High Performance Numerical Solvers

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Excercise 2. Try to find a problem where the sync method converges but the async method does not converge.

We can consider a linear system $Ax = b$, where

$$A = \begin{bmatrix} 2 & -1 & 1 & 0 \\ 1 & 2 & 1 & 0 \\ 1 & 0 & 2 & 1 \\ 1 & 1 & 0 & 2 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}.$$

The T matrix for Jacobi method is

$$T = \begin{bmatrix} 0 & 0.5 & -0.5 & 0 \\ -0.5 & 0 & -0.5 & 0 \\ -0.5 & 0 & 0 & -0.5 \\ -0.5 & -0.5 & 0 & 0 \end{bmatrix}.$$

The sync method converges, since $\rho(T) = 0.5 < 1$; however, the async method does not necessarily converge, since $\rho(|T|) = 1$. Following is a simmulation result using **chazan_miranker.m** with **update_prob**= 0.5, **delay_bound**= 30 and **max_time**= 10000.

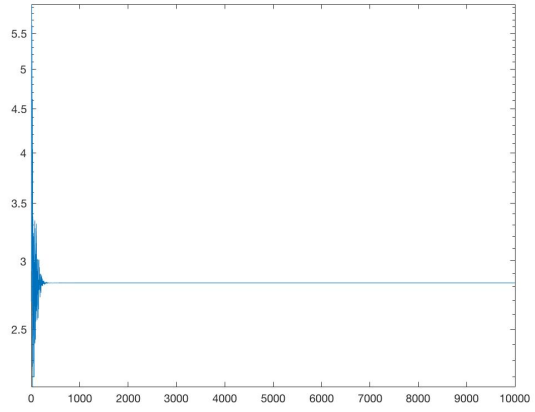

Figure 1: chazan_miranker(A,b,0.5,30,10000)

We can see that the sequence of residuals does not converge to 0.