On the Construction of Environmental Contours

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Northeastern University

Motivation

Re-parameterized Weibull distribution for modeling metocean extremes of multiple hazards with the Rosenblatt transformation

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On the Extrapolation of Distribution Tail for Dependent Data

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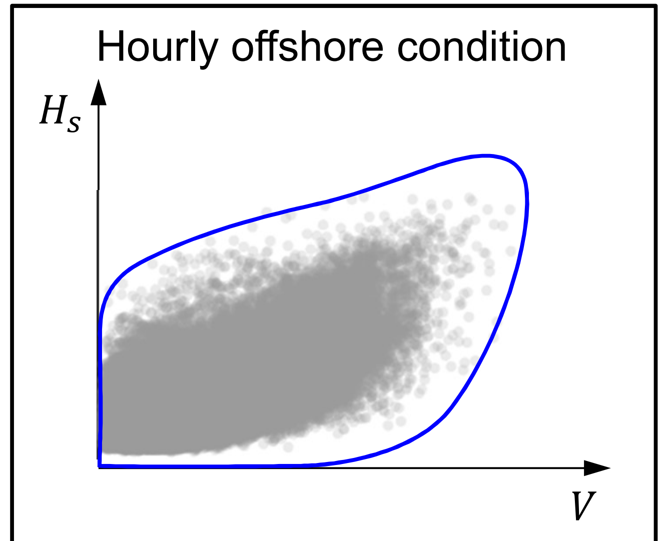
Abstract

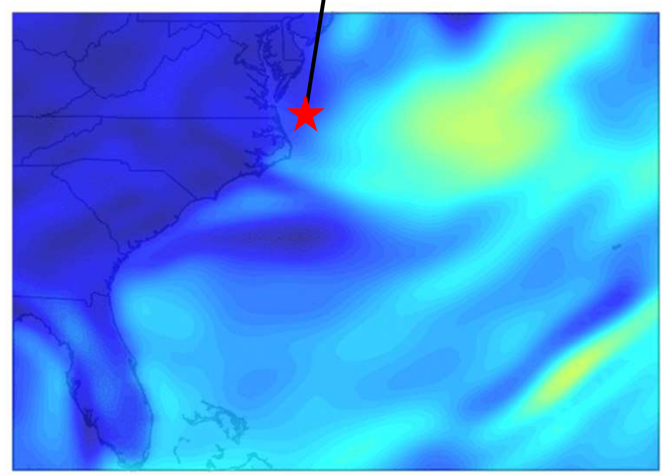
The classical Extreme Value Theory (EVT) uses the Genialized Extreme Value (GEV) distribution to extrapolate long-term conditions from a dataset of independent extremes. Such a dataset is often a subset extracted from a continuous and dependent dataset. In some cases, the continuous dependent data must be extrapolated to estimate long-term conditions in the tails of the joint distribution, however, this creates modeling difficulties because (1) the traditional approach, which is to fit a single distribution using the entire continuous dataset, might be inaccurate in characterizing the tail behavior and (2) the GEV distribution, while accurate for representing the tails, cannot be directly applied to the entire dataset since it is not independent. To remediate this situation, a practical method is proposed, which is to quantify and extrapolate the mean return period ratio between independent extreme values and the continuous dependent values. This ratio relates the results from the classical EVT with distributions from the continuous and dependent dataset. An application of this approach is presented using a 37-year hindcast dataset of wind speeds and wave heights for the U.S. Atlantic coast.

Keywords: Extreme Value; Return Period; Environmental Contour; Offshore Engineering

1. Introduction

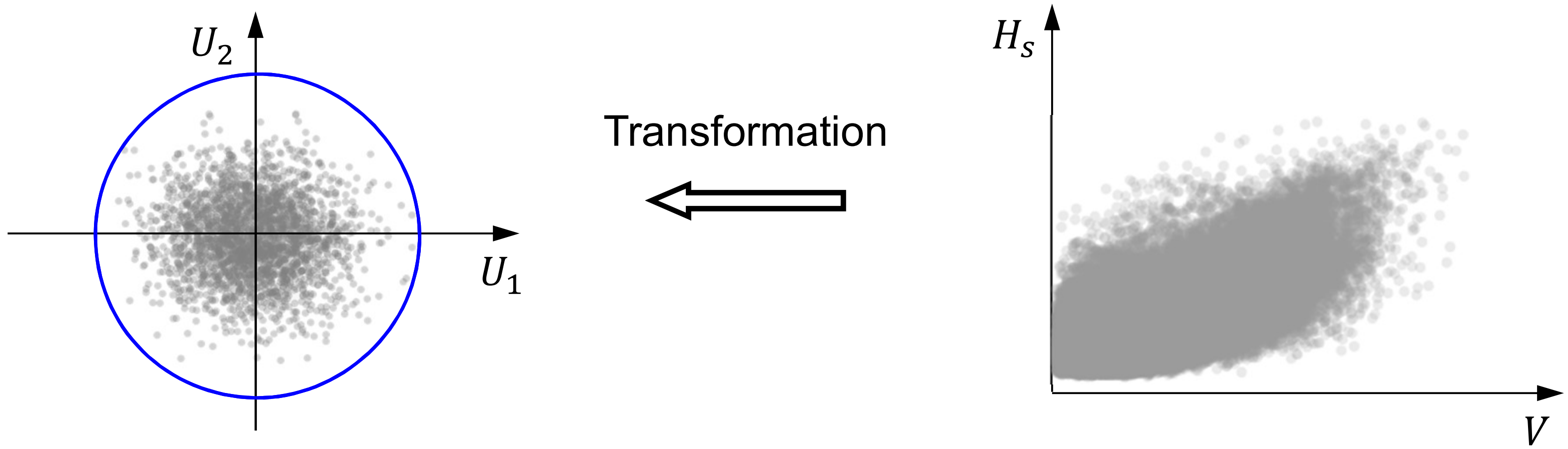
In most design standards for onshore and offshore structures, extreme environmental conditions associated with a particular mean return period (MRP) are used to determine loading for ultimate



