Algorithmic Trading
Session 10
Performance Analysis I
Performance Measurement

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Outline

- **Introduction**

- **Arithmetic vs. Geometric Mean**

- **Why Dollars are More Important Than Percentages**

- **Traditional Performance Measures**

- **Time Weighted vs. Money Weighted Rates of Return**

- **Performance Measurement with Cash Deposits and Withdrawals**

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Introduction
Where Do We Stand in the Algo Prop Trading Framework?

SIGNAL GENERATION

DECIDE WHEN AND HOW TO TRADE

TRADE IMPLEMENTATION

SIZE AND EXECUTE ORDERS, INCL. EXIT

PERFORMANCE ANALYSIS

RETURN, RISK AND EFFICIENCY RATIOS

▫ As we have seen, algorithmic proprietary trading strategies can be broken down into three subsequent steps: Signal Generation, Trade Implementation and Performance Analysis

▫ **Performance Analysis** is conducted after the trade has been closed and used in a backtesting context to judge whether the strategy is successful or not. In general, we can judge the performance according to five different metrics: return, risk, efficiency, trade frequency and leverage

▫ Sessions 10 -12 deal with the question of analyzing performance

  – **Today's Session 10**: Performance Measurement
  – **Session 11 & 12:** Performance Analysis

Introduction
Performance Measurement

◻ Performance measurement is a critical aspect of portfolio management

◻ Proper performance measurement should involve a recognition of both the return and the riskiness of the investment

◻ When two investments' returns are compared, their relative risk must also be considered

◻ People maximize expected utility:

  – A positive function of expected return
  – A negative function of the return variance

Introduction

A Historical Guideline

◻ The 1968 Bank Administration Institute's *Measuring the Investment Performance of Pension Funds* concluded:

- – Performance of a fund should be measured by computing the actual rates of return on a fund's assets
- – These rates of return should be based on the market value of the fund's assets
- – Complete evaluation of the manager's performance must include examining a measure of the degree of risk taken in the fund
- – Circumstances under which fund managers must operate vary so greatly that indiscriminate comparisons among funds might reflect differences in these circumstances rather than in the ability of managers

Key Points of Performance Measurement
Arithmetic vs. Geometric Mean

☐ The arithmetic mean is not a useful statistic in evaluating growth. It might give misleading information as a 50 percent decline in one period followed by a 50 percent increase in the next period does not produce an average return of zero

☐ Consider the following example from the assigned reading. 44 Wall Street and Mutual Shares both had good returns over the 1975 to 1988 period:

Year	44 Wall Street	Mutual Shares	Year	44 Wall Street	Mutual Shares
1975	184.1%	24.6%	1982	6.9	12.0
1976	46.5	63.1	1983	9.2	37.8
1977	16.5	13.2	1984	−58.7	14.3
1978	32.9	16.1	1985	−20.1	26.3
1979	71.4	39.3	1986	−16.3	16.9
1980	36.1	19.0	1987	−34.6	6.5
1981	-23.6	8.7	1988	19.3	30.7
			Mean	**19.3%**	**23.5%**

Change in net asset value, January 1 through December 31.

Review: Why the Arithmetic Mean Is Misleading

☐ The proper measure of average investment return over time is the geometric mean:

$$GM = \left[\prod_{i=1}^{n} R_i \right]^{1/n} - 1$$

where R_i = the return relative in period i

☐ The geometric means in the preceding example are:

 − 44 Wall Street: 7.9 percent
 − Mutual Shares: 22.7 percent

☐ The geometric mean correctly identifies Mutual Shares as the better investment over the 1975 to 1988 period

Key Points of Performance Measurement
Dollars Are More Important Than Percentages

◻ Measuring dollar values clearly shows that Mutual shares significantly outperformed 44 Wall Street:

