Reliability Analysis HW1

Problem 1

Exercise 3.7 The supplier of an electromechanical control for a household appliance ran an accelerated life test on sample controls. In the test, 25 controls were put on test and run until failure or until 30 thousand cycles had been accumulated. Failures occurred at 5, 21, and 28 thousand cycles. The other 22 controls did not fail by the end of the test.

(a) Compute and plot a nonparametric estimate for $F(t)$.

(b) Compute an approximate 95% confidence interval for the probability that an electromechanical device from the same production process, tested in the same way, would fail before 30 thousand cycles. Use the conservative binomial distribution method.

(c) Repeat part (b) using the Jeffrey method.

(d) Repeat part (b) using the Wald method, based on $Z_{\widehat{F}(30)} \sim \text{NORM}(0, 1)$.

(e) Explain why, in this situation, the method in part (b) or (c) would be preferred to the method in part (d).

(f) The appliance manufacturer is really interested in the probability of the number of days to failure for its product. Use rate differs from household to household, but the average rate is 2.3 cycles per day. What can the manufacturer say about the fraction of devices that would fail in 10 years of operation (the expected technological life of the product)?

(g) Refer to part (f). Describe an appropriate model to use when use rate varies in the population of units. To simplify, start by assuming that there are only two different use rates. Discuss, using appropriate expressions.

Problem 2

Exercise 3.12 Weis et al. (1986) report on the results of a life test on silicon photodiode detectors in which 28 detectors were tested at 85°C and 40 volts reverse bias. These conditions, which were more stressful than normal use conditions, were used in order to get failures quickly. Specified electrical tests were made at $0, 10, 25, 75, 100, 500, 750, 1000, 1500, 2000, 2500, 3000, 3500,\ 3600,\ 3700,$ and 3800 hours to determine if the detectors were still performing properly. Failures were found after the inspections at 2500 (1 failure), 3000 (1 failure), 3500 (2 failures), 3600 (1 failure), 3700 (1 failure), and 3800 (1 failure). The other 21 detectors had not failed after 3800 hours of operation. These data are also available in file PhotoDetector.csv. Use these data to estimate the failure-time cdf of such photodiode detectors running at the test conditions.

(a) From the description given above, the data would be useful for making inferences about what particular population or process? Explain your reasoning.

(b) Compute and plot a nonparametric estimate of the cdf for time to failure at the test conditions.

(c) Compute standard errors for the nonparametric estimate in part (b).

(d) Compute pointwise approximate 95% confidence intervals for $F(t)$ and add these to your plot.

(e) Compute nonparametric simultaneous approximate 95% confidence bands for $F(t)$ over the complete range of observation.

(f) Provide a careful explanation of the differences in interpretation and application of the nonparametric pointwise confidence intervals and the nonparametric simultaneous confidence bands.

Problem 3

Exercise 3.21 Consider the model in Section 2.2.1 and the data collection method described in Section 3.5.

(b) Show that in terms of the parameters $\boldsymbol{p} = (p_1, \cdots, p_m)$,

$$L(\boldsymbol{p}) = \prod_{j=1}^{m} p_j^{d_j} (1 - p_j)^{n_j - d_j},$$

where $n_j = n - \sum_{i=0}^{j-1} d_i - \sum_{i=0}^{j-1} r_i$, with the understanding that $d_0 = 0$ and $r_0 = 0$.

(c) Show that the ML estimators of the parameters are

$$\hat{p}_j = \frac{d_j}{n_j}, \ j = 1, \cdots, m.$$

(d) Show that the observed information matrix for the parameters $\boldsymbol{p}$ is diagonal and that diagonal element i of the matrix is equal to

$$-\frac{\partial^2 \log[L(\boldsymbol{p})]}{\partial p_i^2} = \frac{n_i}{p_i(1 - p_i)}$$

evaluated at $\hat{\boldsymbol{p}}$. This shows that, asymptotically (in large samples), the components of $\hat{\boldsymbol{p}}$ are uncorrelated and $Var(\hat{p}_i) = p_i(1 - p_i)/n_i$. Use these results and the delta method to derive Greenwood's formula as given in (3.10).

Problem 4

Exercise 3.22 Consider the relationship $S(t_i) = \exp\left[-H(t_i)\right]$, where $H(t)$ is the cumulative hf. Note that a nonparametric ML estimator (based on the product-limit estimator) of $H(t)$ without assuming a distributional form is

$$\hat{H}(t_i) = -\sum_{j=1}^{i} \log(1 - \hat{p}_j) \approx \sum_{j=1}^{i} \hat{p}_j = \sum_{j=1}^{i} \frac{d_j}{n_j} = \hat{\hat{H}}(t_i).$$

$\hat{\hat{H}}(t_i)$ is known as the Nelson–Aalen estimator of $\hat{H}(t_i)$. Thus, $\hat{\hat{F}}(t_i) = 1 - \exp\left[\hat{\hat{H}}(t_i)\right]$ is another nonparametric estimator for $F(t)$.

(a) Give conditions to assure a good agreement between $\hat{H}(t_i)$ and $\hat{\hat{H}}(t_i)$ and thus between $\hat{F}(t)$ and $\hat{\hat{F}}(t)$.

(b) Use the delta method to compute approximate expressions for $Var\left[\hat{H}(t_i)\right]$ and $Var\left[\hat{\hat{H}}(t_i)\right]$. Comment on the expression(s) you get.

(c) Compute the Nelson–Aalen estimate of $F(t)$ and compare with the estimate computed in Exercise 3.20. Describe similarities and differences.

(d) Shoe that $\hat{\hat{H}}(t_i) < \hat{H}(t_i)$ and that $\hat{\hat{F}}(t_i) < \hat{F}(t_i)$.

(e) Describe suitable modifications of the estimator that can be used when failure and censoring times are grouped into common intervals.

Problem 5

Show that $\hat{K}(t) = \hat{F}(t)$ when there is no censoring (see page 23 of the slides).