Characterizing the Age of Information with Prioritized Streams

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Outline

- Age of Information (AoI)
- System Model and Scenarios
- Related Work and Objective
- Stochastic Hybrid Systems (SHS) - Introduction
- SHS for AoI
- Results and Discussion

Age of Information(AoI)

It matters mainly in the distributed machine type networks

- Performance metric of a system (freshness of data)
  - e.g.Environmental monitoring system, Sensor Networks, Vehicular networks
- Priority
  - e.g. Critical Safety Data vs. Non-safety Data

Age of Information(AoI)

Definition of AoI

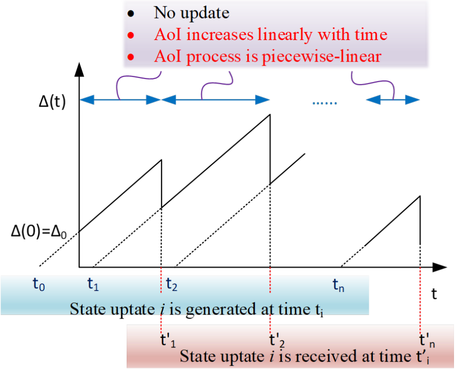
- Instantaneous AoI of a stream at time t

  $\Delta(t) = t - t_i$

  - t_i: time-stamp of the most recently delivered packet
- Total Average AoI

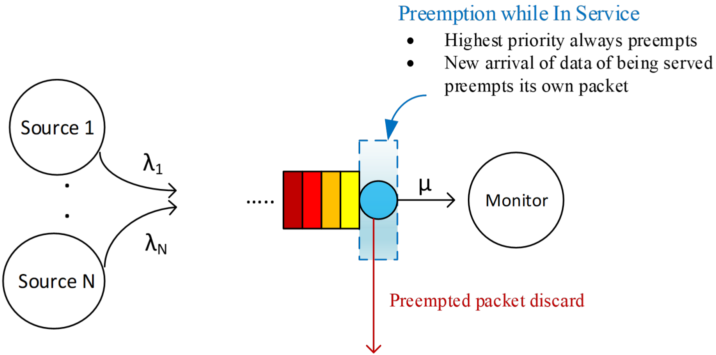
  $\bar{\Delta} = \sum_{k=1}^{N} \bar{\Delta}_k = \sum_{k=1}^{N} \lim_{\tau\to\infty} \frac{1}{\tau} \int_0^{\tau} \Delta_k(t)dt$

  - each with an instantaneous AoI $\Delta_k(t), 1 \leq k \leq N$.



System Models and Scenarios

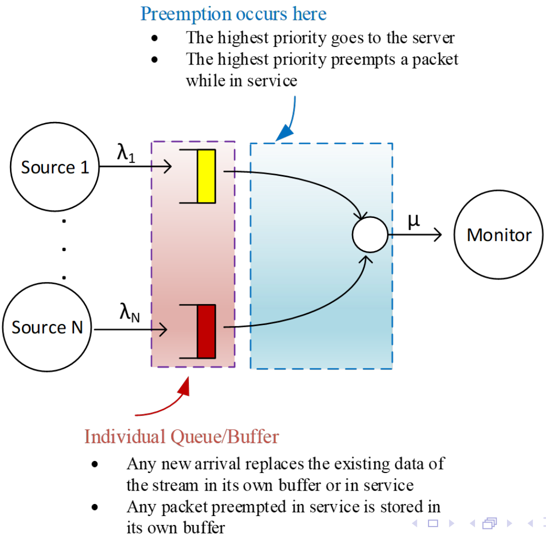
Preemption with No Waiting Room(Packet Discard)



- N information streams share a common service facility
- Each stream from each source has a different priority

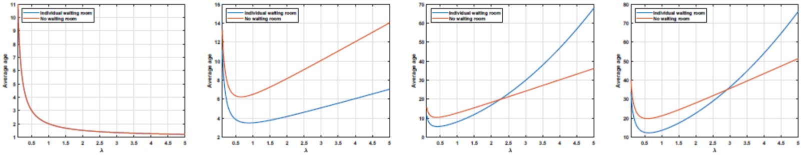
System Models and Scenarios

Preemption with Individual Waiting Room(Store and Resume)



Related Work[1]

- AoI Analysis of N information streams
- with the same arrival rate
- with individual waiting rooms
- compared to a system with no waiting room



(a) Stream 1 (b) Stream 2 (c) Stream 3 (d) Total streams

[1] Ali Maatouk, Mohamad Assad, Anthony Ephremides, "Age of Information With Prioritized Streams: When to Buffer Preempted Packets?", Jan 2019, arxiv preprint available online.

