Development 8 - Theory exercises

Beta-reduction rules:

Variables:

$$\frac{}{x \rightarrow_\beta x} \quad (1)$$

Function application:

$$\frac{}{(\lambda x \rightarrow t)\ u \rightarrow_\beta t[x \mapsto u]} \quad (2)$$

Application:

$$\frac{t \rightarrow_\beta t' \wedge u \rightarrow u' \wedge t'\ u' \rightarrow_\beta v}{t\ u \rightarrow_\beta v} \quad (3)$$

Exercise 1:

$$\begin{aligned}
&x = x \\
&t = \lambda y \rightarrow x\ y \\
&u = \lambda y \rightarrow x\ y \\
&t[x \mapsto u] = \lambda y \rightarrow (\lambda y \rightarrow x\ y)\ y
\end{aligned}$$

Exercise 2:

$$\begin{aligned}
&x = x \\
&t = \lambda x \rightarrow \lambda y \rightarrow y\ x \\
&u = A \\
&t[x \mapsto u] = \lambda x \rightarrow \lambda y \rightarrow y\ x \qquad \text{mind the scope of x!}
\end{aligned}$$

Exercise 3:

$$\begin{aligned}
&x = f \\
&t = \lambda g \rightarrow f \\
&u = \lambda x \rightarrow y\ x \\
&t[x \mapsto u] = \lambda g \rightarrow \lambda x \rightarrow y\ x
\end{aligned}$$

Exercise 4:

$$\begin{aligned}
&x = x \\
&t = \lambda y \rightarrow ((\lambda z \rightarrow z)x) \\
&u = 0 \\
&t[x \mapsto u] = \lambda y \rightarrow ((\lambda z \rightarrow z)\ 0)
\end{aligned}$$

Exercise 5:

$$\begin{array}{l} x = x \\ t = \lambda x\ y \to y\ x \\ u = 0 \\ t[x \mapsto u] = \lambda x\ y \to y\ x \qquad \text{mind the scope of x!} \end{array}$$

Exercise 6:

$$\begin{array}{l} t = (\lambda x\ y \to y)\ 5 \\ u = (\lambda x \to x)\ 3 \\ t' = \lambda y \to y \\ u' = 3 \\ v = \begin{cases} (\lambda y \to y)\ 3 \to_\beta \\ 3 \end{cases} \end{array}$$

Exercise 7:

$$\begin{array}{l} t = (\lambda f\ g \to f)(\lambda x \to x) \\ u = 5 \\ t' = \lambda g \to \lambda x \to x \\ u' = 5 \\ v = \begin{cases} (\lambda g \to \lambda x \to x)\ 5 \to_\beta \\ \lambda x \to x \end{cases} \end{array}$$

Exercise 8:

$$\begin{array}{l} t = ((\lambda f\ g \to g)\ ((\lambda x\ y \to\ y)\ 3)) \\ u = (\lambda f\ g \to f) \\ t' = \begin{cases} (\lambda f\ g \to g)\ ((\lambda x\ y \to\ y)\ 3) \to_\beta \\ (\lambda f\ g \to g)\ (\lambda y \to\ y) \to_\beta \\ \lambda g \to g \end{cases} \\ u' = (\lambda f\ g \to f) \\ v = \begin{cases} (\lambda g \to g)\ (\lambda f\ g \to f) \to_\beta \\ (\lambda f\ g \to f) \end{cases} \end{array}$$

Exercise 9:

$$\begin{array}{l} t = ((\lambda x\ y\ z \to y\ z\ x)\ (\lambda x \to x))(\lambda y\ x \to x\ y) \\ u = 3 \\ t' = \begin{cases} ((\lambda x\ y\ z \to y\ z\ x)\ (\lambda x \to x))(\lambda y\ x \to x\ y) \to_\beta \\ (\lambda y\ z \to y\ z\ (\lambda x \to x))(\lambda y\ x \to x\ y) \to_\beta \\ \lambda z \to ((\lambda y\ x \to x\ y)\ z)\ (\lambda x \to x) \to_\beta \\ \lambda z \to (\lambda x \to x\ z)\ (\lambda x \to x) \\ \lambda z \to (\lambda x \to x)\ z \\ \lambda z \to z \end{cases} \\ u' = 3 \\ v = \begin{cases} (\lambda z \to z)\ 3 \to_\beta \\ 3 \end{cases} \end{array}$$

Exercise 10:

$$
\begin{aligned}
&t = \lambda x\ y \to x \\
&u = ((\lambda f\ g \to g)\ (\lambda x \to x))\ 3 \\
&t' = \lambda x\ y \to x \\
&u' = \begin{cases} ((\lambda f\ g \to g)\ (\lambda x \to x))\ 3 \to_\beta \\ (\lambda g \to g)\ 3 \\ 3 \end{cases} \\
&v = \begin{cases} (\lambda x\ y \to x)\ 3 \to_\beta \\ \lambda y \to 3 \end{cases}
\end{aligned}
$$