Development 8 - Theory exercises

Beta-reduction rules:

Variables:

$$\frac{}{x \to_\beta x}$$

Function application:

$$\frac{}{(\lambda x \to t)\ u \to_\beta t[x \mapsto u]}$$

Application:

$$\frac{t \to_\beta t' \wedge u \to u' \wedge t'\ u' \to_\beta v}{t\ u \to_\beta v}$$

Exercise 1:

Given the following untyped lambda-calculus expression:

$$(\lambda x\ y \to x\ y)\ (\lambda y \to x\ y)$$

replace the requested terms with the elements from the expression in the following lambda-calculus rule that evaluates it:

$$\frac{}{(\lambda x \to t)\ u \to_\beta t[x \mapsto u]}$$

$$\begin{cases} x = \dots \\ t = \dots \\ u = \dots \\ t[x \mapsto u] = \dots \end{cases}$$

Exercise 2:

Given the following untyped lambda-calculus expression:

$$(\lambda x \to \lambda x \to \lambda y \to y\ x)\ A$$

replace the requested terms with the elements from the expression in the following lambda-calculus rule that evaluates it:

$$\frac{}{(\lambda x \to t)\ u \to_\beta t[x \mapsto u]}$$

$$\begin{cases} x = \dots \\ t = \dots \\ u = \dots \\ t[x \mapsto u] = \dots \end{cases}$$

Exercise 3:

Given the following untyped lambda-calculus expression:

$$(\lambda f\ g \to f)(\lambda x \to y\ x)$$

replace the requested terms with the elements from the expression in the following lambda-calculus rule that evaluates it:

$$\frac{}{(\lambda x \to t)\ u \to_\beta t[x \mapsto u]}$$

$$\begin{cases} x = \dots \\ t = \dots \\ u = \dots \\ t[x \mapsto u] = \dots \end{cases}$$

Exercise 4:

Given the following untyped lambda-calculus expression:

$$(\lambda x\ y \to ((\lambda z \to z)\ x))\ 0$$

replace the requested terms with the elements from the expression in the following lambda-calculus rule that evaluates it:

$$\frac{}{(\lambda x \to t)\ u \to_\beta t[x \mapsto u]}$$

$$\begin{cases} x = \dots \\ t = \dots \\ u = \dots \\ t[x \mapsto u] = \dots \end{cases}$$

Exercise 5:

Given the following untyped lambda-calculus expression:

$$(\lambda x \to (\lambda x\ y \to y\ x))\ 0$$

replace the requested terms with the elements from the expression in the following lambda-calculus rule that evaluates it:

$$\frac{}{(\lambda x \to t)\ u \to_\beta t[x \mapsto u]}$$

$$\begin{cases} x = \dots \\ t = \dots \\ u = \dots \\ t[x \mapsto u] = \dots \end{cases}$$

Exercise 6:

Given the following untyped lambda-calculus expression:

$$((\lambda x\ y \to y)\ 5)\ ((\lambda x \to x)\ 3)$$

replace the requested terms with the elements from the expression in the following lambda-calculus rule that evaluates it:

$$\frac{t \to_\beta t' \wedge u \to u' \wedge t'\ u' \to_\beta v}{t\ u \to_\beta v}$$

$$\begin{cases} t = \dots \\ u = \dots \\ t' = \dots \\ u' = \dots \\ v = \dots \end{cases}$$

Exercise 7:

Given the following untyped lambda-calculus expression:

$$((\lambda f\ g \to f)\ (\lambda x \to x))\ 5$$

replace the requested terms with the elements from the expression in the following lambda-calculus rule that evaluates it:

$$\frac{t \to_\beta t' \wedge u \to u' \wedge t'\ u' \to_\beta v}{t\ u \to_\beta v}$$

$$\begin{cases} t = \dots \\ u = \dots \\ t' = \dots \\ u' = \dots \\ v = \dots \end{cases}$$

Exercise 8:

Given the following untyped lambda-calculus expression:

$$(((\lambda f\ g \to g)\ ((\lambda x\ y \to\ y)\ 3))\ (\lambda f\ g \to f))$$

replace the requested terms with the elements from the expression in the following lambda-calculus rule that evaluates it:

$$\frac{t \to_\beta t' \wedge u \to u' \wedge t'\ u' \to_\beta v}{t\ u \to_\beta v}$$

$$\begin{cases} t = \dots \\ u = \dots \\ t' = \dots \\ u' = \dots \\ v = \dots \end{cases}$$

Exercise 9:

Given the following untyped lambda-calculus expression:

$$(((\lambda x\ y\ z \to y\ z\ x)\ (\lambda x \to x))(\lambda y\ x \to x\ y))\ 3$$

replace the requested terms with the elements from the expression in the following lambda-calculus rule that evaluates it:

$$\frac{t \to_\beta t' \wedge u \to u' \wedge t'\ u' \to_\beta v}{t\ u \to_\beta v}$$

$$\begin{cases} t = \dots \\ u = \dots \\ t' = \dots \\ u' = \dots \\ v = \dots \end{cases}$$

Exercise 10:

Given the following untyped lambda-calculus expression:

$$(\lambda x\ y \to x)\ (((\lambda f\ g \to g)\ (\lambda x \to x))\ 3)$$

replace the requested terms with the elements from the expression in the following lambda-calculus rule that evaluates it:

$$\frac{t \to_\beta t' \wedge u \to u' \wedge t'\ u' \to_\beta v}{t\ u \to_\beta v}$$

$$\begin{cases} t = \dots \\ u = \dots \\ t' = \dots \\ u' = \dots \\ v = \dots \end{cases}$$