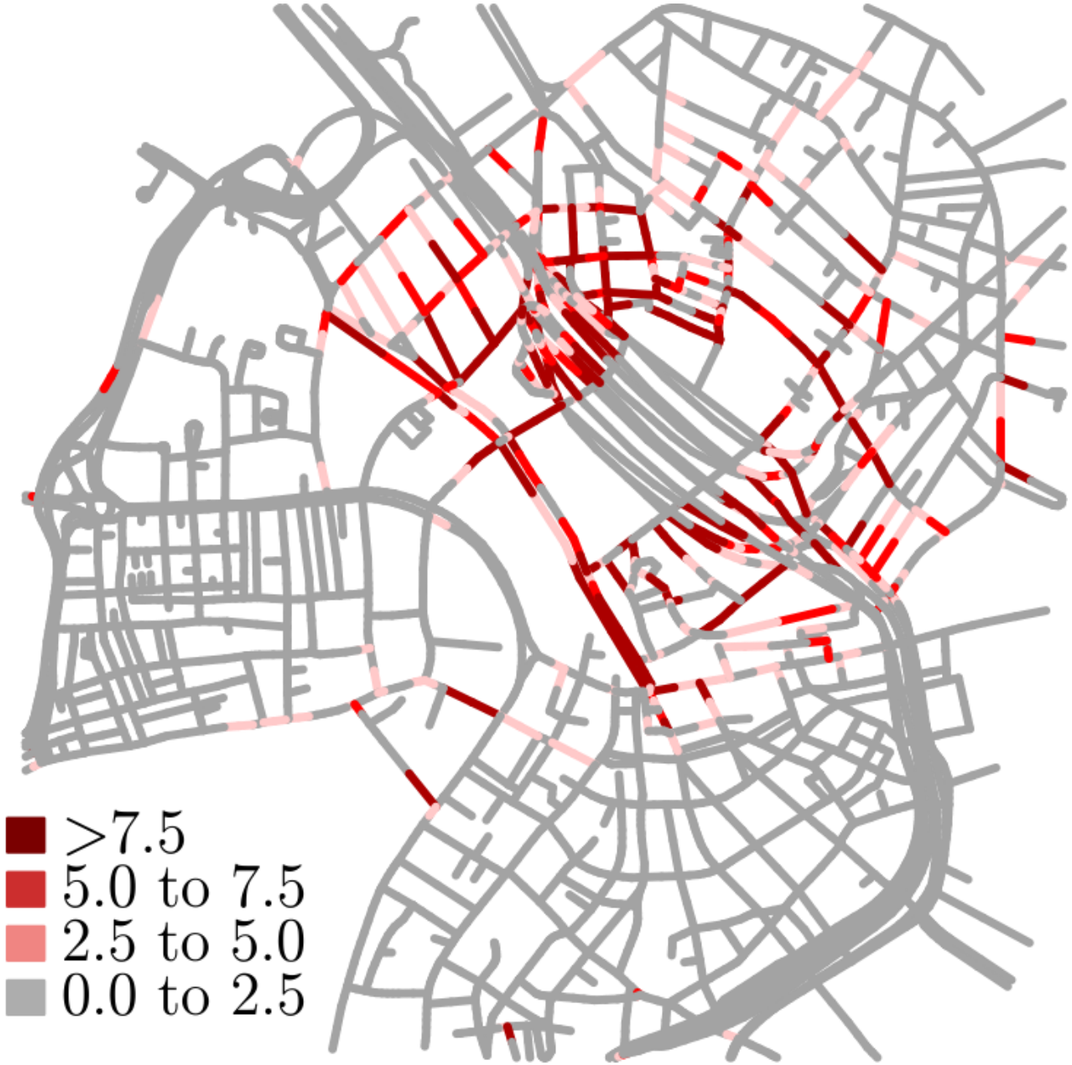
Classical Fish-Eye View

Our Result

Experiments

Classical Fish-Eye View

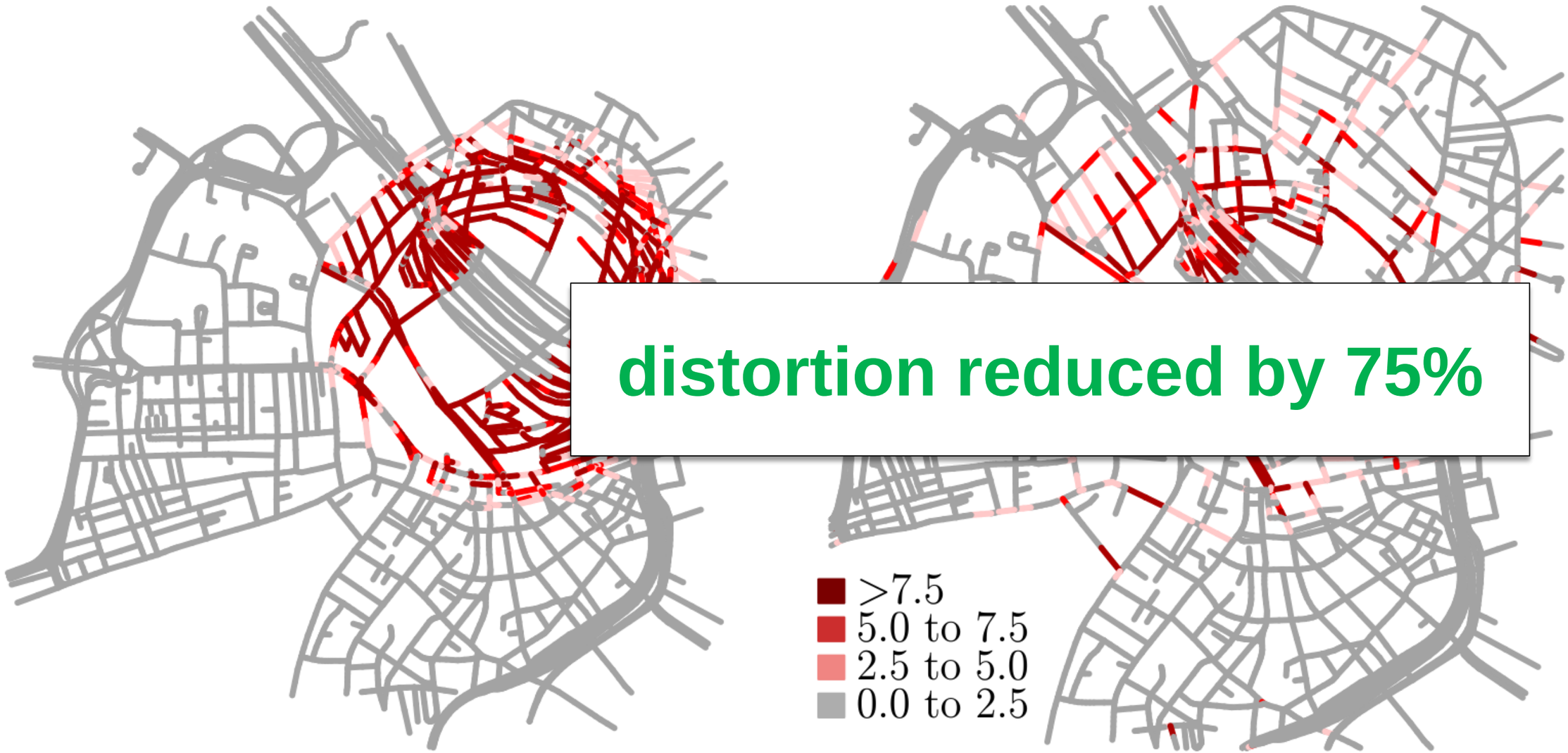
Our Result



Experiments

Classical Fish-Eye View

