Statistical Computing: Homework 2

Due on March 20 (Monday) 23:30

1. Consider a Weibull model with the pdf

$$f(x;\boldsymbol{\theta}) = \frac{\beta}{\alpha}\left(\frac{x}{\alpha}\right)^{\beta-1} e^{-(x/\alpha)^{\beta}}, \quad \boldsymbol{\theta} = (\alpha,\beta)', \quad \alpha > 0, \beta > 0, x > 0.$$

Denote $X \sim f$ and consider the model with $\boldsymbol{\theta} = (1, 1.5)$. Use the Monte Carlo method to evaluate the following quantities. Report your Monte Carlo estimates and their Monte Carlo s.e.

(a) kurtosis: $E\left(\frac{X-EX}{\sqrt{var(X)}}\right)^4$. (bonus point)

(a1) EX^3

(b) Fisher information matrix (2×2):

$$I(\boldsymbol{\theta}) = -E\left[\frac{\partial^2}{\partial\boldsymbol{\theta}\partial\boldsymbol{\theta}'}\log f(X;\boldsymbol{\theta})\right].$$

(c) Evaluate $E[X|X > 2]$ using the importance sampling method.

2. Consider a bi-variate random vector (X, Y) with the cdf:

$$F(x,y) = e^{-V(x,y)}, \quad x > 0, y > 0,$$
$$V(x,y) = \frac{1}{x}\Phi\left\{\frac{1}{2} - \log\left(\frac{x}{y}\right)\right\} + \frac{1}{y}\Phi\left\{\frac{1}{2} - \log\left(\frac{y}{x}\right)\right\},$$

where $\Phi(\cdot)$ is the cdf of N(0,1).

(a) Use the MH algorithm to draw a random sample of size $n = 1000$ from this bi-variate distribution. Provide the sampling procedure and plot your results.

(b) Use the Monte Carlo method to evaluate $\tau \equiv P(X + Y > 5)$. Report Monte Carlo estimate $\hat{\tau}$ and its Monte Carlo s.e.

Hint:

$$\tau = 1 - P(X + Y \le 5) = 1 - \int_0^5 \int_0^{5-x} f(x,y)dydx$$
$$= 1 - \int_0^\infty \int_0^\infty \{I(0 < y \le 5 - x)\, I(0 < x < 5)\}\, f(x,y)\, dydx$$
$$= 1 - E[h(X,Y)],$$

where $I(A)$ is the indicator function taking the value 1 if the event A holds and value 0 otherwise, and

$$h(x, y) = I(0 < y \leq 5 - x)\, I(0 < x < 5).$$