Question 1

My ID is 23399066

The discount factors dti are obtained via cfr (constant forward rate) interpolation of the yield curve.

d0.5 = e-0.5\*0.0099751 = 0.995025

d1.0 = e-1.0\*0.0161980 = 0.983932

d1.5 = e-1.5\*0.0198461 = 0.970670

d2.0 = e-2.0\*0.0224432 = 0.956106

Therefore, I can compute d0.25, d0.75, d1.25, and d1.75

d0.25 = e-0.25\*0.0099751 = 0.997509

d0.75 = e-0.75\*0.0141237 = 0.989463

d1.25 = e-1.25\*0.0183869 = 0.977279

d1.75 = e-1.75\*0.0213302 = 0.96336

in formula (1.1) with my ID 23399066

In formula (1.2) with my ID 23399066 & BFV, after using C++ to compute,

y = 0.0223151

In formula (1.3) with my ID 23399066 & BFV, after using C++ to compute,

c = 4.73823

Question 2

2.1 case 1

Short sell one share of stock get (get 100.5)

Buy the American call option (loss 10)

Sell the American Put (get 10)

The total money in hand is 100.5. We save the money to the bank.

We can exercise these two options at any time .

When and the time , the American put option is in the money, and be exercised.

We pay K=100, and the cash in bank is .

The profit we get is

When and the time , the American call option is in the money, and be exercised.

We pay K=100, and the cash in bank is .

The profit we get is

So no matter what is at time . Our profit is always positive and bigger than 0.5

2.2 case 2

The American bull call spread consists of long and short , hence .

The option prices satisfy the following inequality:

At time :

The inequality is violated and

And there exists , such that

Short the and long the at time

Put the money in a bank.

The cash amount in the bank is

1. Our portfolio at time consists of short one American bull call spread, cash in bank. The total value of portfolio is 0.
2. American options can be exercised at any time, so we must analyze what happens for arbitrary
3. Suppose the spread is exercised at a time t such that
4. The cash in the bank compounds to
5. If the spread is exercised, the value of each American option is equal to its intrinsic value.
6. Because the intrinsic value of an American option is the same as at expiration, the value of the spread, when exercised, will be equal to its value at expiration.
7. Hence if the spread is exercised at the time t, its value satisfies the inequality
8. So we have enough money to cover the spread, with a profit left over
9. If , both options are exercised and out profit is
10. If , only the low strike option is exercised and our profit is
11. If , the spread is worthless (and would not be exercised if ) and our profit
12. Therefore, this is an arbitrary trade
13. In all cases, we start with zero and end with a positive profit
14. Hence the inequality in (2.2) must be satisfied, to avoid arbitrary

2.3 case 3

The American bear put spread consists of long and short , hence

The option prices satisfy the following inequality:

At time :

The inequality is violated and

And there exists , such that

Short the and long the at time

Put the money in the bank

The cash amount in the bank is

1. Our portfolio at time consists of short one American bull call spread, cash in bank. The total value of portfolio is 0.
2. American options can be exercised at any time, hence we must analyze what happens for arbitrary
3. Suppose the spread is exercised at a time t such that
4. The cash in the bank compounds to
5. If the spread is exercised, the value of each American option is equal to its intrinsic value.
6. Because the intrinsic value of an American option is the same as expiration, the value of the spread, when exercised, will be equal to its value at expiration.
7. Hence using the eq. (2.3), if the spread is exercised at the time t, its value satisfies the inequality
8. So we have enough money to cover the spread, with a profit left over
9. If , both options are exercised and out profit is
10. If , only the high strike option is exercised and our profit is
11. If , the spread is worthless (and would not be exercised if t< T) and our profit is
12. Therefore, this is an arbitrary trade
13. In all cases, we start with zero and end with a positive profit
14. Hence the inequality in eq. (2.3) must be satisfied, to avoid arbitrary

Question 3

3.1 European option butterfly spreads (Lecture8 Page 12)

(3.1)

Use eq. (3.1) to prove the following relation:

(3.2)

use eq (3.1)

That is same with the equation we have .