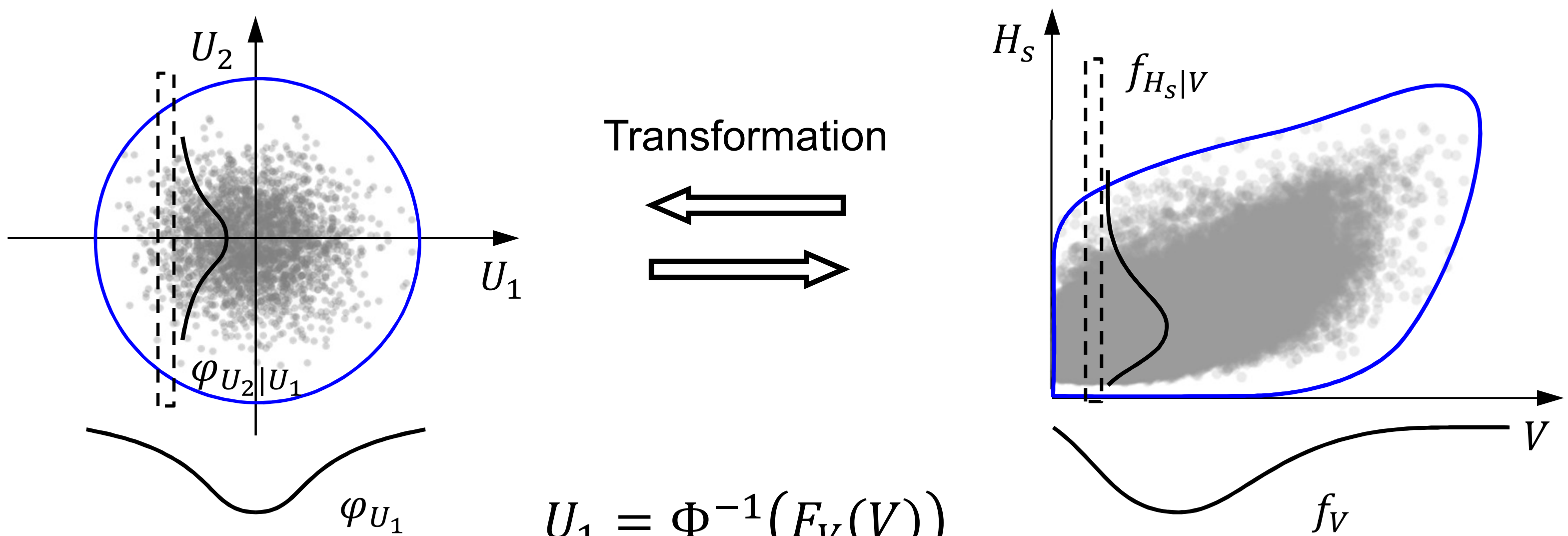


CFSR wind field

Motivation

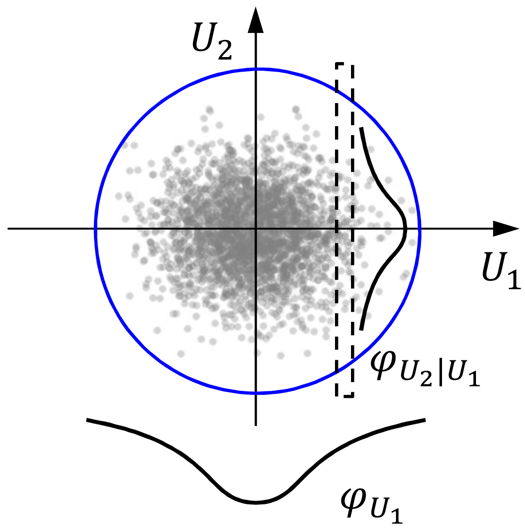


Motivation

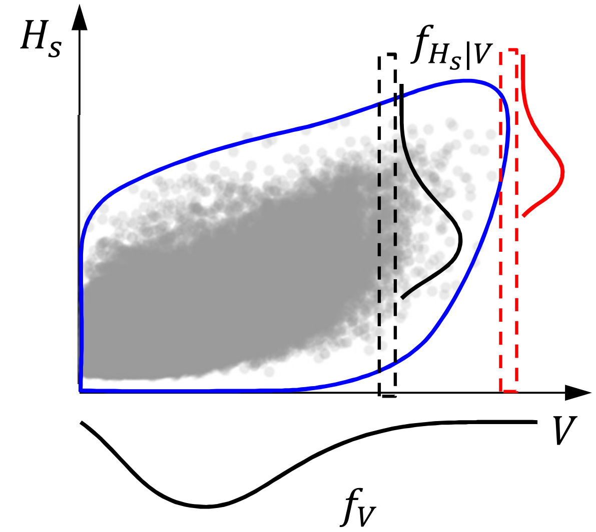


$$U_1 = \Phi^{-1}\left(F_V(V)\right)$$

Motivation



Transformation



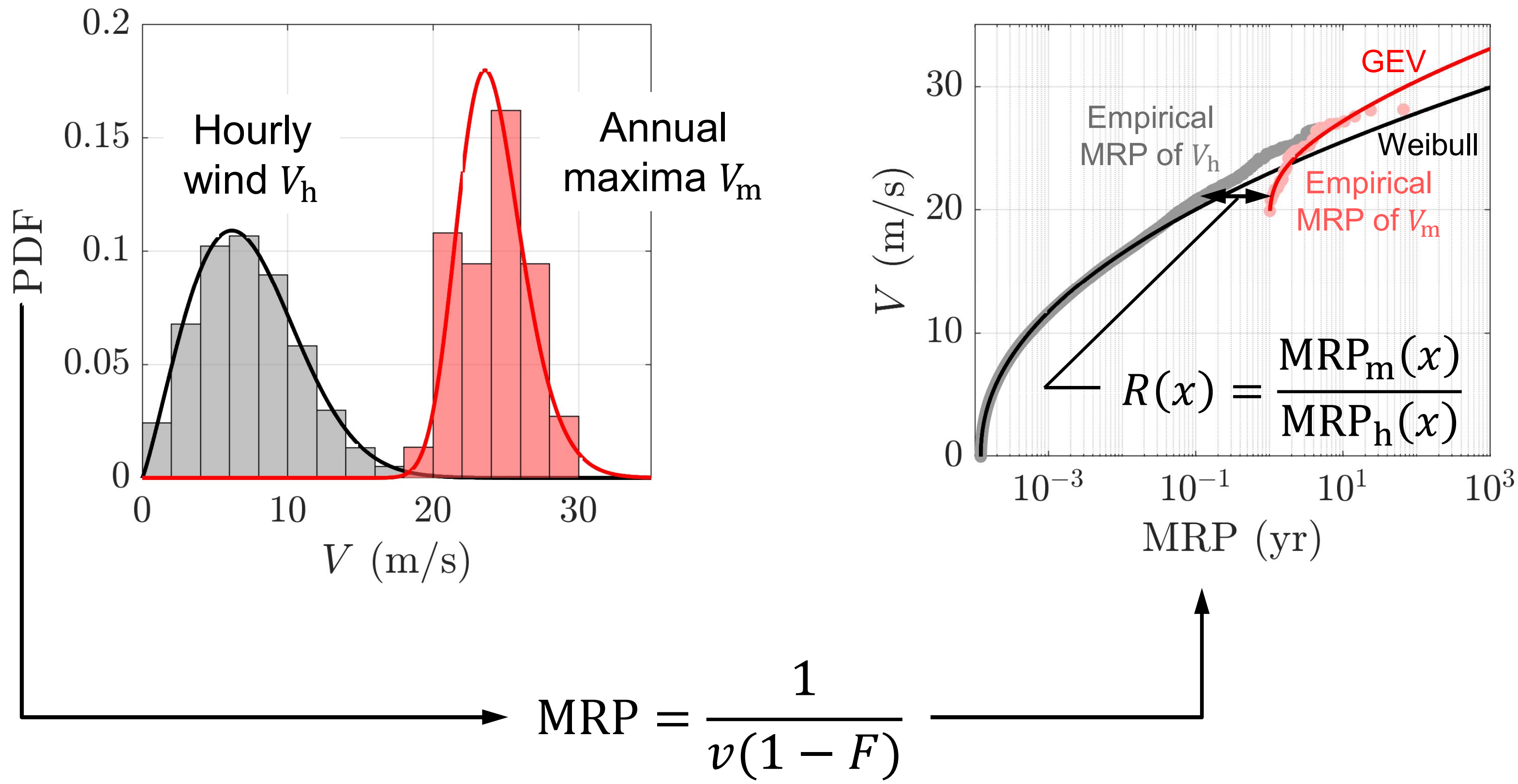
$$U_1 = \Phi^{-1}\left(F_V(V)\right)$$

$$U_2 = \Phi^{-1}\left(F_{H_s|V}(H_s)\right)$$

Q1: How to extrapolate the tail of a continuous variable?

