Age of Information in a Multiclass Prioritized Queueing System with Heterogeneous Arrival Rates

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1 AoI with Heterogeneous Arrival Rates

We follow a similar approach in [4]. Instead of considering the age of information of all streams simultaneously, we consider each streams separately.

2 The Stochastic Hybrid System Set Up

We consider a stream m which can be preempted by $0 \leq i \leq N-1$ streams having higher priority than itself. (i.e., $m = i + 1$)

2.1 The Discrete State Space $\mathcal{Q}$

For such stream, we denote by $\mathcal{Q}_i$ the state space. $\mathcal{Q}_i = \{0, 1\}^i$. A state $\mathbf{q} \in \mathcal{Q}_i$ has $q_k = 1$ if and only if there is a packet of stream k in the waiting room or is being served.

The continuous state $\mathbf{x} = [x_0\ x_1] \in \mathbb{R}^2$, where x_0 is the age of information of stream of interest (i.e., stream m, in our language). x_1 is what the age of information of stream of interest will become if the packet in the server finishes its service.

The arrival rates of the streams that can preempt stream m are $\lambda_1, \lambda_2, \cdots, \lambda_i$, where λ_k is the arrival rate of stream with the kth highest priority. By our assumption, $\lambda_1 > \lambda_2 > \cdots > \lambda_i$. We let λ denote the arrival rate of stream m. That is, $\lambda = \lambda_{i+1}$.

2.2 Set of Possible Transitions

The set of admissible transitions $\mathcal{L}_{\mathbf{q}}$ from state $\mathbf{q}$ is comprised of three different kinds of transitions

(i) A transition from $\mathbf{q}$ to $\mathbf{q} + \mathbf{e}_k$ occurs with rate λ_k, where $q_k = 0$. This means a new packet arrival of stream k arrives.

(ii) A self-transition $\mathbf{q}$ to $\mathbf{q}$ occurs with rate $\sum_{k:\, q_k=1} \lambda_k$. The self-transition represents a packet arrives and preempts either a packet (arriving earlier from the same stream) in the waiting room or in service.

(iii) A transition from $\mathbf{q}$ to $\mathbf{q}-\mathbf{e}_{k^*}$ occurs with rate μ, where $k^* = \arg\min_k \{q_k = 1\}$. This indicates the service of the packet (with highest priority) is finished.

On the other hand, the set of admissible transitions $\mathcal{L}'_{\mathbf{q}}$ into $\mathbf{q}$ can be categorized by whether $\mathbf{q} = \mathbf{0}$ or not.

For $\mathbf{q} \neq \mathbf{0}$, $\mathcal{L}_{\mathbf{q}}$ is comprised of three kinds of transitions

(i) A transition from $\mathbf{q}-\mathbf{e}_k$ to $\mathbf{q}$ occurs with rate λ_k, where $q_k = 1$. This means a stream-k packet arrival in state $\mathbf{q}-\mathbf{e}_k$.

(ii) A self-transition from $\mathbf{q}$ to $\mathbf{q}$ occurs with rate $\sum_{k:\, q_k=1} \lambda_k$. This also represents a packet arrival which preempts either a packet in the waiting room or in service.

(iii) A transition from $\mathbf{q}+\mathbf{e}_k$ to $\mathbf{q}$ occurs with rate μ, where $k > \arg\min_j \{q_j = 1\}$. This indicates a service completion (of a stream-k packet) in state $\mathbf{q}+\mathbf{e}_k$.

When $\mathbf{q} = \mathbf{0}$, there are three kinds of transitions as well.

(i) A self-transition from $\mathbf{q}$ to $\mathbf{q}$ occurs with rate λ. This is a (new) packet arrival of stream of interest.

(ii) Another self-transition from $\mathbf{q}$ to $\mathbf{q}$ occurs with rate μ. This represents a "fake update" in which there is no change of age of information. Due to the memoryless property (of exponential distribution), introducing fake update reduces the state space and the possible transitions.

(iii) A transition from $\mathbf{q}+\mathbf{e}_k$ to $\mathbf{q}$ occurs with rate λ_k. This indicates a service completion of a stream-k packet in state $\mathbf{q}+\mathbf{e}_k$.

2.3 The b vectors

Since $\mathbf{x} = [x_0\ x_1]$, where x_0 is the age of information of stream and x_1 is the age of information once the current service is complete, x_0 and x_1 both grows at unit rate. In other words, $\mathbf{b}_{\mathbf{q}} = [1\ 1]$ for all $\mathbf{q} \in \mathcal{Q}_i$.

2.4 The Linear Reset Maps

For $\mathbf{q} \neq \mathbf{0}$, the set of linear reset maps are

(i) A transition from $\mathbf{q} - \mathbf{e}_k$ to $\mathbf{q}$ has its reset map

$$\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

(ii) A self-transition from $\mathbf{q}$ to $\mathbf{q}$ has its transition map

$$\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

(iii) A transition from $\mathbf{q} + \mathbf{e}_k$ to $\mathbf{q}$ has its transition map

$$\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

When $\mathbf{q} = \mathbf{0}$, the set of linear reset maps are

(i) A self-transition from $\mathbf{q}$ to $\mathbf{q}$ indicating an arrival of packet of stream of interest has its reset map

$$\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

(ii) A self-transition from $\mathbf{q}$ to $\mathbf{q}$ indicating a fake update has its reset map

$$\mathbf{A} = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$$

(iii) A transition from $\mathbf{q} + \mathbf{e}_k$ to $\mathbf{q}$ indicating a service completion of stream-k packet has it reset map

$$\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

References

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