Mutual Fund Performance
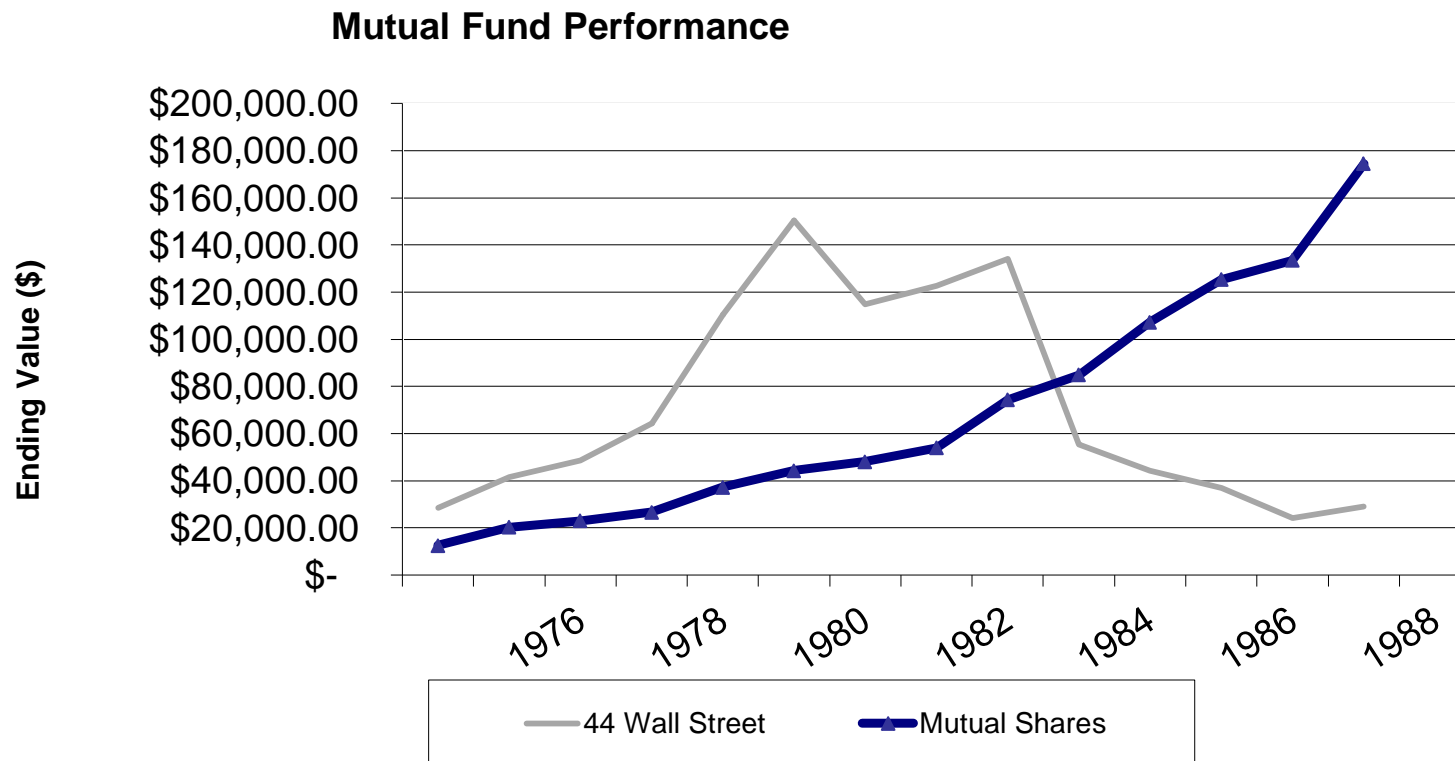
Key Points of Performance Measurement
Dollars Are More Important Than Percentages

◻ Assume two funds managed by the same portfolio manager:

    ◻ Fund A has $40 million in investments and earned 12 percent last period

    ◻ Fund B has $250,000 in investments and earned 44 percent last period

◻ The correct way to determine the return of both funds combined is to weigh the funds' returns by the dollar amounts:

$$\left(\frac{\$40,000,000}{\$40,250,000} \times 12\%\right) + \left(\frac{\$250,000}{\$40,250,000} \times 44\%\right) = 12.10\%$$

◻ In fact, 99.38 percent of the $40.25 million managed by this person earned 12 percent. Only 0.62 percent earned the higher rate

Traditional Performance Measures
Sharpe and Treynor Measures

◻ The Sharpe and Treynor Measures are calculated as follows:

$$\text{Sharpe measure} = \frac{\bar{R} - R_f}{\sigma}$$

$$\text{Treynor measure} = \frac{\bar{R} - R_f}{\beta}$$

$$\begin{aligned} \text{where } \bar{R} &= \text{ average return} \\ R_f &= \text{risk-free rate} \\ \sigma &= \text{ standard deviation of returns} \\ \beta &= \text{ beta} \end{aligned}$$

◻ The **Sharpe measure** evaluates return relative to total risk. Hence, it is appropriate for a well-diversified portfolio, but not for individual securities

◻ The **Treynor measure** evaluates the return relative to beta, a measure of systematic risk. Hence, it ignores any unsystematic risk and is therefore also not appropriate for individual securities