Q2: How to improve the accuracy when extrapolating the parameters of $f_{H_s|V}$?

Tail extrapolation of dependent variables

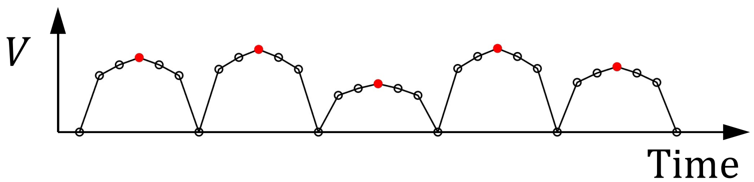


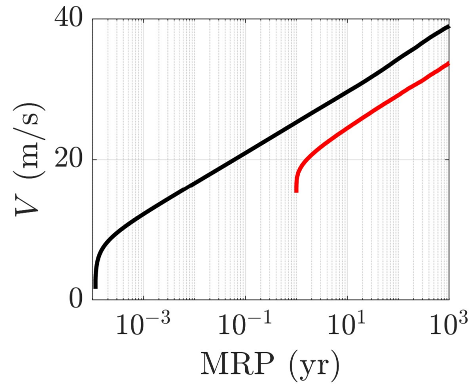
MRP: Mean Return Period

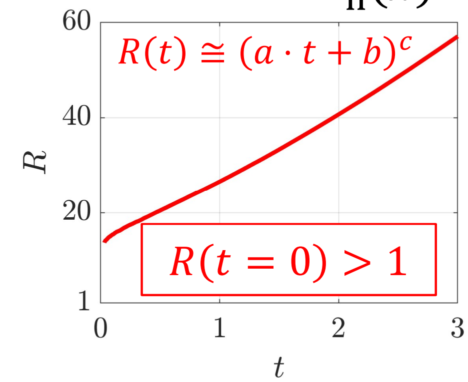
Tail extrapolation of dependent variables

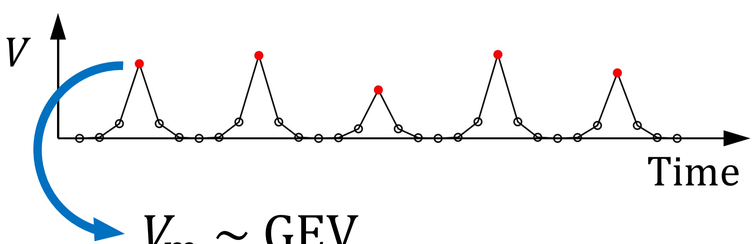
- Numerical experiment

$$R(x) = \frac{\mathrm{MRP_m}(x)}{\mathrm{MRP_h}(x)}$$





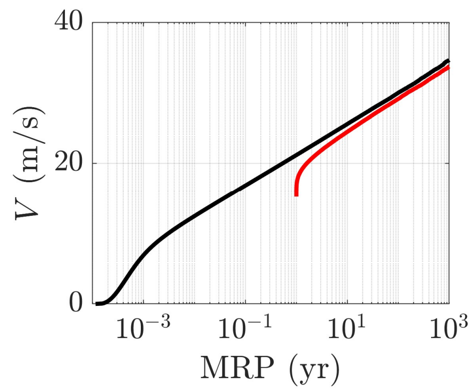


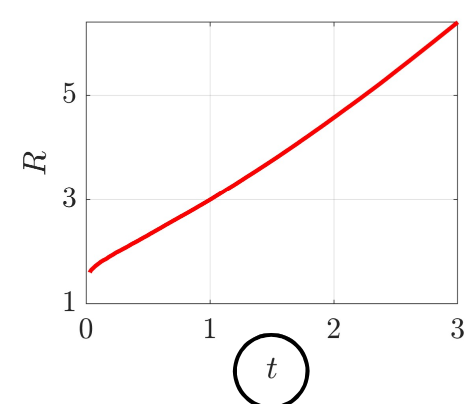


$V_\mathrm{m} \sim \mathrm{GEV}$

$$F(x) = e^{-t(x)}$$

$$t(x) = \begin{cases} \left(1 + \xi\left(\frac{x-\mu}{\sigma}\right)\right)^{-1/\xi}, & \xi \neq 0 \\ e^{-(x-\mu)/\sigma}, & \xi = 0 \end{cases}$$