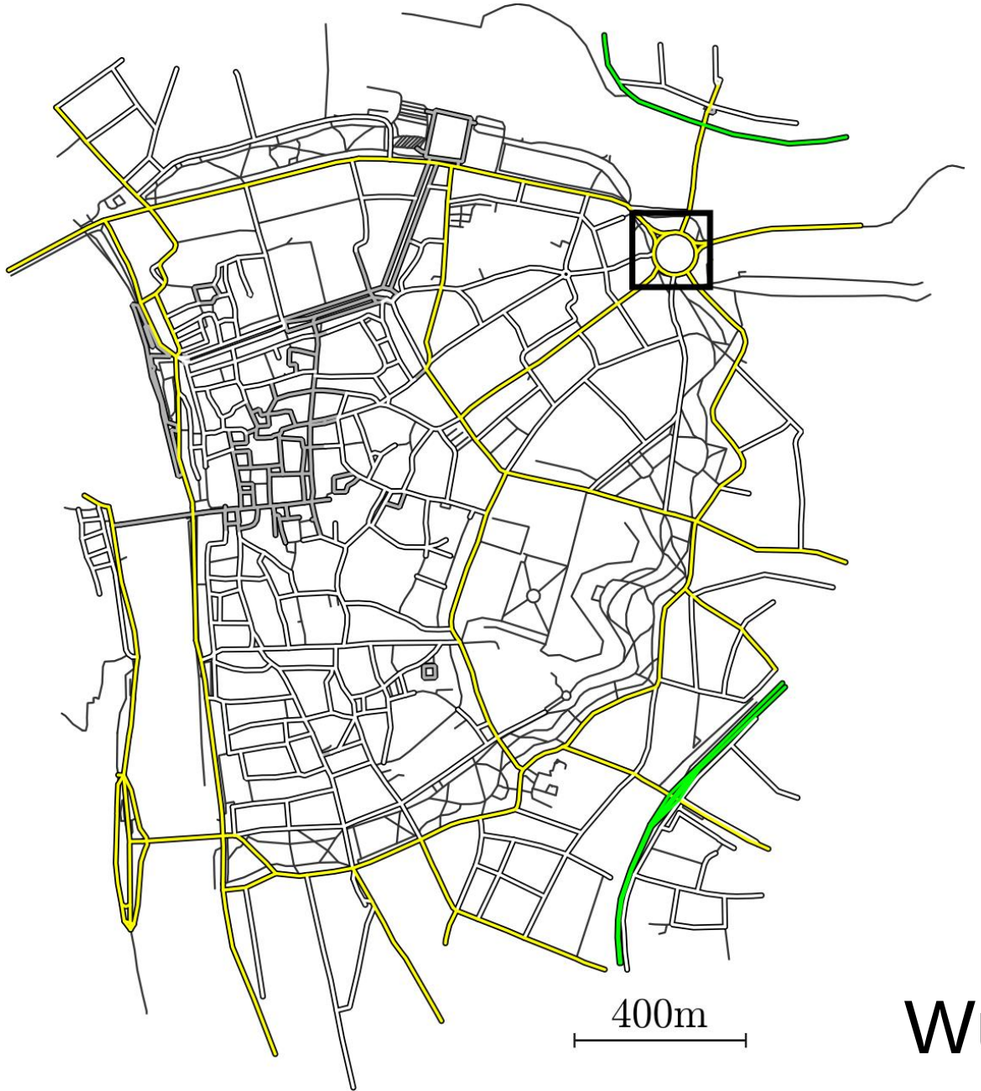
Our Result



distortion reduced by 75%

Experiments

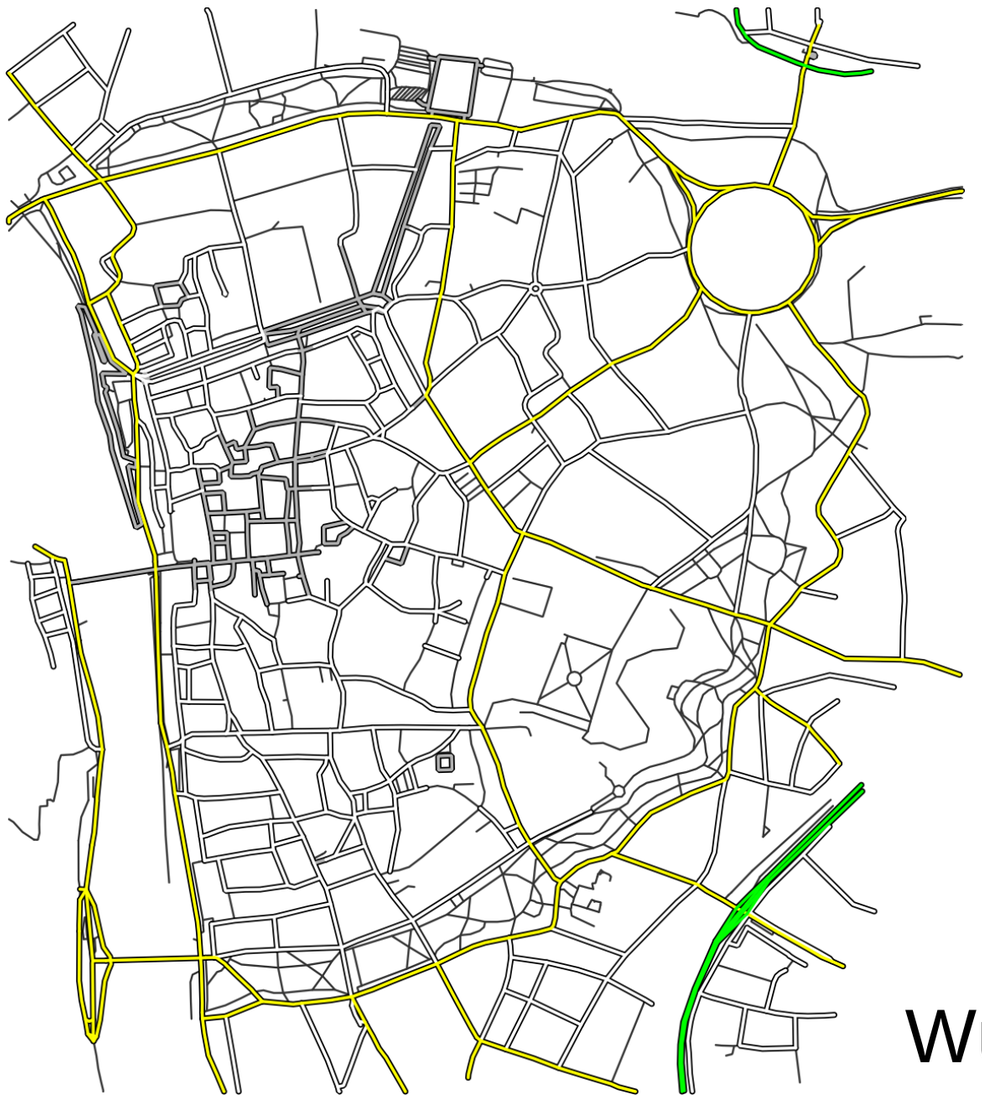
# nodes	# edges	solution time
465	513	1.48 s
2511	***2965***	***7.08 s***
15941	18158	61.70 s



