Statistical Computing: Homework 4

Due on May 8 (Monday) 23:30

1. Consider the following normal model:

$$\begin{aligned} &Y_i \sim N(\mu_i, \sigma_i^2), \quad i = 1, 2, ..., n; \\ &\mu_i = \beta_0 + \beta_1 x_i, \quad \sigma_i^2 = e^{\alpha_0 + \alpha_1 x_i}, \quad \boldsymbol{\alpha} = (\alpha_0, \alpha_1)' \in R^2,\ \boldsymbol{\beta} = (\beta_0, \beta_1)' \in R^2. \end{aligned}$$

   Using DataA, solve the MLE of $(\boldsymbol{\beta}, \boldsymbol{\alpha})$ based on the (block) coordinate descent method. (You may treat $\boldsymbol{\beta}$ as a block and $\boldsymbol{\alpha}$ as a block.)

   <u>DataA (extract from Boston Housing data):</u>
   $Y_i = \log(\text{medv}_i)$ for the ith suburb region of Boston; and $x_i = \text{lstat}_i$. Detail variable descriptions can be found in "Boston {ISLR2}".

2. Consider an alternative fused lasso problem:

$$\min_{\boldsymbol{\beta}} \left\{ \frac{1}{2} \sum_{t=1}^{n} (y_t - \beta_t)^2 + \tau \sum_{t=3}^{n} |\beta_t - 2\beta_{t-1} + \beta_{t-2}| \right\}, \quad \boldsymbol{\beta} = (\beta_1, ..., \beta_n)',$$

   which can be reformulated as

$$\min_{\boldsymbol{\beta}} \left\{ \frac{1}{2} \|\boldsymbol{y} - \boldsymbol{\beta}\|^2 + \tau \sum_{t=3}^{n} |\delta_t| \right\},$$

   where

$$\delta_t \equiv \beta_t - 2\beta_{t-1} + \beta_{t-2} = \big(0, ..., 0, 1, -2, \underbrace{1}_{t\text{-th}}, 0, ..., 0\big)\boldsymbol{\beta}, \quad t = 3, ..., n;$$

$$\underbrace{\boldsymbol{\delta}}_{(n-2)\times 1} \equiv (\delta_3, \delta_4, ..., \delta_n)' = \underbrace{D}_{(n-2)\times n} \boldsymbol{\beta}, \quad D = \begin{bmatrix} 1 & -2 & 1 & \cdots & 0 \\ 0 & 1 & -2 & 1 & \cdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 1 & -2 & 1 \end{bmatrix}.$$

   Using ADMM to solve $\{\beta_t\}$ for DataB.

   <u>DataB:</u> $\{(t, y_t) : t = 1, 2, ..., 100\}$ (same data shown in R Lab7-2 Example 5)

3. Consider the following function:

$$f(\boldsymbol{x}) = -\sum_{j=1}^{10} \sin(\pi x_j) \left[\sin(j\pi x_j^2)\right]^{20}, \quad \boldsymbol{x} = (x_1, x_2, \cdots, x_{10})', \quad 0 \le x_j \le 1, \quad j = 1, 2, ..., 10.$$

   Use any optimization method to find the minimum of $f(\boldsymbol{x})$ for $\boldsymbol{x} \in [0, 1]^{10}$.