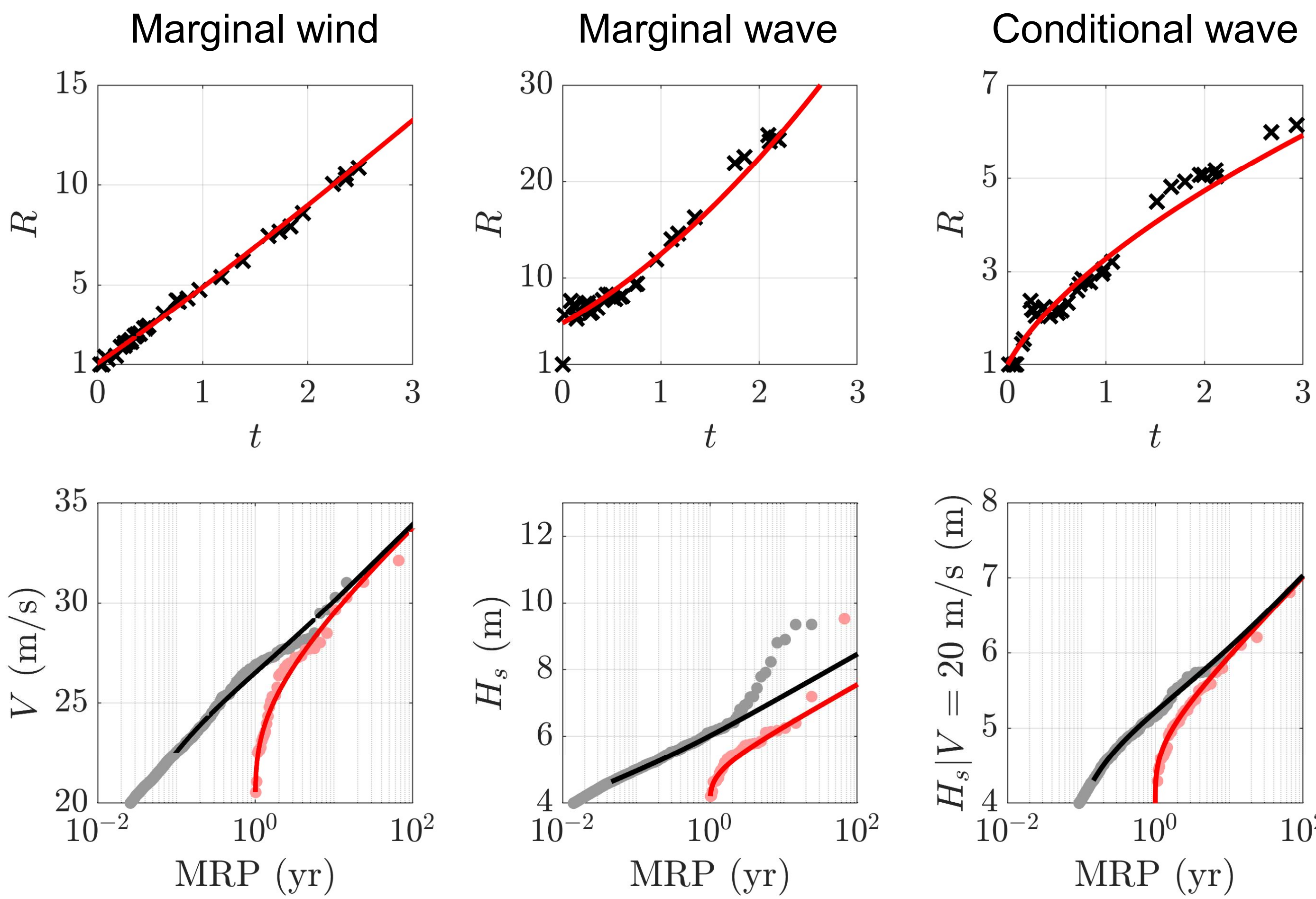


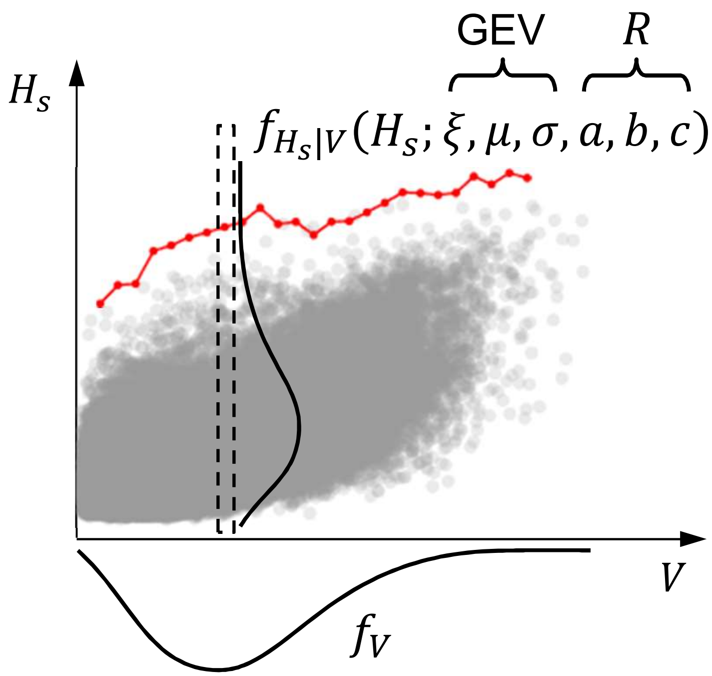
1. Same expression for three types of GEV
2. When $x \to +\infty$, $t \to 0$. Easy to observe $R(t=0)$

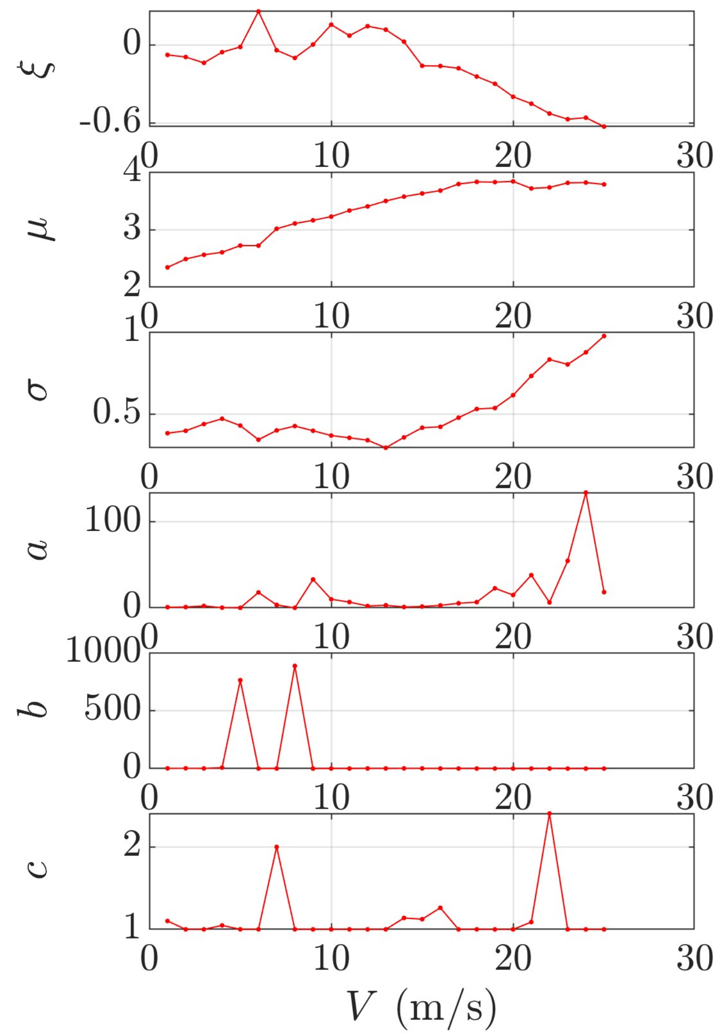
Tail extrapolation of dependent variables