Objective

To minimize total average AoI in individual waiting room scenario

- In the related work,
  - having individual waiting rooms is clearly not beneficial for all streams(i.e. stream 3, lower priority)
  - due to the fact that higher priority streams keeps preempting lower priority streams
- In this project, we consider having heterogeneous arrival rates,
  - specifically lower arrival rate for higher priority streams
  - higher arrival rates for lower priority streams
- We explore if this would be beneficial to all streams in terms of average AoI when there are individual waiting rooms

Introduction to Stochastic Hybrid Systems (SHS)[2]

- In class: processes with discrete states

[2] J.P.Hespanha,"Modeling and Analysis of Stochastic Hybrid Systems," IEEE Proc.-Control Theory Appl., vol.153, no.5., pp.520-535, Sep.2006

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- SHS: hybrid states - a state is partitioned into
  - a discrete component $q(t) \in \mathcal{Q} = \{0, 1, 2, \dots, m\}$
  - a continuous component $\mathbf{x}(t) = [x_0(t) \dots x_n(t)] \in \mathbb{R}^{n+1}$

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- While in discrete state $q \in \mathcal{Q}$, $\mathbf{x}(t)$ evolves according to

$$\dot{\mathbf{x}} = f(q, \mathbf{x}, t)$$

for mapping $f : \mathcal{Q} \times \mathbb{R}^{n+1} \times [0, \infty) \to \mathbb{R}^{n+1}$

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$$\dot{\mathbf{x}} = f(q, \mathbf{x}, t) \; + \; g(q, \mathbf{x}, t)\dot{\mathbf{z}}$$

for mapping $f : \mathcal{Q} \times \mathbb{R}^{n+1} \times [0, \infty) \to \mathbb{R}^{n+1}$ and $g : \mathcal{Q} \times \mathbb{R}^{n+1} \times [0, \infty) \to \mathbb{R}^{(n+1)\times k}$ and $k-$vector $\mathbf{z}(t)$ of independent Brownian motion processes

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- For each $\ell \in \mathcal{L}$, define a discrete reset map ϕ_ℓ such that the transition

$$(q', \mathbf{x}') = \phi_\ell(q, \mathbf{x}, t)$$

happens with intensity

$$\lambda^{(\ell)}(q, \mathbf{x}, t)$$

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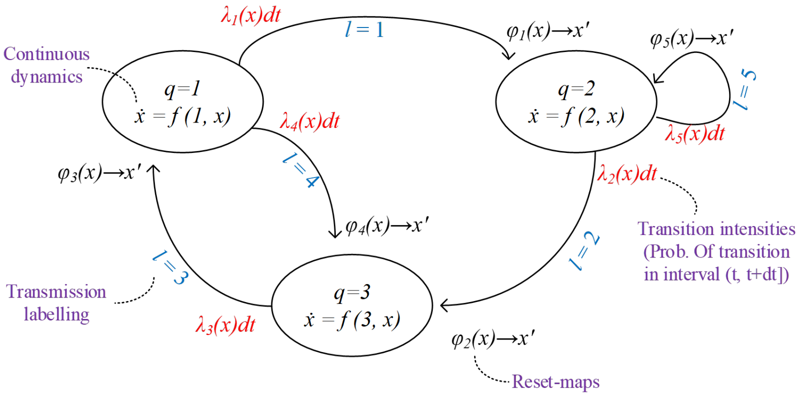

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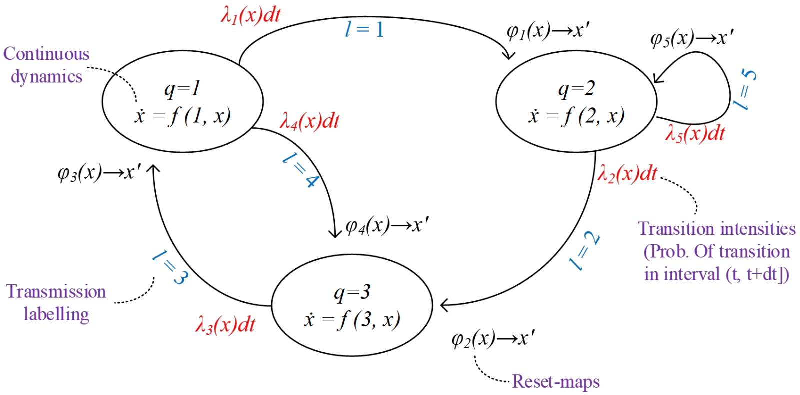

