Reliability Analysis HW5

Problem 1

Exercise 9.11 Revisit the bearing-cage Weibull analysis done in Exercise 8.27.

(a) Obtain 1000 bootstrap samples from these data using the fractional-random-weight bootstrap sampling method based on the Dirichlet distribution (see Section B.3.12). For each of the samples, compute bootstrap ML estimates $\widehat{\mu}_i^*$ and $\widehat{\sigma}_i^*$, $i = 1, \ldots, 1000$ of the SEV distribution parameters μ and σ corresponding to the logs of the times.

(b) Using the results in part (a), compute bootstrap ML estimates $t_{0.10}^*$ for each of the 1000 bootstrap samples.

(c) Using the results in part (b), compute a simple "percentile bootstrap" confidence interval for $t_{0.10}$. How does it compare with the Wald and LR-based intervals computed in Exercise 8.27?

Problem 2

Exercise 17.3 McCool (1980) gives the results of a life test on rolling contact fatigue of ceramic ball bearings. Ten specimens were tested at each of four levels of stress. The ordered failure times are in the following table and in file CeramicBearing02.csv. McCool indicates that it is customary to model such data with the two-parameter Weibull distribution with a shape parameter that does not depend on stress.

Stress (10^6 psi)	Ordered lifetimes (10^6 revolutions)
0.87	$1.67, 2.20, 2.51, 3.00, 3.90, 4.70, 7.53, 14.70, 27.80, 37.40$
0.99	$0.80, 1.00, 1.37, 2.25, 2.95, 3.70, 6.07, 6.65, 7.05, 7.37$
1.09	$0.012, 0.18, 0.20, 0.24, 0.26, 0.32, 0.32, 0.42, 0.44, 0.88$
1.18	$0.073, 0.098, 0.117, 0.135, 0.175, 0.262, 0.270, 0.350, 0.386, 0.456$

(a) Plot the failure times versus stress on log-log axes (or, alternatively, take logs and plot on linear axes).

(b) It is often suggested that median failure time is proportional to a power transformation of stress: that is, $t_{0.50} = \exp(\beta_0) \times (\text{ stress })^{\beta_1}$ or $\log(t_{0.50}) = \beta_0 + \beta_1 \log(\text{stress})$. Is the suggestion reasonable in this case? Plot the sample medians on the graph in part (a) to help answer this question.

(c) Make separate Weibull plots for the data at each level of stress, plotting them all on the same graph. What does this plot suggest about the customary assumption that the Weibull shape parameter is the same for all levels of stress? Provide possible explanations for the observed differences in the estimates of the Weibull shape parameter at each level of stress.

(d) Use ML to fit the Weibull regression model (17.3) to the rolling contact fatigue data.

(e) Compute estimates for the median time to failure at 1.05×10^6 psi, the 0.01 quantile at 1.05×10^6 psi, and the 0.01 quantile at 0.85×10^6 psi.