Development 8 - Theory exercises

Beta-reduction rules:

Variables:

$$\frac{}{x \to_\beta x} \quad (1)$$

Function application:

$$\frac{}{(\lambda x \to t)\ u \to_\beta t[x \mapsto u]} \quad (2)$$

Application:

$$\frac{t \to_\beta t' \wedge u \to u' \wedge t'\ u' \to_\beta v}{t\ u \to_\beta v} \quad (3)$$

Exercise 1:

$$\begin{aligned}
&x = x\\
&t = \lambda y \to x\ y\\
&u = \lambda y \to x\ y\\
&t[x \mapsto u] = \lambda y \to (\lambda y \to x\ y)\ y
\end{aligned}$$

Exercise 2:

$$\begin{aligned}
&x = x\\
&t = \lambda x \to \lambda y \to y\ x\\
&u = A\\
&t[x \mapsto u] = \lambda x \to \lambda y \to y\ x \qquad \text{mind the scope of x!}
\end{aligned}$$

Exercise 3:

$$\begin{aligned}
&x = f\\
&t = \lambda g \to f\\
&u = \lambda x \to y\ x\\
&t[x \mapsto u] = \lambda g \to \lambda x \to y\ x
\end{aligned}$$

Exercise 4:

$$\begin{aligned}
&x = x\\
&t = \lambda y \to ((\lambda z \to z)x)\\
&u = 0\\
&t[x \mapsto u] = \lambda y \to ((\lambda z \to z)\ 0)
\end{aligned}$$

Exercise 5:

$$\begin{aligned}
&x = x\\
&t = \lambda x\ y \to y\ x\\
&u = 0\\
&t[x \mapsto u] = \lambda x\ y \to y\ x \qquad \text{mind the scope of x!}
\end{aligned}$$

Exercise 6:

$$\begin{aligned}
&t = (\lambda x\ y \to y)\ 5\\
&u = (\lambda x \to x)\ 3\\
&t' = \lambda y \to y\\
&u' = 3\\
&v = \begin{cases} (\lambda y \to y)\ 3 \to_\beta \\ 3 \end{cases}
\end{aligned}$$

Exercise 7:

$$\begin{aligned}
&t = (\lambda f\ g \to f)(\lambda x \to x)\\
&u = 5\\
&t' = \lambda g \to \lambda x \to x\\
&u' = 5\\
&v = \begin{cases} (\lambda g \to \lambda x \to x)\ 5 \to_\beta \\ \lambda x \to x \end{cases}
\end{aligned}$$

Exercise 8:

$$\begin{aligned}
&t = ((\lambda f\ g \to g)\ ((\lambda x\ y \to\ y)\ 3))\\
&u = (\lambda f\ g \to f)\\
&t' = \begin{cases} (\lambda f\ g \to g)\ ((\lambda x\ y \to\ y)\ 3) \to_\beta \\ (\lambda f\ g \to g)\ (\lambda y \to\ y) \to_\beta \\ \lambda g \to g \end{cases}\\
&u' = (\lambda f\ g \to f)\\
&v = \begin{cases} (\lambda g \to g)\ (\lambda f\ g \to f) \to_\beta \\ (\lambda f\ g \to f) \end{cases}
\end{aligned}$$

Exercise 9:

$$\begin{aligned}
&t = ((\lambda x\ y\ z \to y\ z\ x)\ (\lambda x \to x))(\lambda y\ x \to x\ y)\\
&u = 3\\
&t' = \begin{cases} ((\lambda x\ y\ z \to y\ z\ x)\ (\lambda x \to x))(\lambda y\ x \to x\ y) \to_\beta \\ (\lambda y\ z \to y\ z\ (\lambda x \to x))(\lambda y\ x \to x\ y) \to_\beta \\ \lambda z \to ((\lambda y\ x \to x)\ z)\ (\lambda x \to x) \to_\beta \\ \lambda z \to (\lambda x \to x)\ (\lambda x \to x) \to_\beta \\ \lambda z \to \lambda x \to x \end{cases}\\
&u' = 3\\
&v = \begin{cases} (\lambda z \to \lambda x \to x)\ 3 \to_\beta \\ (\lambda x \to x) \end{cases}
\end{aligned}$$

Exercise 10:

$$
\begin{aligned}
&t = \lambda x\ y \to x\\
&u = ((\lambda f\ g \to g)\ (\lambda x \to x))\ 3\\
&t' = \lambda x\ y \to x\\
&u' = \begin{cases} ((\lambda f\ g \to g)\ (\lambda x \to x))\ 3 \to_\beta \\ (\lambda g \to g)\ 3 \\ 3 \end{cases}\\
&v = \begin{cases} (\lambda x\ y \to x)\ 3 \to_\beta \\ \lambda y \to 3 \end{cases}
\end{aligned}
$$