Würzburg

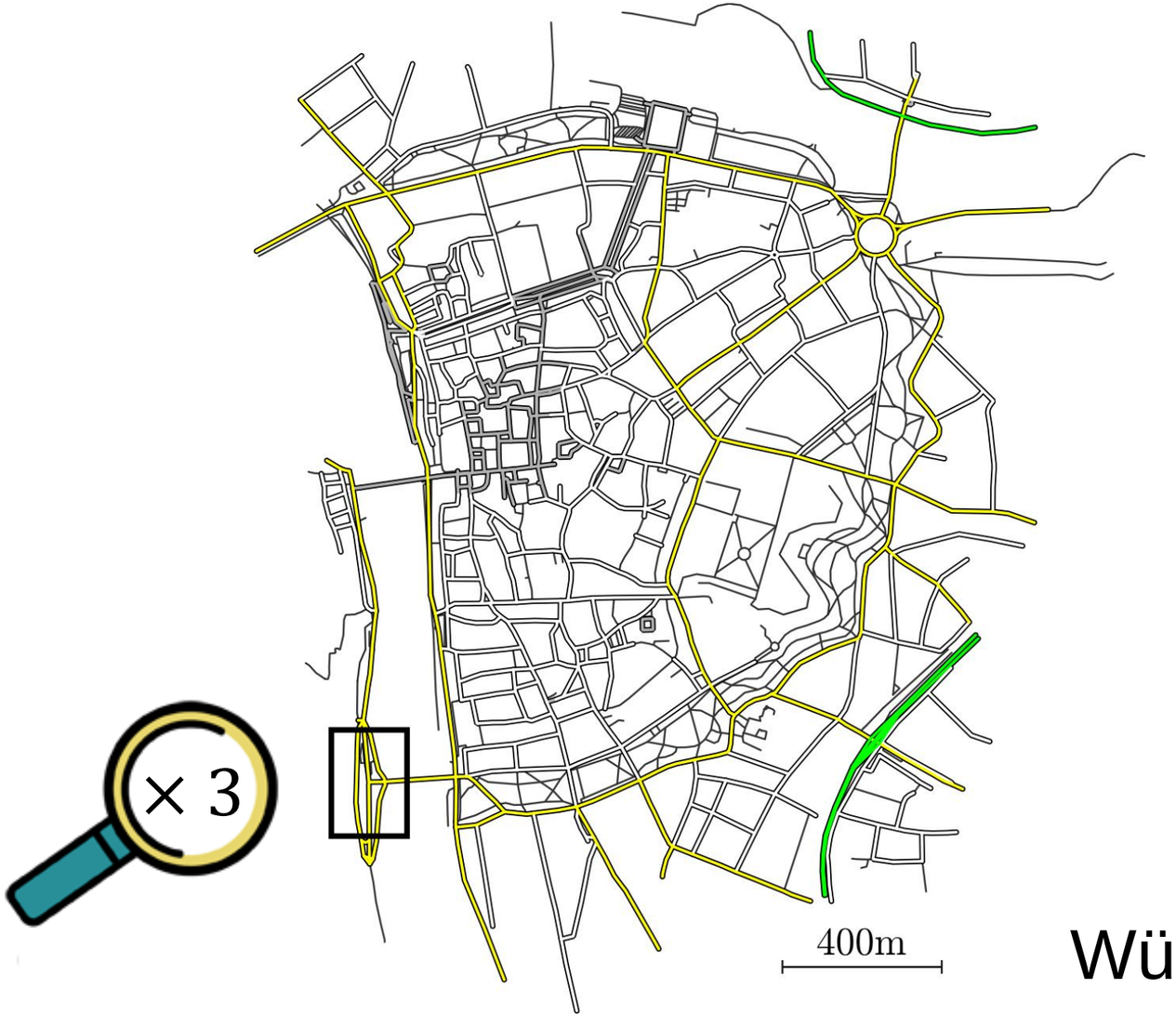
Experiments

# nodes	# edges	solution time
465	513	1.48 s
2511	***2965***	***7.08 s***
15941	18158	61.70 s



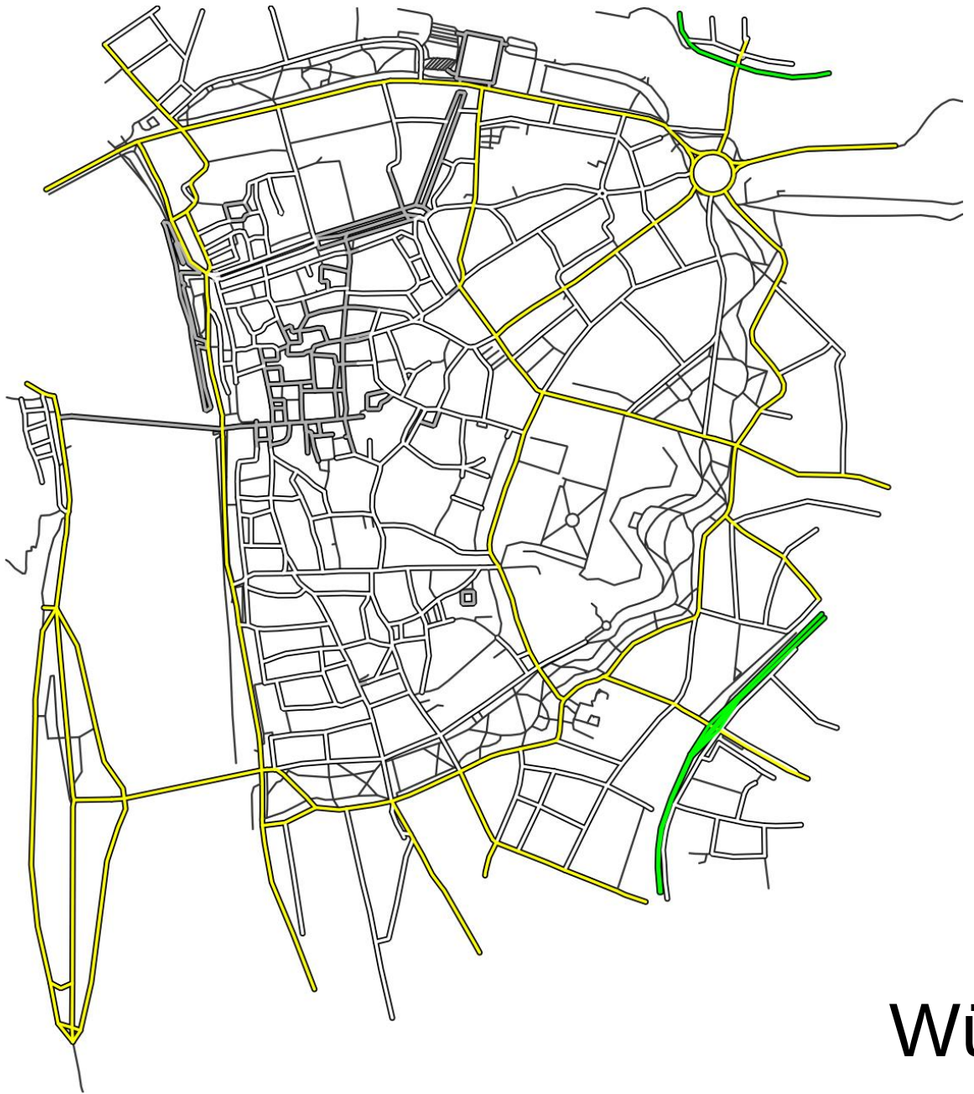
Würzburg

Experiments



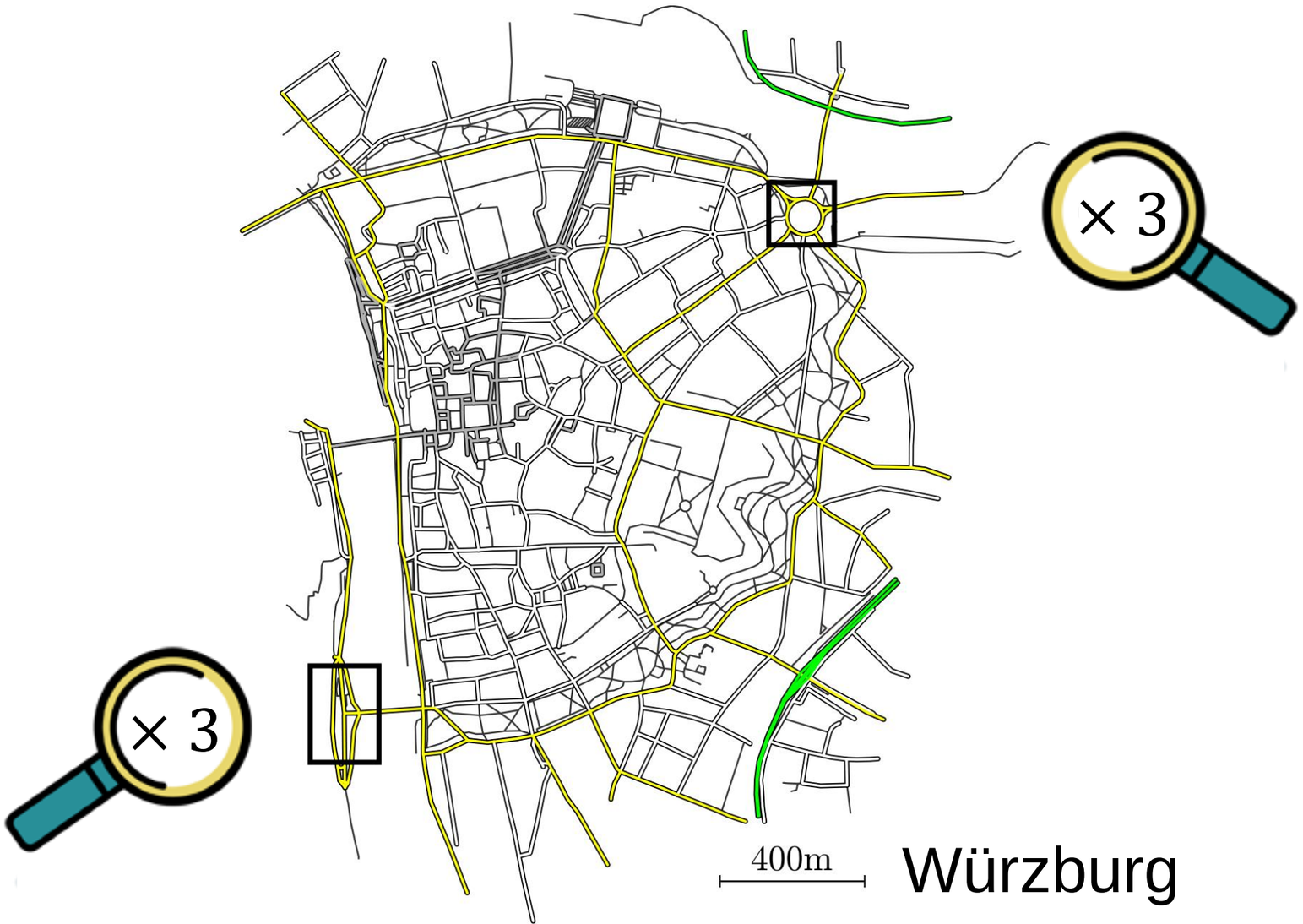
Würzburg

Experiments

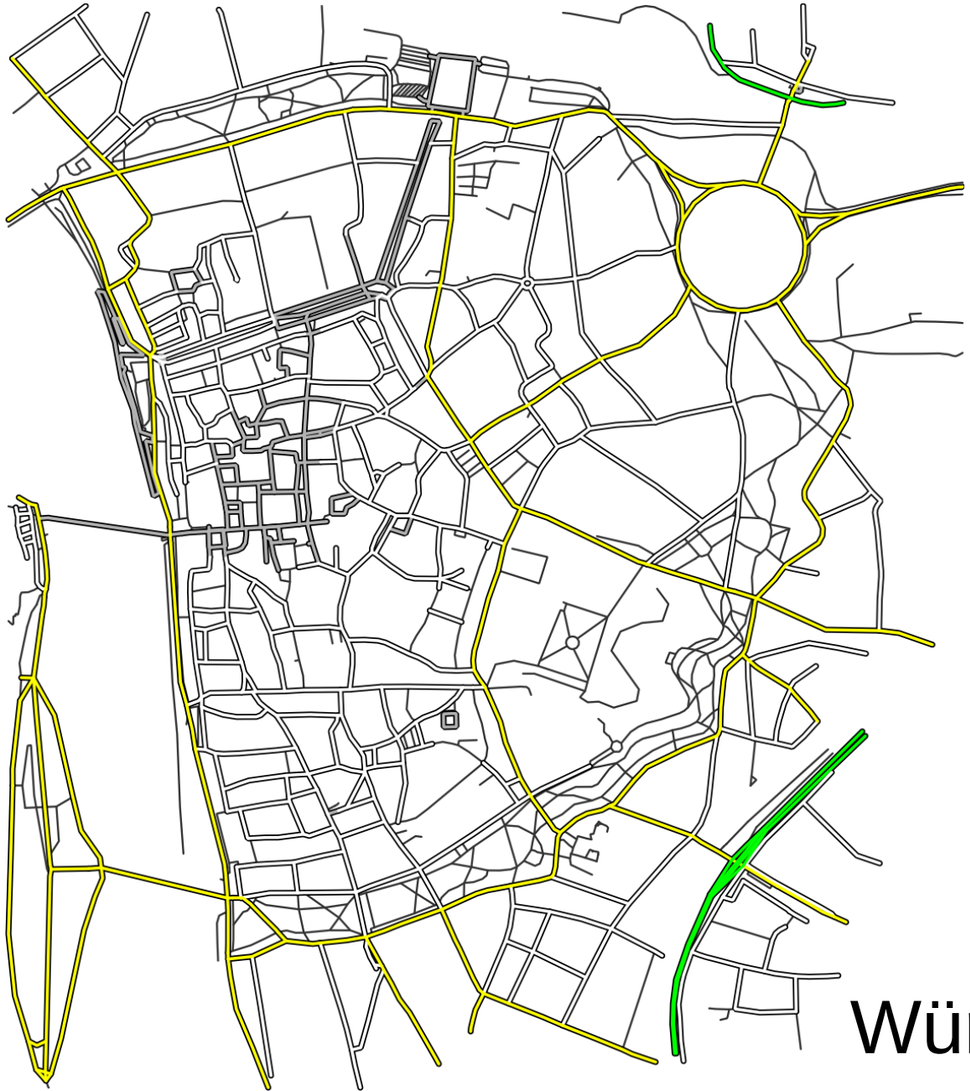


Würzburg

Experiments



Experiments

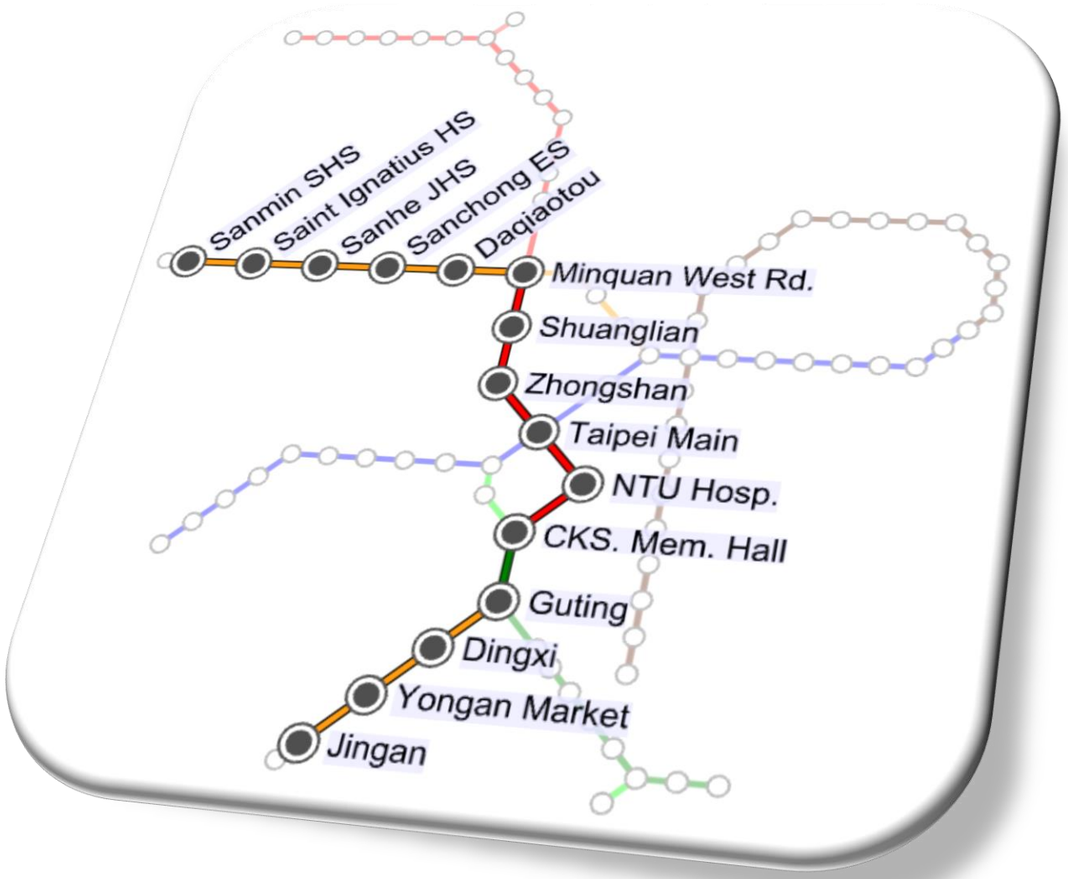


Würzburg

Conclusion

- New method for ***focus-and-context maps***
  - ***graph*** based
  - ***optimization*** based
  - ***deterministic*** and ***efficient***

  - results ***less distorted*** than with classical fish-eye views
  - ***speed up needed*** for application in real-time systems



Focus+Context Metro Maps

Motivation

- Metro maps of urban cities require large space for clear visualization.