$$R(x) = \frac{\mathrm{MRP_m}(x)}{\mathrm{MRP_h}(x)}$$

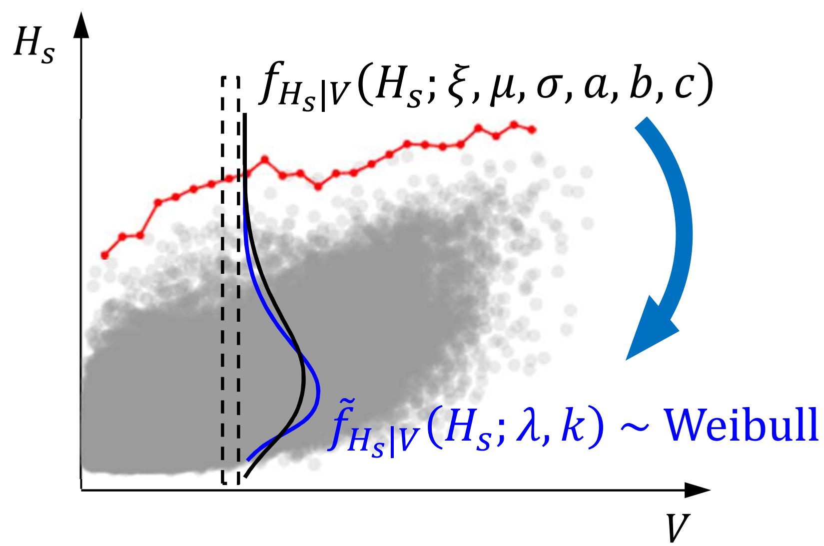
- Applications in hindcast data

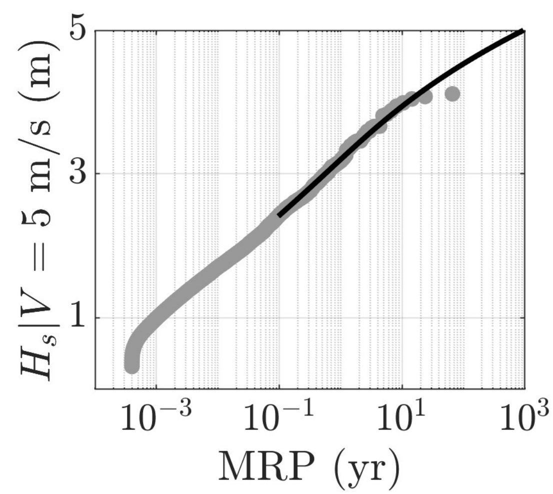


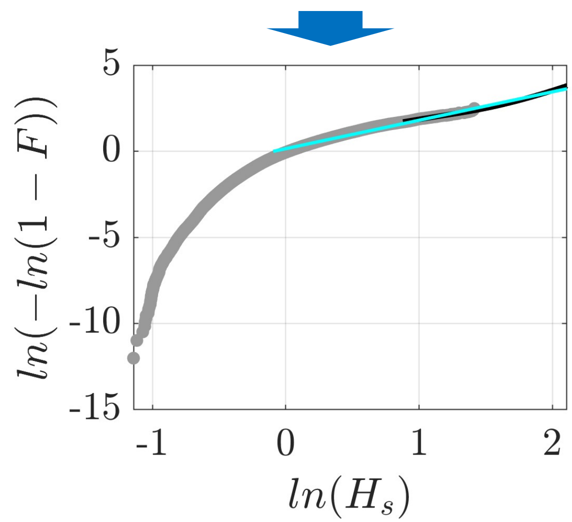

GEV
R
H_s
$f_{H_s|V}(H_s; \xi, \mu, \sigma, a, b, c)$
V
f_V


ξ
μ
σ
a
b
c
V (m/s)

