SC-HW3

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due on 04/03

Power function for Alternative Model II

Jarque-Bera(JB) test statistic:

$$\text{JB} = \frac{n}{6}(\text{skewness}^2 + \frac{1}{4}(\text{kurtosis-3})^2),$$

where

$$\begin{aligned} \text{skewness} &= \frac{\frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^3}{s^3} \\ \text{kurtosis} &= \frac{\frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^4}{s^4} \\ \text{,where } s^2 &= \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2. \end{aligned}$$

Setting:

- Alternative model II: $t(v)$ with df $v = 1/\theta, 0 < \theta \leq 1$.
- $\theta = (0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9)$.
- Replication M=10000
- $\alpha = 0.05$.

(a) data with size n = 20

For each θ_i,the testing results:

```
JB <- function(sample){
  n = length(sample)
  skew_upp <- sum((sample-mean(sample))^3)/(n-1)
  skew <- skew_upp/(sd(sample))^3
  kurt_upp <- sum((sample-mean(sample))^4)/(n-1)
  kurt <- kurt_upp/(sd(sample))^4
  return(n/6*(skew^2+1/4*(kurt-3)^2))
} # JB depend on sample
experiment7 = function(n, theta){
```

```
  x = rt(n, df=1/theta)
  JB(x)
} #generate JB
n = 20 # small
M = 10000 #MC replication
reject.rate = rep(0,9)
testing.result = matrix(, nrow = 9, ncol = 2)
testing.result = as.data.frame(testing.result)
colnames(testing.result) <- c("non-reject","reject")
sd.MC <- rep(0,9)
the <- c(0.1,0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9)
set.seed(1)
for (i in 1:9){
  y = replicate(M, experiment7(n,theta=the[i]))
  testing.result[i,] <- c(table((y >qchisq(p = 0.95,df = 2))))
  reject.rate[i] = mean(y >qchisq(p = 0.95,df = 2))
  sd.MC[i] <- sqrt(reject.rate[i]*(1-reject.rate[i])/M)
}
theta_col <-  paste0("theta","=",seq(0.1, 0.9, length=9))
knitr::kable(cbind(theta_col,testing.result,reject.rate,sd.MC),
             col.names = c("","non-reject","reject","reject.rate","sd.MC"))
```

	non-reject	reject	reject.rate	sd.MC
theta=0.1	9330	670	0.0670	0.0025002
theta=0.2	8590	1410	0.1410	0.0034802
theta=0.3	7427	2573	0.2573	0.0043715
theta=0.4	6387	3613	0.3613	0.0048038
theta=0.5	5404	4596	0.4596	0.0049837
theta=0.6	4411	5589	0.5589	0.0049652
theta=0.7	3580	6420	0.6420	0.0047941
theta=0.8	2985	7015	0.7015	0.0045760
theta=0.9	2393	7607	0.7607	0.0042666

(b) data with size n = 100

For each θ_i,the testing results:

```
n = 100 # large
reject.rate1 = rep(0,9)
testing.result1 = matrix(, nrow = 9, ncol = 2)
```

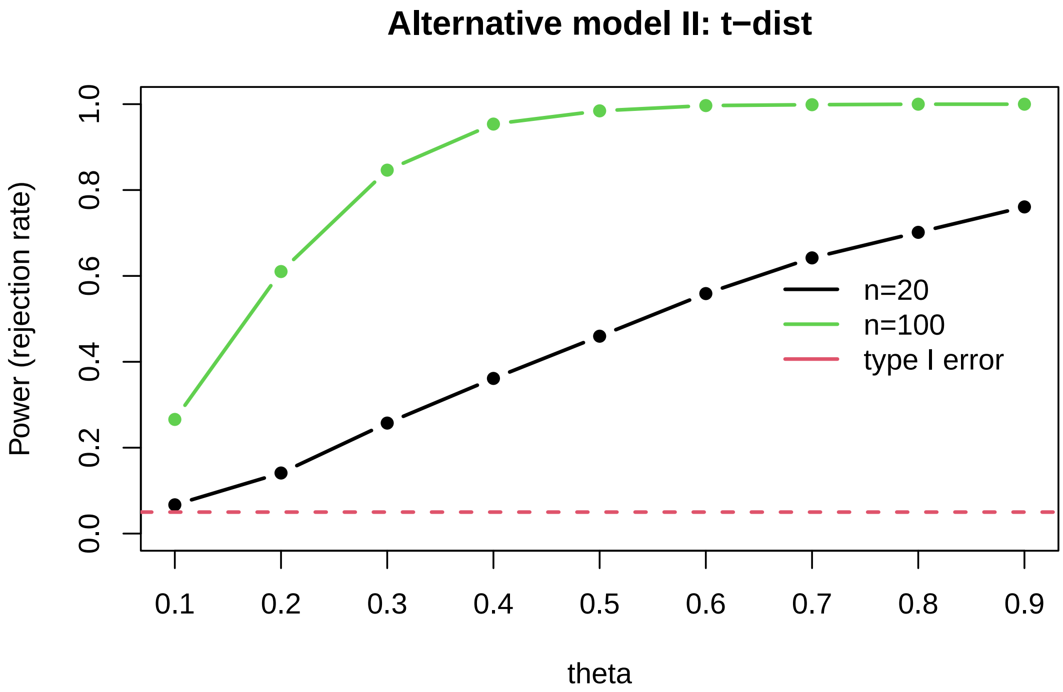
```r
testing.result1 = as.data.frame(testing.result1)
colnames(testing.result1) <- c("non-reject","reject")
sd.MC <- rep(0,9)
set.seed(6)
for (i in 1:9){
  y = replicate(M, experiment7(n,theta =the[i]))
  count = as.vector(table((y >qchisq(p = 0.95,df = 2))))
  testing.result1[i,] <- count
  reject.rate1[i] = mean(y >qchisq(0.95,2))
  sd.MC[i] <- sqrt(reject.rate1[i]*(1-reject.rate1[i])/M)
}
theta_col <-  paste0("theta","=",seq(0.1, 0.9, length=9))
knitr::kable(cbind(theta_col,testing.result1,reject.rate1,sd.MC),
             col.names = c("","non-reject","reject","reject.rate","sd.MC"))
```

	non-reject	reject	reject.rate	sd.MC
theta=0.1	7342	2658	0.2658	0.0044176
theta=0.2	3898	6102	0.6102	0.0048770
theta=0.3	1537	8463	0.8463	0.0036066
theta=0.4	464	9536	0.9536	0.0021035
theta=0.5	156	9844	0.9844	0.0012392
theta=0.6	33	9967	0.9967	0.0005735
theta=0.7	13	9987	0.9987	0.0003603
theta=0.8	1	9999	0.9999	0.0001000
theta=0.9	1	9999	0.9999	0.0001000

Power function plot

```r
par(mfcol=c(1,1))
plot(sort(the),reject.rate,type="b", pch=16, axes=F,
     lwd=2, ylim=c(0,1),
     xlab="theta", ylab="Power (rejection rate)")
title("Alternative model II: t-dist", line = 1.5)
axis(1, labels = sort(the), at=sort(the))
axis(2)
box()
abline(h=0.05,lty=2,col=2, lwd=2)
lines(sort(the),reject.rate1,
      col=3,lwd=2, pch=16, type="b")
```

```r
legend(0.65,0.65,legend=c("n=20","n=100","type I error"),
       lty=1, col=c(1,3,2), lwd=2, bty="n")
```



As the sample size increases, power increases significantly.