Then relation is proved.

3.2 American calls

Draw a graph of the intrinsic value of the butterfly spread

Show that

Butterfly Spread American Call

Depend on the Lecture 8.12 Figure 6:

Butterfly Spread American Call intrinsic value

Formulate an arbitrary trade if

Long American calls butterfly spread

1. Our portfolio contains long , long short two
2. Since , so after long the American call butterfly spread, the total value of portfolio is , save the money in the bank
3. American options can be exercised at any time t such that
4. The cash in the bank compounds to is
5. If the spread is exercised, the value of each American option is equal to its intrinsic value
6. If , these three options are exercised and out profit is

.

Since , So and out profit is still positive

1. If , only the low strike options are exercised and out profit is

.

Since . So we replace with and then get

.

We already know the , so the out profit is always positive during

1. If , only the low strike option is exercised and out profit is

Since so out profit is always positive during .

1. If , the butterfly spread is worthless (and would not be exercised if ) and our profit is .
2. Therefore, this is an arbitrary trade
3. In call cases we start with zero and end with a positive profit
4. Hence, must be satisfied, to avoid arbitrary

3.3 American puts

Draw a graph of the intrinsic value of the butterfly spread

Show that

Butterfly Spread American Put

Depend on the Lecture 8.12 Figure 6:

Butterfly Spread American Call intrinsic value

Formulate an arbitrary trade if .

Long American puts butterfly spread

1. Hence our portfolio contains long , long short two
2. Since , So After long the American put butterfly spread, the total value of portfolio is , save the money in the bank
3. American options can be exercised at any time t such that
4. The cash in the bank compounds to is
5. If the spread is exercised, , the value of each American option is equal to its intrinsic value
6. If , these three options are exercised and out profit is

.

Since , so and out profit is still positive.

1. If , only the high strike options are exercised and out profit is

.

Since , so we replace with and get .

We already know , so the out profit is always positive during

.

1. If , only the high strike option is exercised and out profit is

.

Since , so the out profit is always positive during

1. If , the butterfly spread is worthless (and would not be exercised if ) and our profit is positive
2. Therefore, this is an arbitrary trade
3. In all cases we start with zero and end with a positive profit
4. Hence the inequality must be satisfied, to avoid arbitrary

Question 4

In coding file

Question 5

In coding file

Question 6

Depend on Question 1 risk free rate r = 0.0223

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Sigma | C Binomial Binary | P Binomial Binary | C BSM Binary | P BSM Binary |
| 0.1 | 0.139 | 0.839 | 0.137 | 0.841 |
| 0.2 | 0.260 | 0.718 | 0.263 | 0.715 |
| 0.3 | 0.305 | 0.673 | 0.304 | 0.674 |
| 0.4 | 0.309 | 0.669 | 0.316 | 0.661 |
| 0.5 | 0.313 | 0.665 | 0.317 | 0.661 |
| 0.6 | 0.318 | 0.660 | 0.312 | 0.666 |
| 0.7 | 0.300 | 0.678 | 0.303 | 0.675 |
| 0.8 | 0.283 | 0.695 | 0.292 | 0.686 |
| 0.9 | 0.287 | 0.691 | 0.280 | 0.698 |
| 1.0 | 0.271 | 0.707 | 0.267 | 0.711 |

Plot a graph of the binary call option

Plot a graph of the binary put option

Question 7

Prove that for a zero coupon convertible bond.

The terminal , so the convertible bond value V is always bigger than the intrinsic value

The fair value is never less than the intrinsic value, so the fair value of V is always bigger and equal than the present value of K,

(The fair value of the convertible bond. After the threshold = 130 the line go straight.)

My CUNY ID is 23399066

Day 0:

implied volatility (Sigma\_0) = 0.3736

Money0 on day 0 is -61.17

Day 1:

implied volatility (Sigma\_1) = 0.3744

Money1 on day 1 is -61.12

Day 2:

implied volatility (Sigma\_2) = 0.3894

Money2 on day 2 is -71.39

The total profit is 0.47