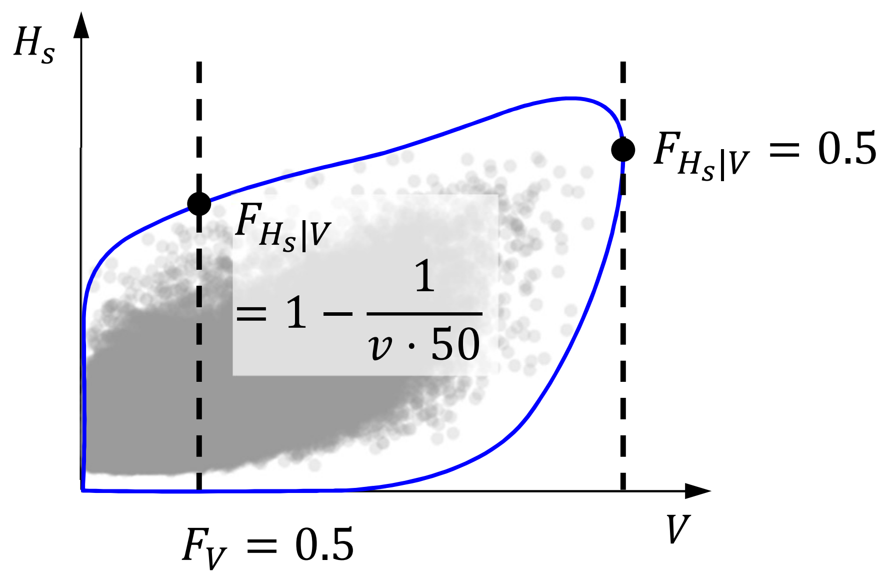
Re-parameterized Weibull

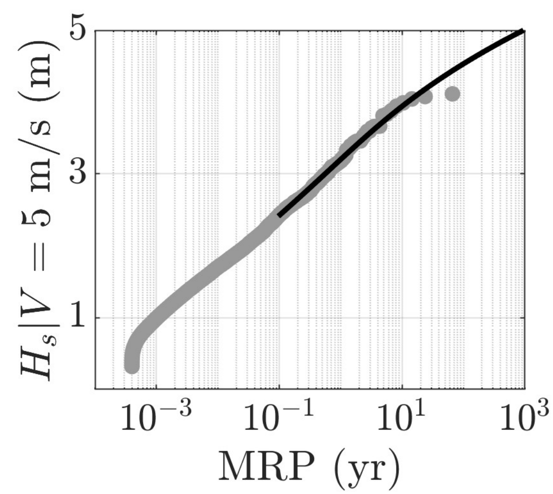


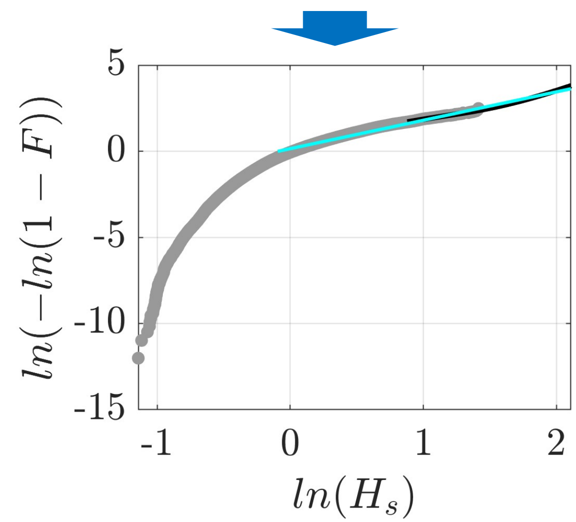




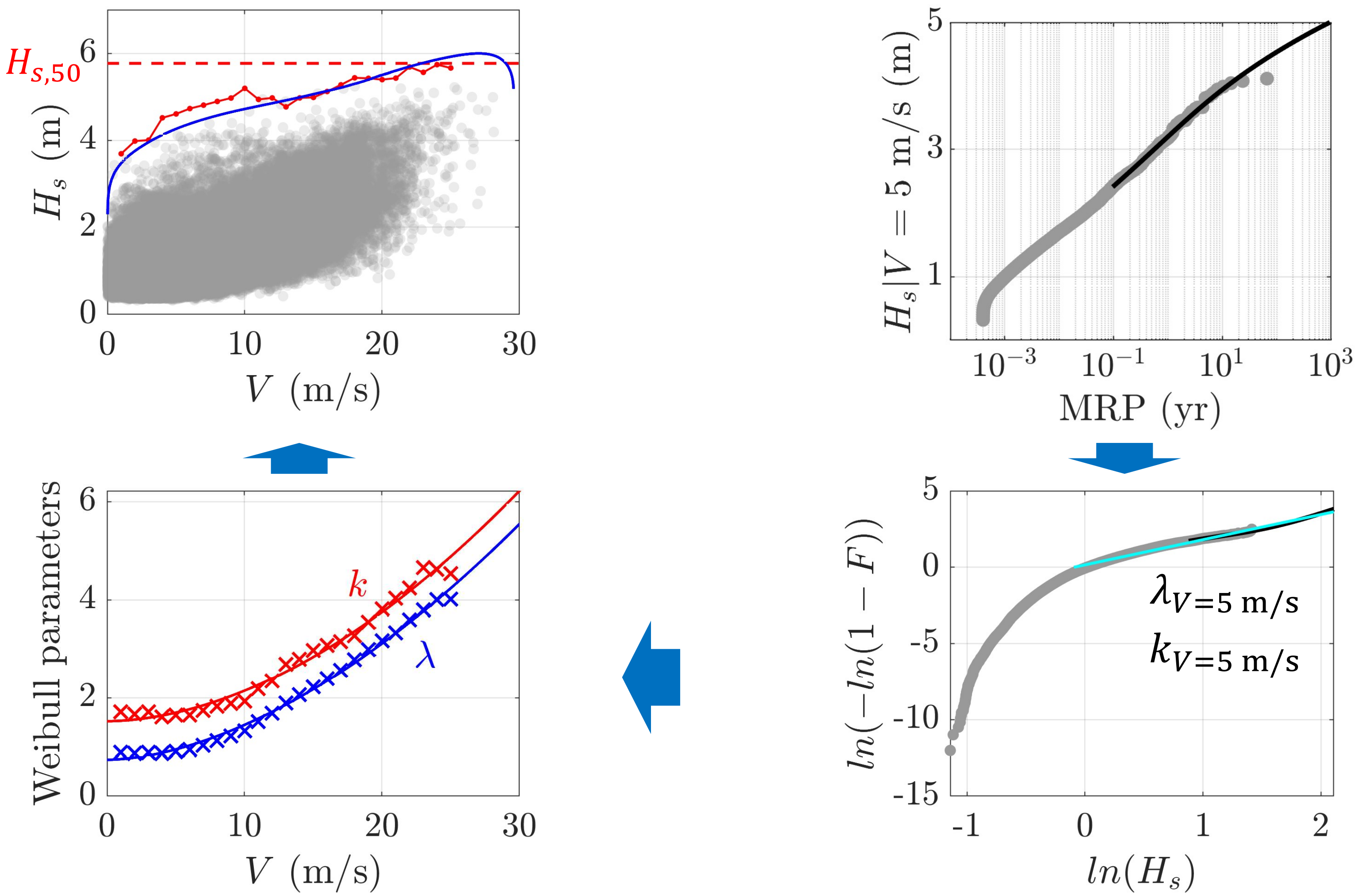
Re-parameterized Weibull





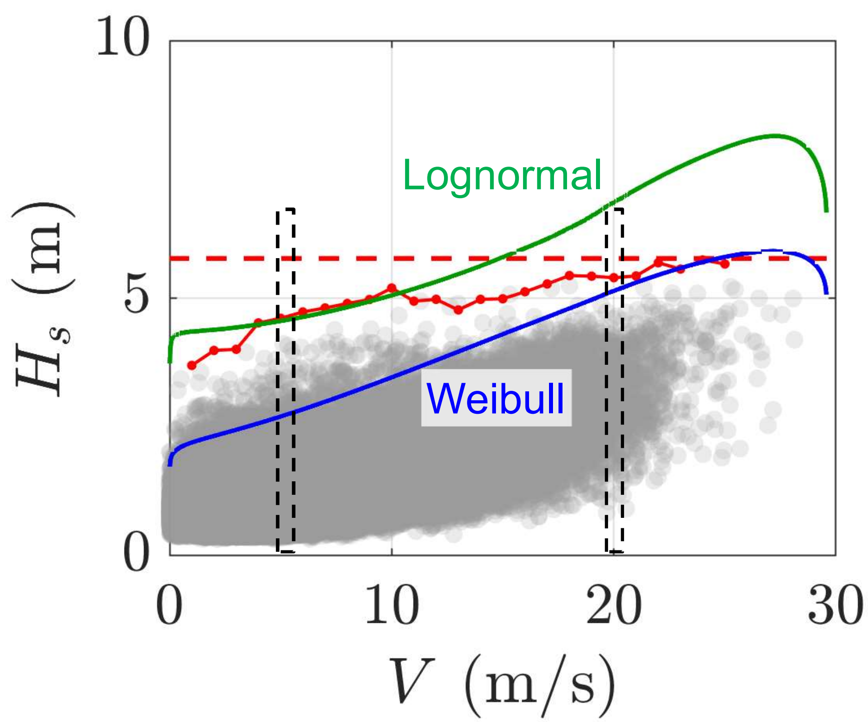


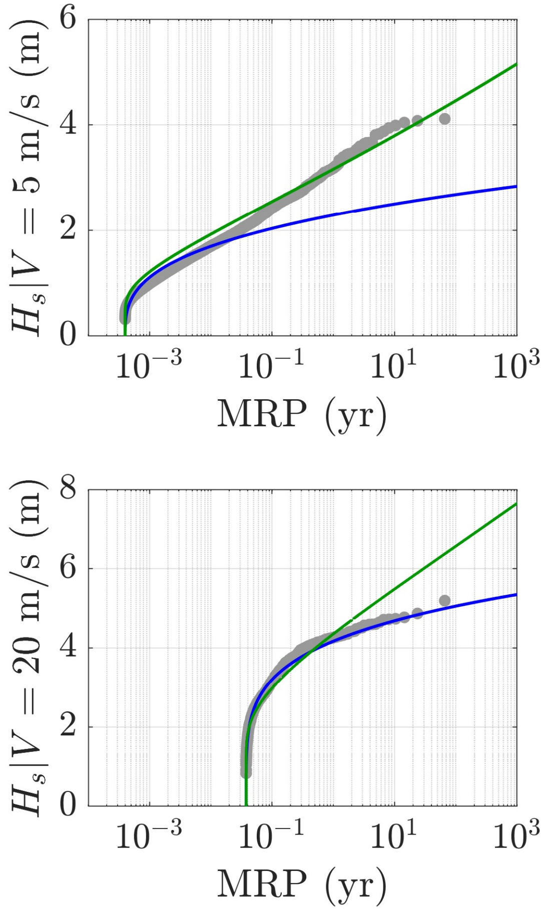
Re-parameterized Weibull



Re-parameterized Weibull

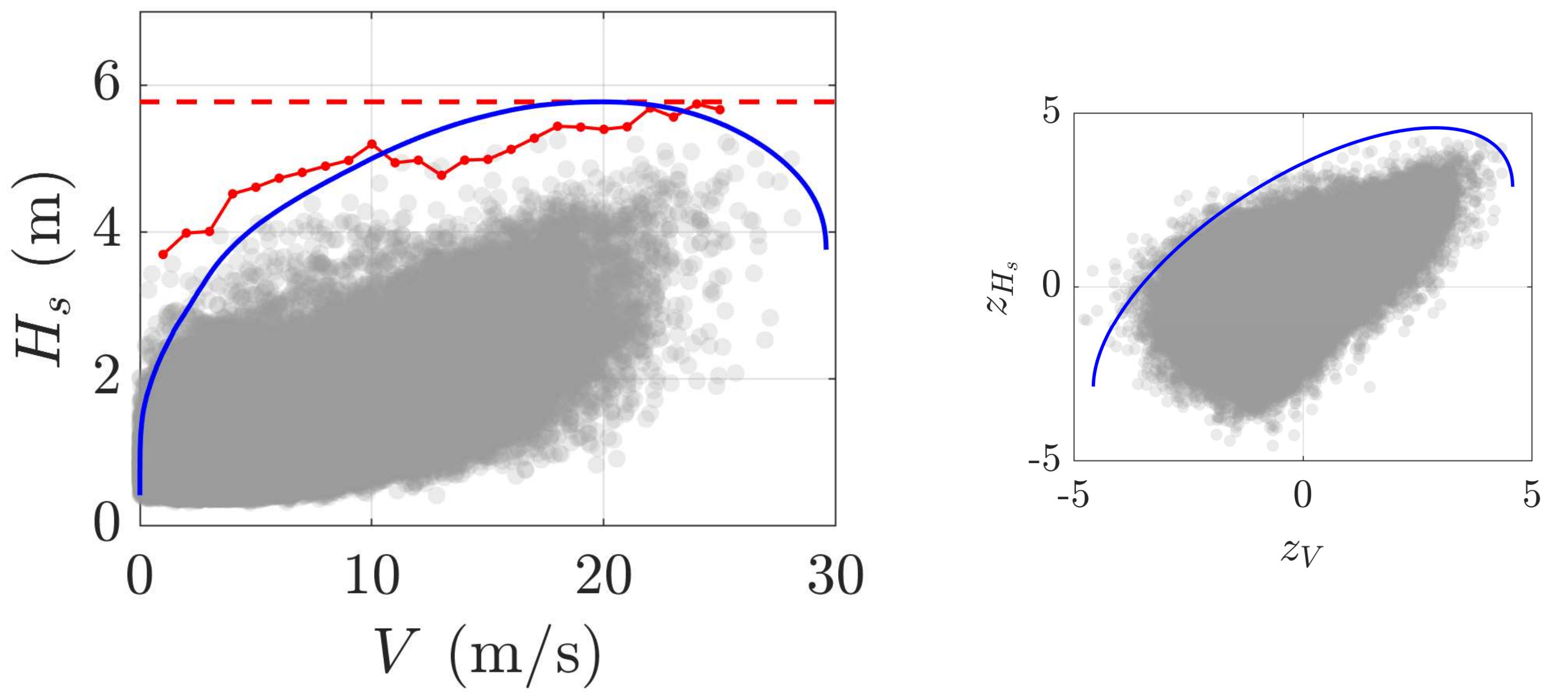
- Traditional method 1: Prescribed $f_{H_s|V}$





