Reliability Analysis HW3

Problem 1

Exercise 4.26 In some applications, a sample of failure times comes from a mixture of subpopulations.

(a) Write down the expression for the cdf $F(t)$ for a mixture of two exponential distributions with means $\theta_1 = 1$ and $\theta_2 = 10$ (subpopulations 1 and 2 respectively) with ξ being the proportion from subpopulation 1.

(b) For $\xi = 0,\ 0.1,\ 0.5,\ 0.9$, and 1, compute the mixture $F(t)$ for a number of values of t ranging between 0 and 30. Plot these distributions on one graph.

(c) Plot $\log(t)$ versus $\log\left(-\log\left[1 - F(t)\right]\right)$ for each $F(t)$ computed in part (b). Comment on the shapes of the mixtures of exponential distributions, relative to a pure exponential distribution or a Weibull distribution.

(d) Plot the hf $h(t)$ of the mixture distributions in part (b).

(e) Qualitatively, what do the Weibull plots in part (c) suggest about the hf of a mixture of two exponential distributions?

Problem 2

Exercise 6.6 The supplier of an electromechanical control for a household appliance ran an accelerated life test on sample controls. In the test, 25 controls were put on test and run until failure or until 30 thousand cycles had been accumulated. Failures occurred at 5, 21, and 28 thousand cycles. The other 22 controls did not fail by the end of the test.

(a) Make a Weibull plot displaying the device failure data.

(b) Use the plotted points to estimate the proportion of devices that will fail before 10,000 hours of operation.

(c) Comment on whether the Weibull distribution fits the data well.

(d) Use the slope and location of this line to estimate the Weibull distribution parameters.

(e) Use the plotted points to estimate the proportion of devices that will fail before 100,000 hours. Comment on the usefulness of this estimate.

(f) Make a lognormal plot with a confidence band on it.

(g) Make a LEV plot with a confidence band on it.

Problem 3

Exercise 6.13 Suppose that T has cdf

$$F(t) = 1 - \left[1 + \left(\frac{\log(t) - \mu}{\sigma}\right)\right]^{-1}, \; t > \exp(\mu), \; -\infty < \mu < \infty, \; \sigma > 0.$$

(a) Find the p quantile t_p of the distribution.

(b) Find the probability scales that will linearize all the cdfs in this distribution family.

(c) Generate a simple random sample of size $n = 20$ from a population with $\mu = 3$ and $\sigma = 2$ and construct a probability plot using the scales derived in part (b). This is accomplished by ordering the sample points in increasing order $t_{(1)} \leq \cdots \leq t_{(20)}$. Then plot $t_{(i)}$ versus $(i - 0.5)/n$ on the derived axes.

(d) What do μ and σ represent in the probability plot?