ECO 394D: Probability and Statistics

Midterm Practice Questions

Questions

1. If an aircraft is present in a certain area, a radar correctly registers its presence with probability 0.99. If it is not present, the radar falsely registers an aircraft presence with probability 0.10. Suppose that on average across all days, an aircraft is present with probability 0.05.

   Let the events A and R be defined as follows: A = an aircraft is present, R = the radar registers an aircraft presence. Show how to calculate $P(A \mid R)$, the conditional probability that an aircraft is present, given that the radar registers an aircraft presence. (Do plug in actual numbers to your expression, but don't bother trying any tedious calculations by hand. The important thing is the expression with numbers plugged in, not the final answer.)

2. Define a discrete random variable X as follows:

   - $X = 2$ with probability 1/10
   - $X = 3$ with probability 1/10
   - $X = 5$ with probability 8/10

   Draw a picture of the CDF of X over the interval $x \in [0, 8]$. What are $P(2 < X \leq 4.5)$ and $P(2 \leq X < 4.5)$?

3. Alice and Bob are scheduled to meet at Quack's coffee shop at a set time, but they are often late for appointments. Each will arrive at Quack's with a delay between 0 and 1 hour, with all pairs of delays being equally likely. Moreover, both Alice and Bob shun the conveniences of the modern world, and thus they do not have cell phones to contact each other. The first person to arrive will wait for 20 minutes (1/3 of an hour) but will get impatient and leave in disgust if the other has not yet arrived by then. What is the probability that they will meet? Show/explain how you arrived at your answer.

   You can derive the answer any way you want. You can even draw a picture and argue convincingly from that, as long as you are precise in your statements.

4. Markov lives in Austin, 2 miles from campus. During the winter, if the weather outside is cold, then Markov prefers to wear a fur cap with ear flaps and to walk to school. But if the weather is warm, like many winter days in Austin, Markov leaves the fur cap at home, wears a helmet instead, and rides to on campus one of those green electric rental scooters that he has

begun to notice on every street corner. If the weather is cold (which happens with probability 0.4 in the winter in Austin), Markov walks the 2 miles to school at a brisk speed of V = 5 miles per hour. Otherwise he travels by scooter at a speed of V = 10 miles per hour.

Markov wants to calculate the expected time T that it takes him to get to class on a random winter day. He reasons as follows. His expected speed V is equal to

$$E(V) = 0.4 \cdot 5 + 0.6 \cdot 10 = 2 + 6 = 8\,.$$

Therefore, since he must travel two miles to class, his expected time T to get to class is $E(2/V) = 2/E(V) = 2/8$ hours, or 15 minutes.

Do you agree with Markov's reasoning that $E(T) = 2/8$ hours? Explain why or why not, and if you don't agree, show how you would calculate $E(T)$ correctly.

5. A non-negative continuous random variable X is said to follow an exponential distribution with parameter $\lambda > 0$ if its PDF is given by

$$f(x) = \lambda e^{-\lambda x}$$

for $x \geq 0$, and is zero otherwise. The parameter λ is usually referred to as the "rate."

Now suppose that $X_1, \ldots, X_N$ are a set of N i.i.d. samples from an exponential distribution with rate parameter λ.

Let $Y_N = \max\{X_1, \ldots, X_N\}$ be the maximum value in your sample. Derive the PDF of Y_N for fixed N.

Here are some hints:

- For any y, we know that $\max\{X_1, \ldots, X_N\} \leq y$ if and only if $X_i \leq y$ for all $i = 1, \ldots N$.
- The following integral will help:

$$\int_0^x \lambda e^{-\lambda t}\, dt = 1 - e^{-\lambda x}\,.$$