Re-parameterized Weibull

- Traditional method 2: Nataf transformation

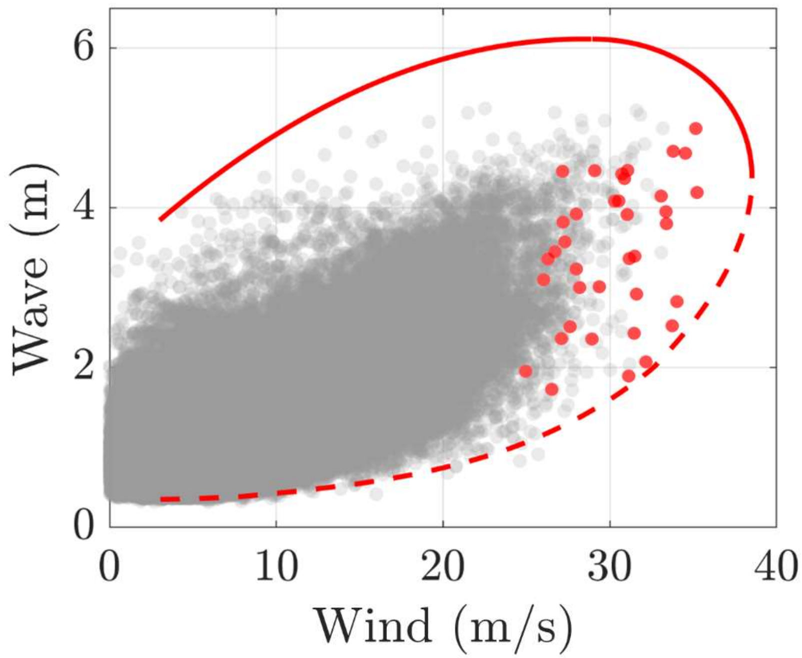


Conclusions

- Extended Extreme Value Theory to dependent variables.
- A re-parameterization method for the Rosenblatt transformation to simplify the conditional distribution.
- Construct environmental contours with much higher accuracy

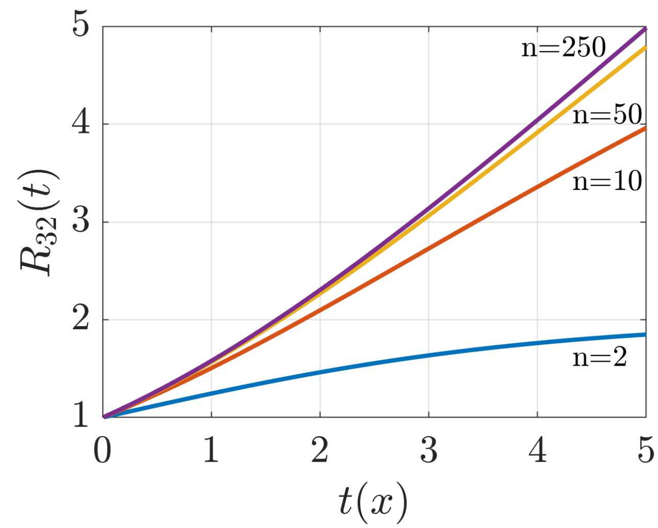

Thank you

Why continuous data for contours?