□ The Jensen measure stems directly from the CAPM:

$$R_{it} - R_{ft} = \alpha + \left[\beta_i \left(R_{mt} - R_{ft} \right) \right]$$

□ The constant term should be zero. Securities with a beta of zero should have an excess return of zero according to classical finance theory

□ According to the Jensen measure, if a portfolio manager is better-than-average, the **alpha** of the portfolio will be positive

□ However, the use of Treynor and Jensen Measure relies on measuring the market return and CAPM

   □ Difficult to identify and measure the return of the market portfolio

   □ Evidence continues to accumulate that may ultimately displace the CAPM, but Arbitrage pricing model, multi-factor CAPMs, inflation-adjusted CAPM could help

Traditional Performance Measures
Fama's Return Decomposition

- "Fama's return decomposition" can be used to assess why an investment performed better or worse than expected:

  - The return the investor chose to take

  - The added return the manager chose to seek

  - The return from the manager's good selection of securities

- Diversification is the difference between the return corresponding to the beta implied by the total risk of the portfolio and the return corresponding to its actual beta

- Net selectivity measures the portion of the return from selectivity in excess of that provided by the "diversification" component
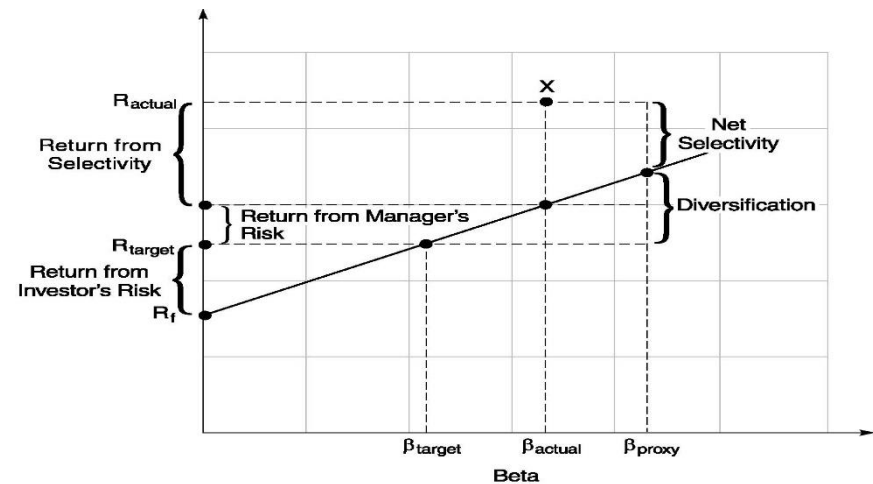
Source: Adapted from Eugene Fama, "Components of Investment Performance," *Journal of Finance*, June 1972, 551–567.

Dollar Weighted vs. Time Weighted Rates of Returns
Overview

□ The **dollar-weighted rate of return** is analogous to the internal rate of return in corporate finance. It is the rate of return that makes the present value of a series of cash flows equal to the cost of the investment

$$\text{cost} = \frac{C_1}{(1+R)} + \frac{C_2}{(1+R)^2} + \frac{C_3}{(1+R)^3}$$

□ The **time-weighted rate of return** measures the compound growth rate of an investment. It eliminates the effect of cash inflows and outflows by computing a return for each period and linking them (like the geometric mean return):

$$\text{time-weighted return} = (1+R_1)(1+R_2)(1+R_3)(1+R_4) - 1$$

□ The time-weighted rate of return and the dollar-weighted rate of return will be equal if there are no inflows or outflows from the portfolio

Performance Measurement with Cash Deposits and Withdrawals
Overview

◻ The owner of a fund often takes periodic distributions from the portfolio, and may occasionally add to it

◻ The established way to calculate portfolio performance in this situation is via a time-weighted rate of return:

– Daily valuation method
– Modified Bank Administration Institute (BAI) method

◻ The daily valuation method:

– Calculates the exact time-weighted rate of return
– Is cumbersome because it requires determining a value for the portfolio each time any cash flow occurs. This might be interest, dividends, or additions to or withdrawals

◻ The modified BAI method:

– Approximates the internal rate of return for the investment over the period in question
– Can be complicated with a large portfolio that might conceivably have a cash flow every day

Performance Measurement with Cash Deposits and Withdrawals
Daily Valuation Method

◻ The daily valuation methods solves for R:

$$R_{\text{daily}} = \prod_{i=1}^{n} S_i - 1$$

$$\text{where } S = \frac{MVE_i}{MVB_i}$$

◻ MVE_i = market value of the portfolio at the end of period *i before any cash flows in period i but including accrued income for the period*