- For AoI, we consider a restricted class of SHS called the piecewise-linear SHS

Piecewise Linear SHS and AoI

SHS and AoI

SHS is a **good candidate** for AoI

1. "Random" nature of the system (e.g., random packet arrival, random service requirement...)
2. AoI is **continuous** and grows **linearly** with respect to t.
   - In particular, choose $\mathbf{x}(t)$ as AoI.

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Definition (Piecewise Linear SHS)

An SHS is said to be **peicewise linear** if $\mathbf{f}(q, \mathbf{x}, t)$ is a constant (vector) and the reset maps are linear, i.e., $\phi_\ell(q, \mathbf{x}, t) = (q'_\ell, \mathbf{x}\mathbf{A})$

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Assumption

The piecewise linear SHS is *ergodic*.

Piecewise Linear SHS and AoI

Distribution & Correlation

Define $\boldsymbol{\pi} = \{\pi_q \colon q \in \mathcal{Q}\}$ and $\{\mathbf{v}_q \colon q \in \mathcal{Q}\}$ as

$$\pi_q(t) = \mathbb{P}(q(t) = q) = \mathbb{E}[\mathbf{1}\{q(t) = q\}]$$
$$\mathbf{v}_q(t) = \mathbb{E}[\mathbf{x}(t)\mathbf{1}\{q(t) = q\}]$$

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Clearly,

$$\mathbb{E}[\mathbf{x}(t)] = \mathbb{E}[\sum_{q \in \mathcal{Q}} \mathbf{x}(t)\mathbf{1}\{q(t) = q\}] = \sum_{q \in \mathcal{Q}} \mathbf{v}_q(t)$$

So,

$$\text{Average AoI} = \lim_{t \to \infty} \frac{1}{t} \int_0^t \mathbf{x}(\tau) d\tau \underbrace{=}_{\text{Ergodicity}} \lim_{t \to \infty} \mathbb{E}[\mathbf{x}(t)] = \lim_{t \to \infty} \sum_{q \in \mathcal{Q}} \mathbf{v}_q(t)$$

Piecewise Linear SHS and AoI

The problem now reduces to

The limiting behavior of $\mathbf{v}_q(t)$

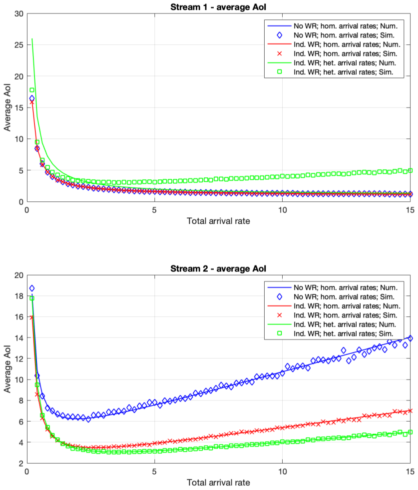
What is $\lim_{t \to \infty} \mathbf{v}_q(t)$?

See

- R. D. Yates and S. K. Kaul: The age of information: Real-time status updating by multiple sources, CoRR, vol.abs/1608.08622, 2016.[Online]. Available: http://arxiv.org/abs/1608.08622

for further details.

Results and Discussion



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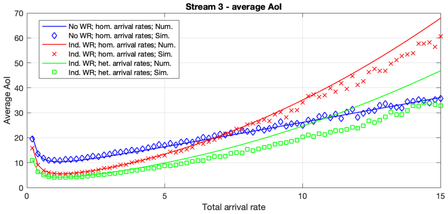

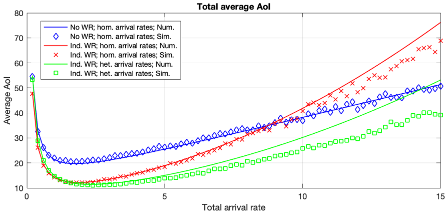

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