- But mobile phones are usually small due to carry issues.

Motivation

Can you see the station names?

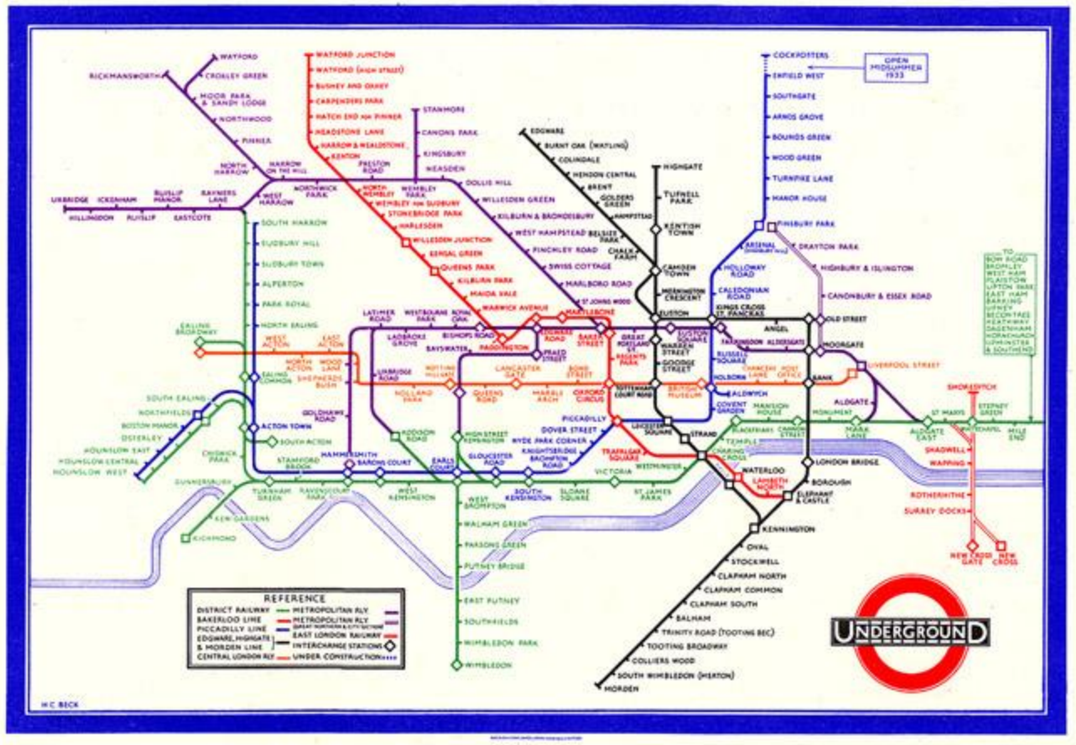
Yes, you can!

Why Focus+Context?

- Display large datasets in a small area
- Due to the limited space,
  - Focal stations are highlighted for route navigation
  - Contextual stations show the principle geographic information of transportation lines

Metro map characteristics

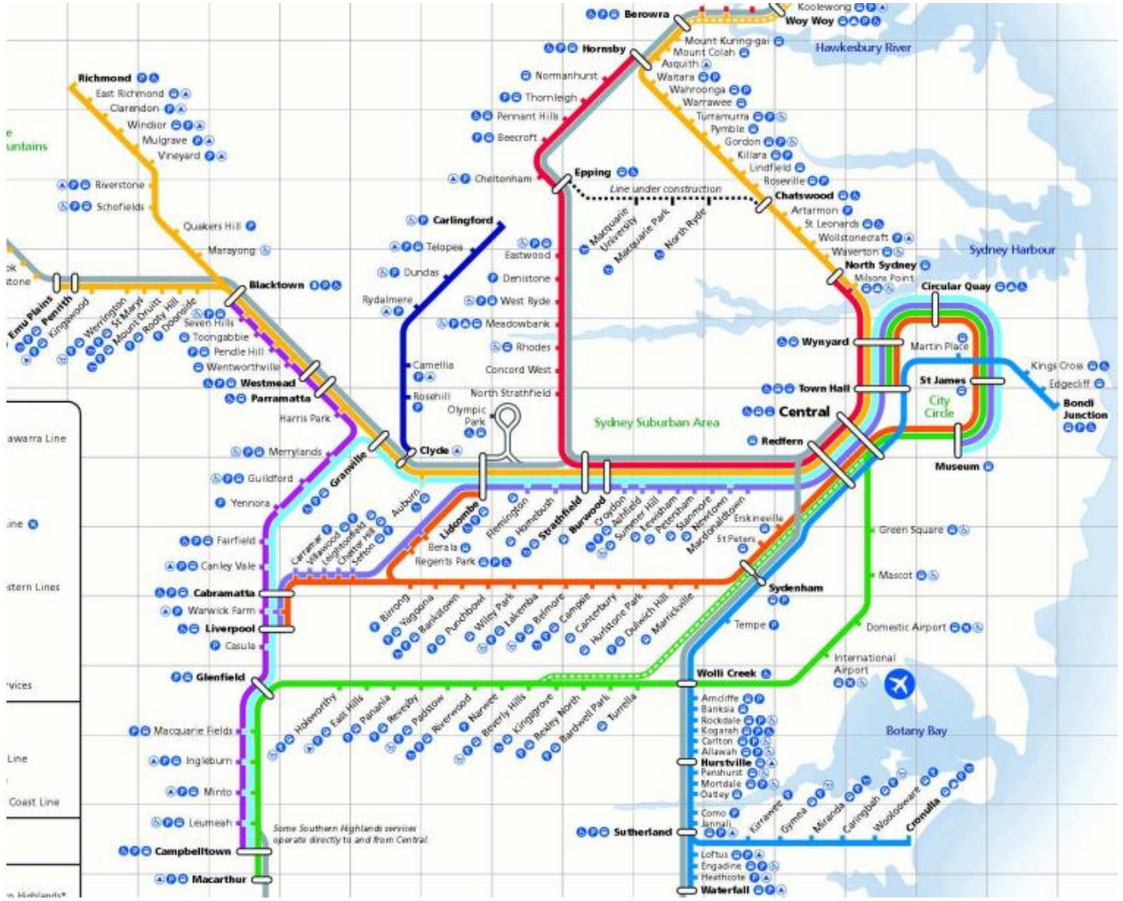
- Schematic metro map design
  - Henry Beck’s map - London Underground, 1933



[Beck 1933]

