

Development 8 - Theory exercises

Beta-reduction rules:

Variables:

$$\overline{x \rightarrow_{\beta} x}$$

Function application:

$$\overline{(\lambda x \rightarrow t) u \rightarrow_{\beta} t[x \mapsto u]}$$

Application:

$$\frac{t \rightarrow_{\beta} t' \wedge u \rightarrow u' \wedge t' u' \rightarrow_{\beta} v}{t u \rightarrow_{\beta} v}$$

Exercise 1:

Given the following untyped lambda-calculus expression:

$$(\lambda x y \rightarrow x y) (\lambda y \rightarrow x y)$$

replace the requested terms with the elements from the expression in the following lambda-calculus rule that evaluates it:

$$\overline{(\lambda x \rightarrow t) u \rightarrow_{\beta} t[x \mapsto u]}$$

$$\left\{ \begin{array}{l} x = \dots \\ t = \dots \\ u = \dots \\ t[x \mapsto u] = \dots \end{array} \right.$$

Exercise 2:

Given the following untyped lambda-calculus expression:

$$(\lambda x \rightarrow \lambda x \rightarrow \lambda y \rightarrow y x) A$$

replace the requested terms with the elements from the expression in the following lambda-calculus rule that evaluates it:

$$\overline{(\lambda x \rightarrow t) u \rightarrow_{\beta} t[x \mapsto u]}$$

$$\left\{ \begin{array}{l} x = \dots \\ t = \dots \\ u = \dots \\ t[x \mapsto u] = \dots \end{array} \right.$$

Exercise 3:

Given the following untyped lambda-calculus expression:

$$(\lambda f\ g \rightarrow f)(\lambda x \rightarrow y\ x)$$

replace the requested terms with the elements from the expression in the following lambda-calculus rule that evaluates it:

$$\overline{(\lambda x \rightarrow t)\ u \rightarrow_{\beta} t[x \mapsto u]}$$

$$\left\{ \begin{array}{l} x = \dots \\ t = \dots \\ u = \dots \\ t[x \mapsto u] = \dots \end{array} \right.$$

Exercise 4:

Given the following untyped lambda-calculus expression:

$$(\lambda x\ y \rightarrow ((\lambda z \rightarrow z)\ x))\ 0$$

replace the requested terms with the elements from the expression in the following lambda-calculus rule that evaluates it:

$$\overline{(\lambda x \rightarrow t)\ u \rightarrow_{\beta} t[x \mapsto u]}$$

$$\left\{ \begin{array}{l} x = \dots \\ t = \dots \\ u = \dots \\ t[x \mapsto u] = \dots \end{array} \right.$$

Exercise 5:

Given the following untyped lambda-calculus expression:

$$(\lambda x\ y \rightarrow ((\lambda z \rightarrow z)\ x))\ 0$$

replace the requested terms with the elements from the expression in the following lambda-calculus rule that evaluates it:

$$\overline{(\lambda x \rightarrow t)\ u \rightarrow_{\beta} t[x \mapsto u]}$$

$$\left\{ \begin{array}{l} x = \dots \\ t = \dots \\ u = \dots \\ t[x \mapsto u] = \dots \end{array} \right.$$

Exercise 6:

Given the following untyped lambda-calculus expression:

$$(\lambda x \rightarrow (\lambda x\ y \rightarrow y\ x))\ 0$$

replace the requested terms with the elements from the expression in the following lambda-calculus rule that evaluates it:

$$\overline{(\lambda x \rightarrow t) u \rightarrow_{\beta} t[x \mapsto u]}$$

$$\left\{ \begin{array}{l} x = \dots \\ t = \dots \\ u = \dots \\ t[x \mapsto u] = \dots \end{array} \right.$$

Exercise 7:

Given the following untyped lambda-calculus expression:

$$((\lambda x y \rightarrow y) 5) ((\lambda x \rightarrow x) 3)$$

replace the requested terms with the elements from the expression in the following lambda-calculus rule that evaluates it:

$$\frac{t \rightarrow_{\beta} t' \wedge u \rightarrow u' \wedge t' u' \rightarrow_{\beta} v}{t u \rightarrow_{\beta} v}$$

$$\left\{ \begin{array}{l} t = \dots \\ u = \dots \\ t' = \dots \\ u' = \dots \\ v = \dots \end{array} \right.$$

Exercise 8:

Given the following untyped lambda-calculus expression:

$$((\lambda f g \rightarrow f) (\lambda x \rightarrow x)) 5$$

replace the requested terms with the elements from the expression in the following lambda-calculus rule that evaluates it:

$$\frac{t \rightarrow_{\beta} t' \wedge u \rightarrow u' \wedge t' u' \rightarrow_{\beta} v}{t u \rightarrow_{\beta} v}$$

$$\left\{ \begin{array}{l} t = \dots \\ u = \dots \\ t' = \dots \\ u' = \dots \\ v = \dots \end{array} \right.$$

Exercise 9:

Given the following untyped lambda-calculus expression:

$$(((\lambda f g \rightarrow g) ((\lambda x y \rightarrow y) 3)) (\lambda f g \rightarrow f))$$

replace the requested terms with the elements from the expression in the following lambda-calculus rule that evaluates it:

$$\frac{t \rightarrow_{\beta} t' \wedge u \rightarrow u' \wedge t' u' \rightarrow_{\beta} v}{t u \rightarrow_{\beta} v}$$

$$\left\{ \begin{array}{l} t = \dots \\ u = \dots \\ t' = \dots \\ u' = \dots \\ v = \dots \end{array} \right.$$

Exercise 10:

Given the following untyped lambda-calculus expression:

$$(((\lambda x y z \rightarrow y z x) (\lambda x \rightarrow x))(\lambda y x \rightarrow x y)) 3$$

replace the requested terms with the elements from the expression in the following lambda-calculus rule that evaluates it:

$$\frac{t \rightarrow_{\beta} t' \wedge u \rightarrow u' \wedge t' u' \rightarrow_{\beta} v}{t u \rightarrow_{\beta} v}$$

$$\left\{ \begin{array}{l} t = \dots \\ u = \dots \\ t' = \dots \\ u' = \dots \\ v = \dots \end{array} \right.$$

Exercise 11:

Given the following untyped lambda-calculus expression:

$$(\lambda x y \rightarrow x) ((\lambda f g \rightarrow g) (\lambda x \rightarrow x) 3)$$

replace the requested terms with the elements from the expression in the following lambda-calculus rule that evaluates it:

$$\frac{t \rightarrow_{\beta} t' \wedge u \rightarrow u' \wedge t' u' \rightarrow_{\beta} v}{t u \rightarrow_{\beta} v}$$

$$\left\{ \begin{array}{l} t = \dots \\ u = \dots \\ t' = \dots \\ u' = \dots \\ v = \dots \end{array} \right.$$