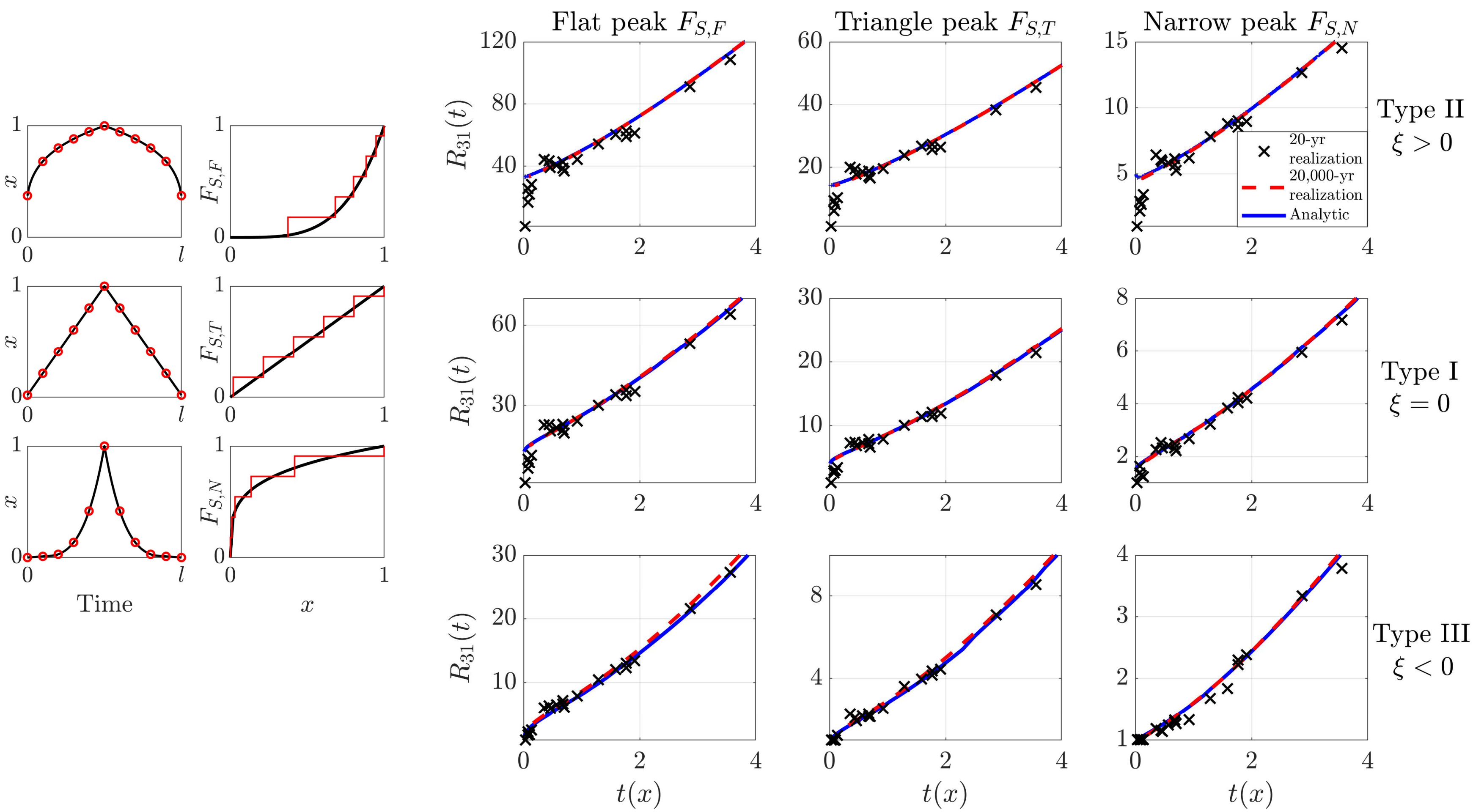


R for independent data

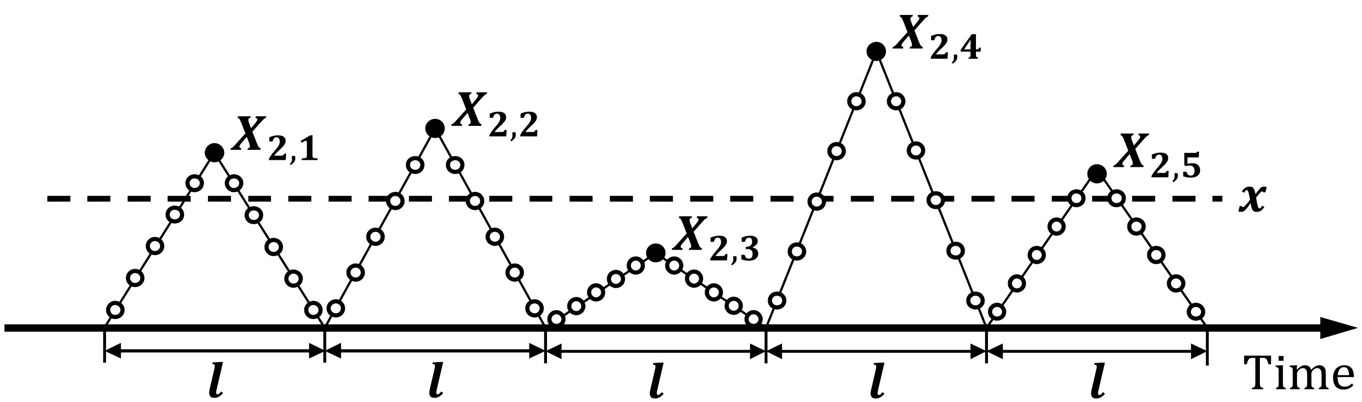
- If X_2 ~ IID, then $X_3 = \max\{X_{2,1}, X_{2,2}, \dots, X_{2,n}\}$ ~ GEV.
- $F_3 = e^{-t(x)}$, $F_2 = F_3^{1/n} = e^{-t(x)/n}$
- $R_{32}(t) = \frac{1/[v_3(1-F_3)]}{1/[v_2(1-F_2)]} = \frac{n\left(1-e^{-t(x)/n}\right)}{1-e^{-t(x)}}$



R for dependent data



Explanation 1



Explanation 1

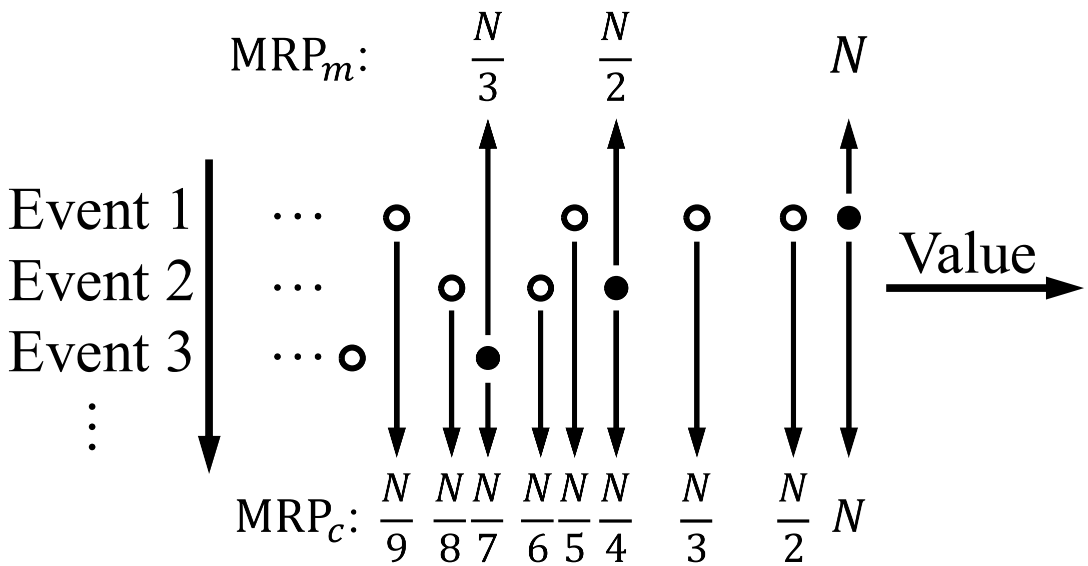
- Considering a 50-year continuous dataset and some corresponding set of event maxima, the second largest event maximum is equaled or exceeded *exactly* twice during the 50-year period and thus it corresponds to an empirical MRP_m of 25 years; however, for the continuous dataset, the same value will be reached or exceeded *at least* twice, leading the corresponding empirical MRP_c to be *at most* 25 years. As such,

$$\mathrm{MRP}_m(x) \geq \mathrm{MRP}_c(x)$$

- This can be equivalently interpreted in an alternative way by noting that the second largest event maximum corresponds to MRP_m equal to 25 years, and the second largest continuous data corresponds to MRP_c also equal to 25 years. Since the second largest continuous data might not be included in the event maxima dataset, and if that is the case, it will be larger than the second largest event maximum, we have

$$X_m(N) \leq X_c(N)$$

Explanation 2



Contours for MA & NY