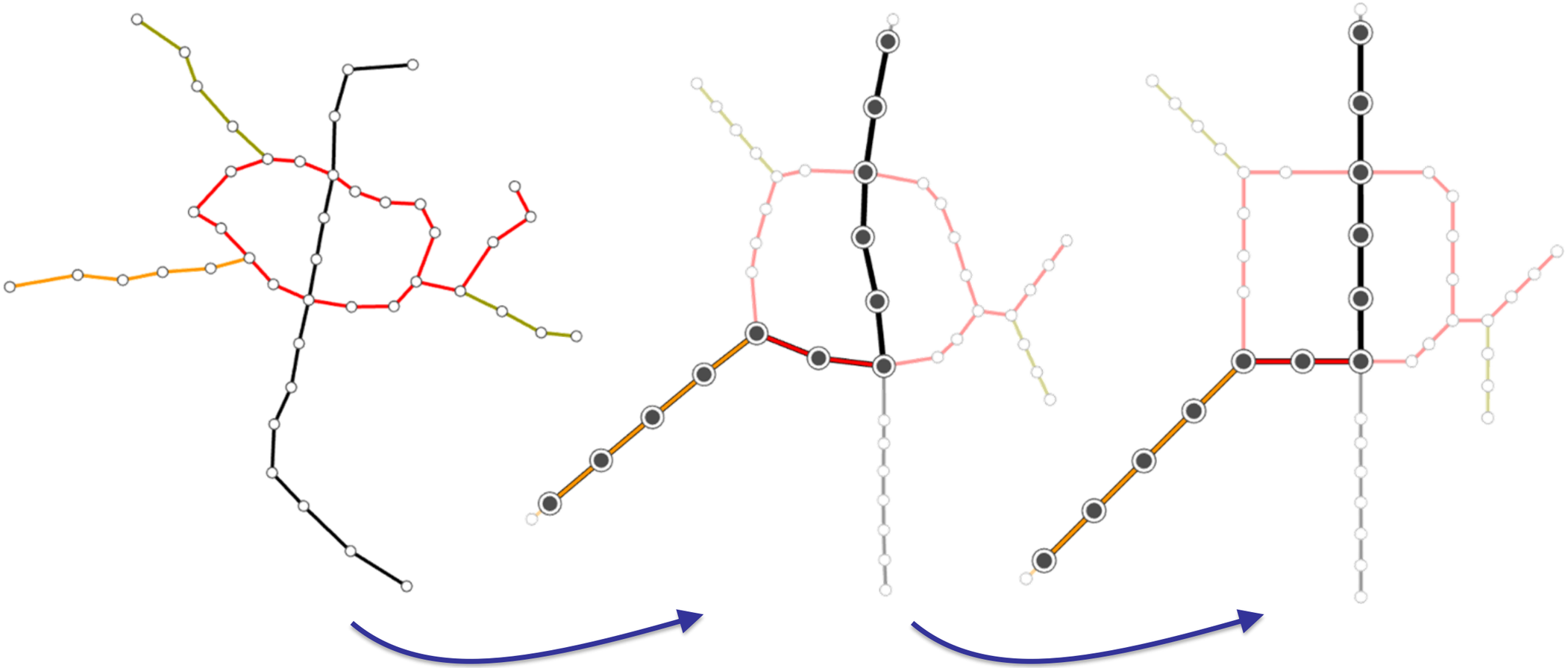
Metro map characteristics

- Straight lines
- Evenly spaced stations
- Maximal angles of incident edges
- Octilinear edge directions
  - Discrete
  - Highly non-linear



official layout of Sydney metro map

Two-steps graph deformation



Smooth deformation

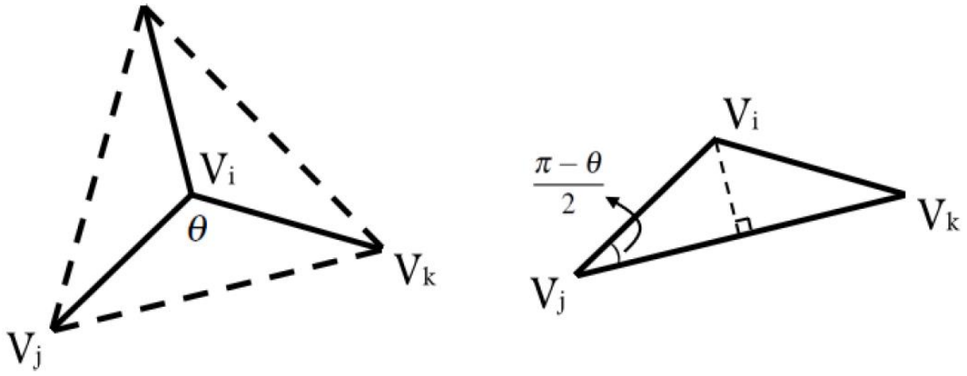
- Regular edge lengths
  - to achieve high regularity

$$\Omega_\ell = \sum_{\{i,j\}\in E} \left|(\mathbf{v}'_i - \mathbf{v}'_j) - s_{ij}\mathbf{R}_{ij}(\mathbf{v}_i - \mathbf{v}_j)\right|^2, \quad \text{where}$$

$$\mathbf{v}'_i \in \mathbf{V}', \quad s_{ij} = \frac{D}{|\mathbf{v}_i - \mathbf{v}_j|}, \quad \mathbf{R}_{ij} = \begin{bmatrix} cos\theta_{ij} & -sin\theta_{ij} \\ sin\theta_{ij} & cos\theta_{ij} \end{bmatrix}, \; D = \{\alpha, \beta\}$$

Smooth deformation

- Maximal angles of incident edges
  - to distinguish different transportation lines



$$\Omega_m = \sum_{\mathbf{v}'_i \in \mathbf{V}'} \sum_{\{\{i,j\},\{i,k\}\} \in N(i)} \left| \mathbf{v}'_i - (\mathbf{v}'_j + \mathbf{u}'_{jk} + \tan(\frac{\pi - \theta}{2})\mathbf{u}'_{jk}) \right|^2,$$

$$\text{where} \quad \mathbf{u}'_{jk} = \frac{1}{2}(\mathbf{v}'_k - \mathbf{v}'_j).$$

Smooth deformation

- Straight transportation lines
  - Maximal angles of two incident edges

$$\Omega_m = \sum_{\mathbf{v}'_i \in \mathbf{V}'} \sum_{\{\{i,j\},\{i,k\}\} \in N(i)} \left| \mathbf{v}'_i - (\mathbf{v}'_j + \mathbf{u}'_{jk} + \tan(\overset{0}{\cancel{\frac{\pi - \theta}{2}}})\mathbf{u}'_{jk}) \right|^2 ,$$

$$\text{where} \quad \mathbf{u}'_{jk} = \frac{1}{2}(\mathbf{v}'_k - \mathbf{v}'_j).$$

$$\Omega_m = \sum_{\mathbf{v}'_i \in \mathbf{V}'} \sum_{\{i,j\},\{i,k\} \in E} \left| \mathbf{v}'_i - \frac{1}{2}(\mathbf{v}'_j + \mathbf{v}'_k) \right|^2$$

Smooth deformation

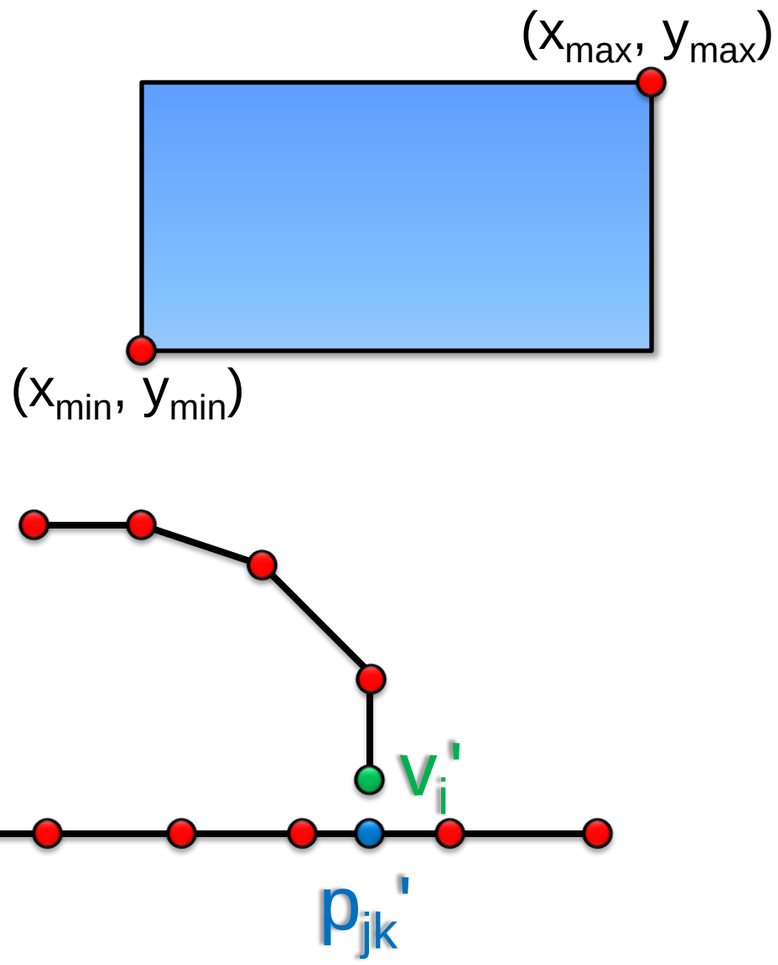
- Positional constraints
  - to retain basic geographic information

$$\Omega_g = \sum_{\mathbf{v}'_i \in \mathbf{V}'} |\mathbf{v}'_i - \mathbf{v}_i|^2.$$

Smooth deformation

- By integrating Ω_m, Ω_ℓ, Ω_g, we solve

$$\Omega = \Omega_m + w_\ell \Omega_\ell + w_g \Omega_g \quad \text{subject to}$$
$$x_{min} < \mathbf{v}'_{i,x} < x_{max}, \quad y_{min} < \mathbf{v}'_{i,y} < y_{max},$$
$$\text{and} \quad \mathbf{v}'_i - \mathbf{p}'_{jk} \geq \varepsilon, \quad \mathbf{v}'_i \in \mathbf{V}', \{j,k\} \in E,$$