◻ MVB_i = market value of the portfolio at the beginning of period *i including any cash flows at the end of the previous subperiod and including accrued income*

Performance Measurement with Cash Deposits and Withdrawals
BAI method

☐ The BAI methods solves for R:

$$MVE = \sum_{i=1}^{n} F_i (1+R)^{w_i}$$

where F = the sum of the cash flows during the period

MVE = market value at the end of the period,

including accrued income

F_0 = market value at the start of the period

$$w_i = \frac{CD - D_i}{CD}$$

CD = total number of days in the period

D_i = number of days since the beginning of the period

in which the cash flow occurred

Performance Measurement with Cash Deposits and Withdrawals
Example

□ An investor has an account with a mutual fund and "dollar cost averages" by putting $100 per month into the fund

□ The following table shows the activity and results over a seven-month period

Date	Description	$ Amount	Price	Shares	Total Shares	Value
January 1	balance forward		$7.00		1,080.011	$7,560.08
January 3	purchase	100	$7.00	14.286	1,094.297	$7,660.08
February 1	purchase	100	$7.91	12.642	1,106.939	$8,755.89
March 1	purchase	100	$7.84	12.755	1,119.694	$8,778.40
March 23	liquidation	5,000	$8.13	-615.006	504.688	$4,103.11
April 3	purchase	100	$8.34	11.900	516.678	$4,309.09
May 1	purchase	100	$9.00	11.111	527.789	$4,750.10
June 1	purchase	100	$9.74	10.267	538.056	$5,240.67
July 3	purchase	100	$9.24	10.823	548.879	$5,071.64
August 1	purchase	100	$9.84	10.163	559.042	$5,500.97

Performance Measurement with Cash Deposits and Withdrawals
Example: Daily Valuation Method

◻ The daily valuation method returns a time-weighted return of 40.6 percent over the seven-month period

Date	Sub Period	MVB	Cash Flow	Ending Value	MVE	MVE/MVB
January 1				$7,560.08		
January 3	1	$7,560.08	100	$7,660.08	$7,560.08	1.00
February 1	2	$7,660.08	100	$8,755.89	$8,655.89	1.13
March 1	3	$8,755.89	100	$8,778.40	$8,678.40	0.991
March 23	4	$8,778.40	5,000	$4,103.11	$9,103.11	1.037
April 3	5	$4,103.11	100	$4,309.09	$4,209.09	1.026
May 1	6	$4,309.09	100	$4,750.10	$4,650.10	1.079
June 1	7	$4,750.10	100	$5,240.67	$5,140.67	1.082
July 3	8	$5,240.67	100	$5,071.64	$4,971.64	0.949
August 1	9	$5,071.64	100	$5,500.97	$5,400.97	1.065

Product of MVE/MVB values = 1.406; → R = 40.6%

Performance Measurement with Cash Deposits and Withdrawals
Example: BAI Method

- The BAI method returns a time-weighted return of 42.1 percent over the seven-month period. However, it requires a function like solver in Excel

Date	Day	Weight (214-days)/214	Cash Flow	$(1.421)^{\text{weight}}$ x cashflow
January 1	0	1.000		
January 3	2	0.9907	$7,560.06	$10,741.36
February 1	31	0.8551	$100	$141.62
March 1	60	0.7196	$100	$135.03
March 23	83	0.6121	$5,000	$128.75
April 3	94	0.5607	$100	($6,199.20)
May 1	123	0.4252	$100	$121.77
June 1	153	0.2850	$100	$116.17
July 3	185	0.1355	$100	$104.87
August 1	214	0.0000	$100	$100
			Total	$5,500.84

Summary and Questions

▫ Performance evaluation is a critical part of the portfolio management process. The central issue is coupling a measure of risk with the return of a portfolio. The measurement of risk is often neglected

▫ Average returns over time should be measured using a geometric growth rate. The arithmetic mean gives misleading results and should not be used to compare competing investment funds or strategies

▫ The Sharpe and Treynor measures are the two leading classical performance indicators. Their calculations are similar, except that the Sharpe measure uses the standard deviation of returns as a risk measure whereas the Treynor measure uses beta. Jensen's measure is not that common anymore, although his definition of alpha is still used for outperformance

▫ When a portfolio has frequent cash deposits and withdrawals, it is best to calculate performance via a time-weighted rate of return

▫ Questions?

Sources

□ Portfolio Construction, Management, and Protection by Robert A. Strong