Statistical Computing: Homework 1

Due: Feb 27 (Monday) 23:30

Develop a general algorithm to draw random samples from the following distribution families. Your write-up should include the following parts:

- Give the algorithm in detail steps for a <u>general parameter value</u>.
- Draw a sample of size 1000 for the distribution with a <u>specific parameter value</u> and compare the empirical distribution of your data to the target pdf.
- Evaluate the efficiency (theoretically and empirically) of your sampling algorithm. Does the efficiency change with the parameter values? Make some comments on this issue.
- Make nice plots and summary tables to show your sampling results and conclusions.
- Submit your summary and code in the following format:
  - Rmd file + output in pdf
  - R code file + hw report in pdf
  - please consult with TA for other formats

(1) Weibull distribution:

$$F(x) = 1 - e^{-(x/\theta)^\beta}, \quad \theta > 0,\, \beta > 0,\, x > 0.$$

(2) pareto distribution:

$$f(x) = \frac{\beta}{\theta(1 + x/\theta)^{\beta+1}}, \quad \theta > 0,\, \beta > 0,\, x > 0.$$

(3) skewed distribution I:

$$f(x) = \frac{2}{\gamma + \frac{1}{\gamma}}\, \phi\left(x\gamma^{-sign(x)}\right), \quad x \in R,\, \gamma > 0,$$

where $\phi(x)$ is the pdf of $N(0,1)$. The parameter γ controls the degrees of asymmetry. In particular, $f(x)$ becomes symmetric when $\gamma = 1$.

(4) skewed distribution II:

$$f(x) = 2\,h(x)\,G(\alpha x), \quad x \in R,\, \alpha \in R,$$

where $G(\cdot)$ is a cdf defined on R and symmetric around zero, and $h(\cdot)$ is also a symmetric pdf (symmetric with respect to zero) defined on R.
If $G'(x) = h(x)$, $f(x)$ is called the skew-"G" distribution. For example, when $G(\cdot)$ and $h(\cdot)$ are cdf and pdf of the same normal distribution, $f(x)$ is called skew-normal distribution. When $G(\cdot)$ and $h(\cdot)$ are cdf and pdf of the t_ν distribution, $f(x)$ is called skew-t distribution. In this homework, you need to generate random samples from this skew-t distribution with parameter (α, ν).

(5) a 2-dimension distribution:

$$f(x, y) = 2(1-x)(1-y)(1-xy)^{-3}, \quad 0 < x < 1,\, 0 < y < 1.$$