Route octilinearity

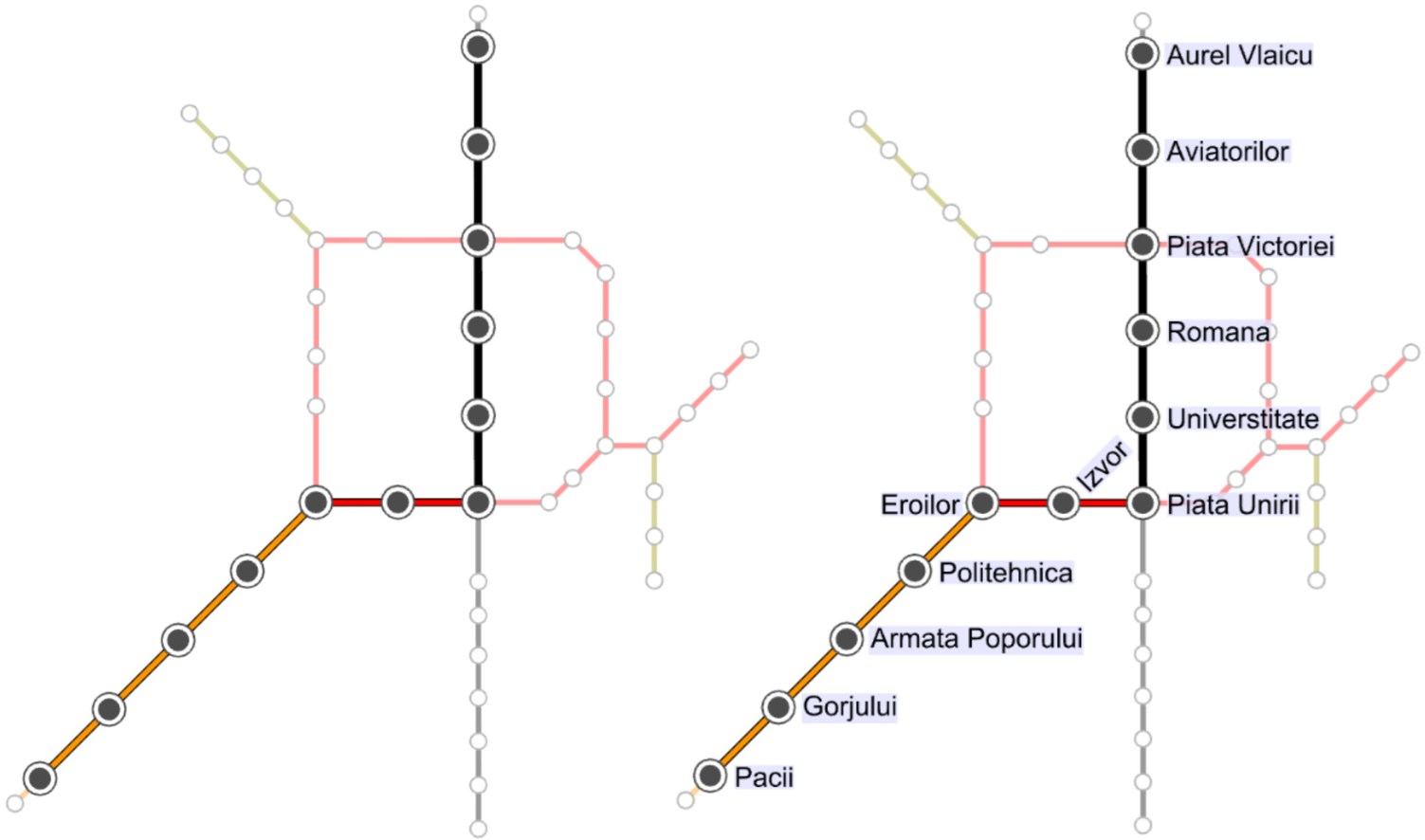
- Rotate each edge to the closest octilinear direction and minimize $\Omega_g+\Omega_o$, where

$$\Omega_o = \sum_{\{i,j\}\in E} |(\tilde{\mathbf{v}}_i - \tilde{\mathbf{v}}_j) - f(\mathbf{v}'_i - \mathbf{v}'_j)|^2$$

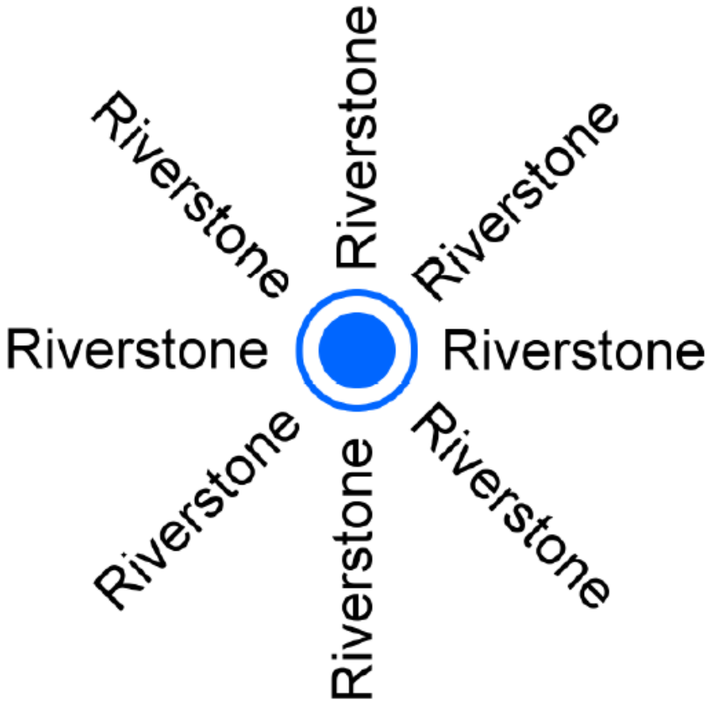
- Note that, Ω_m, Ω_ℓ, are not included in this step because $f(v_i'-v_j')$ already implies regular lengths and the right directions.

