On the random behavior of asynchronous computing

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1 Introduction

In this project, we investigate the random behavior of asynchronuous computing. Given a system $Ax = b$, an iterative method solve it by letting

$$x^{(k+1)} = Tx^{(k)} + c,$$

where T is an iteration matrix, and c is a constant vector.

According to [1], asynchronous method is guaranteed to converge if and only if the spectrum, $\sigma(|T|)$, is strictly less than one, which is different from the criterion of synchronous computing scheme, $\sigma(T) < 1$. However, it does not imply that an asynchronous method will always diverge when $\sigma(|T|) \geq 1$.

An interesting question then arises naturally. How likely is an asychronous scheme going to converge when its iteration matrix does not satisfy the crierion in [1]?

2 Main Result

Consider $A_\alpha = \alpha^{-1}D + R$. The iteration matrix for solving $A_\alpha x = b$ with Jacobi's method is

$$T_\alpha = -(\alpha^{-1}D)^{-1}R = -\alpha D^{-1}R \quad \text{where } \alpha > 0.$$

We can see that the parameter α controlls the spectrums of $|T_\alpha|$; the spectrums increase as α increases. In addition, α also affects the diagonal of A_α; the magnitude of iagonals decrease as α increases.

(i)

$$A_\alpha = \begin{bmatrix} 7/\alpha & 1 & 2 & 3 & -1 \\ 3 & 13/\alpha & -2 & -1 & -7 \\ 2 & 1 & 5/\alpha & -2 & 0 \\ 2 & -1 & 5 & 11/\alpha & -3 \\ -2 & -3 & -1 & 1 & 7/\alpha \end{bmatrix}$$

(ii)

$$A_\alpha = \begin{bmatrix} 3/\alpha & -1 & -1 & 1 & 0 \\ 2 & 17/\alpha & -5 & 6 & -4 \\ 1 & 2 & 5/\alpha & -1 & -1 \\ -3 & -5 & -7 & 19/\alpha & -4 \\ -3 & 1 & -2 & 1 & 7/\alpha \end{bmatrix}$$

(iii)

$$A_\alpha = \begin{bmatrix} 23/\alpha & 2 & -11 & -9 & -1 \\ 9 & 17/\alpha & 1 & -1 & 6 \\ 12 & -2 & 29/\alpha & 3 & -12 \\ -29 & 1 & 0 & 31/\alpha & 1 \\ -1 & 1 & 1 & 34 & 37/\alpha \end{bmatrix}$$

3 Conclusion

According these results, a less diagonally dominated A_α should perform relatively poor using asynchronous scheme; this property is very similar to the Gauss-Seidel's method. In fact, we can consider Gauss-Seidel's method as an asynchronous computing process with a very *regular* pattern.

References

[1] D. Chazan, W. Miranker. *Chaotic relaxation.* Linear Algebra and its Applications, Vol 2, Issue 2, April 1969, Pages 199-222.

[2] G.M. Baudet. *Asynchronous Iterative Methods for Multiprocessors* Journal of the ACM, Vol 25, Issue 2, April 1978, Pages 226-244.