Labeling

- Preferential position
- Label occlusion
- Spaciousness
- Coherence

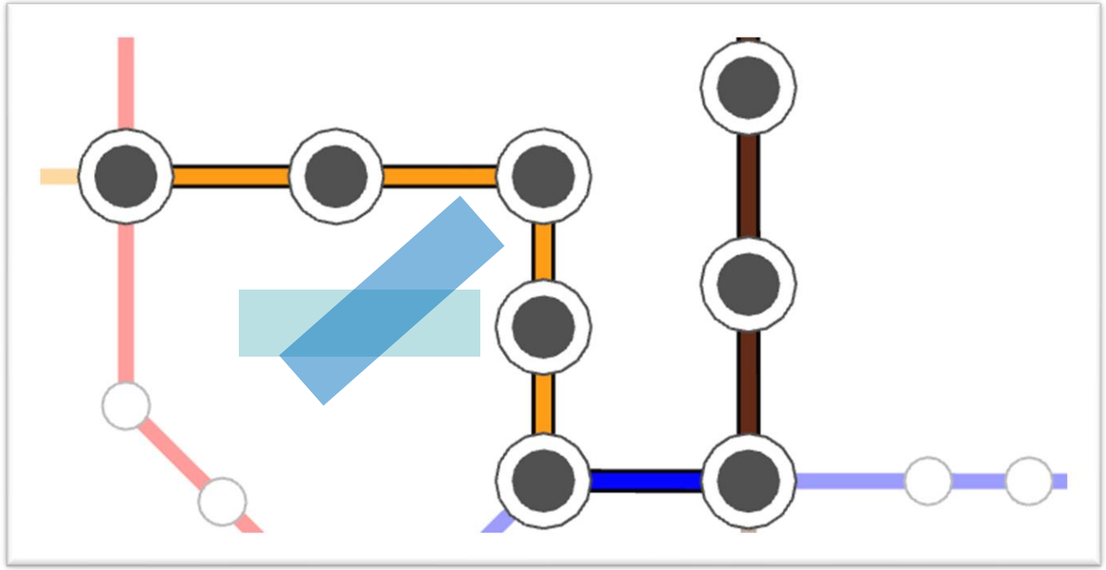


Preferential label direction



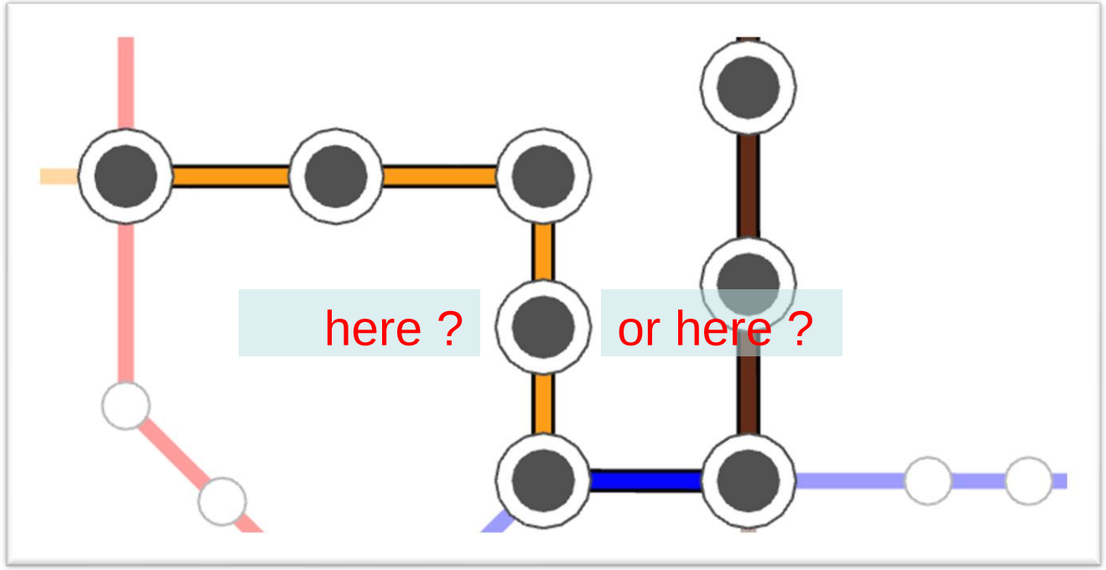
$\mathbf{h}_i$	$\Psi_p(\mathbf{h}_i)$
east	0
west	0
north	10
south	10
north-east	2
south-east	5
south-west	5
north-west	2

Label occlusion



$$\Psi_q(\mathbf{h}_i, \mathbf{h}_j) = \begin{cases} 1 & \text{if } \mathbf{q}_i \text{ intersects } \mathbf{q}_j \\ 0 & \text{else} \end{cases}$$

Spaciousness



$$\Psi_e(\mathbf{h}_i) = \begin{cases} 1 & \text{if } t < 0.5 \\ 1 - 2(t - 0.5) & \text{if } t > 0.5 \\ 0 & \text{if } t > 1 \end{cases}, \text{ where } t = \frac{|\mathbf{c}_i - \tilde{\mathbf{v}}_i|}{\mathbf{qq}_{i,d}},$$

For each label h_i , we magnify its represented quad q_i two times larger along the octilinear direction, and denote it as qq_i. c_i is the closest intersection point to the node $\tilde{v}_i$ and $qq_{i,d}$ is the diagonal length of qq_i.

Coherence

- Labels should appear at coherent positions
  - to improve readability

$$\Psi_s(\mathbf{h}_i, \mathbf{h}_j) = \begin{cases} 0 & \text{if } \mathbf{h}_i = \mathbf{h}_j \\ 1 & \text{else} \end{cases}, \quad \text{where } \{i, j\} \in E,$$

Multi-label optimization problem

- Solve for label positions by minimizing

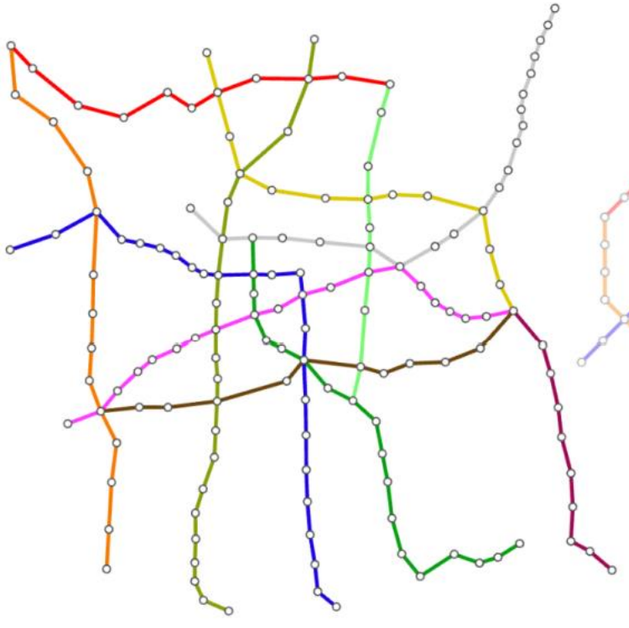
$$\Psi = \sum_{\mathbf{h}_i \in \mathbf{H}} (w_p \Psi_p(\mathbf{h}_i) + w_b \Psi_b(\mathbf{h}_i) + w_e \Psi_e(\mathbf{h}_i)) + \sum_{\mathbf{h}_i, \mathbf{h}_j \in \mathbf{H}} w_q \Psi_q(\mathbf{h}_i, \mathbf{h}_j) + \sum_{\{i,j\} \in \mathbf{E}} w_s \Psi_s(\mathbf{h}_i, \mathbf{h}_j).$$

- For the optimization details, please refer to

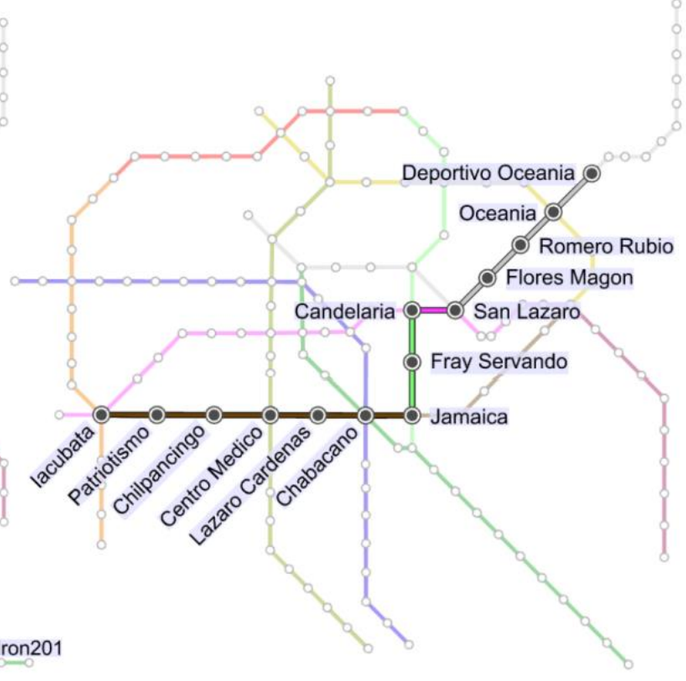
  Fast approximate energy minimization via graph cuts

Solna Centrum
Vastra Skogen
Stadshagen
Fridhemsplan
Radhuset
T-Centralen
Gamla Stan
Mariatorget
Slussen
Zinkensdamm
Hornstull
Liljeholmen
Midsommarkransen
Telefonplan

Fridhemsplan
Radhuset
T-Centralen
Thorildsplan
Kristineberg
Alvik
Stora Mossen
Abrahamsberg
Brommaplan
Akeshov
Angbyplan
Islandstorget
Blackeberg



Indios Verdes
Deportivo18de Marzo
Potrero
La Raza
Misterios
Valle Gomez
Consulado
Canaldel Norte
Morelos
Candelaria
Fray Servando
Jamaica
Santa Anita
Coyuya
Iztacalco
Apatlaco
Aculco
Escuadron201


Deportivo Oceania
Oceania
Romero Rubio
Flores Magon
Candelaria
San Lazaro
Fray Servando
Jamaica
Iacubata
Patriotismo
Chilpancingo
Centro Medico
Lazaro Cardenas
Chabacano

Scalability problem

- The passenger’s route is too long, or the displaying area is too small.
  - Highlight only the stations that transportation lines incident with each other