Reliability Analysis-HW1

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Problem 1

(a)

可以採用 Chapter 1,Example 4(Heat Exchanger Tube Crack Data) 的例子來解釋這個事情。由於每次檢查時，將不足以造成核電廠安全問題的極少數有裂縫管子都給維修好，這對於工廠的運作是沒效率的。所以在建造機器前，會考慮機器內可容忍最低比例的裂縫管子使運作安全而建造。通常不會考慮到能容忍一半以上的失敗品同時存在，這會造成安全性的問題。

(b)

通常可靠度資料會有 censored data ，所以在大部分的可靠度資料中，沒辦法從期望值和變異數得知想要的資訊。

(c)

比如機車使用的電瓶壽命，這個產品可能會依照使用頻率與氣候而有所不同的壽命，考慮使用其 mean time to failure 來分析想知道的問題。

Problem 2

The CDF of the lifetime T to the transmission for the Model X automobile is

$$F(t) = 1 - \exp\left[-\left(\frac{t}{130}\right)^{2.5}\right], t > 0,$$

where time is measured in thousands of miles. A Model X automobile with 120 thousand miles of previous service is being offered for sale.

(a)

The desired probability is

$$P(T < 150|T > 120) = \frac{F(150) - F(120)}{1 - F(120)} = 1 - \exp\left[-((15/13)^{2.5} - (12/13)^{2.5})\right] = 0.457446$$

```r
#2.a
d <- (15/13)^(2.5) - (12/13)^(2.5)
1-exp(-d)
```

```
## [1] 0.457446
```

(b)

Its remaining-life time is defined by U=T-120 condition on T > 120. The CDF of $U|\{T > 120\}$ is

$$G(u|T > 120) = P(T - 120 \leq u|T > 120) = \frac{F(u + 120) - F(120)}{1 - F(120)}, u > 0.$$

Let m be the desired median of $U|\{T > 120\}$,we have

$$0.5 = G(m|T > 120) = 1 - \exp(-\frac{(m + 120)^{2.5} - 120^{2.5}}{130^{2.5}})$$

$$m = (130^{2.5} \times \ln(2) + 120^{2.5})^{1/2.5} - 120 = 33.36985$$

```r
#2.b
(130^(2.5)*log(2)+120^(2.5))^(1/2.5)-120
```

```
## [1] 33.36985
```

Problem 3

$$T \sim F(t) = \frac{t}{2}, 0 \leq t \leq 2.$$

(a)

The pdf of T, f(t) = $dF(t)/dt$ = 1/2,0<t<2.

The hf of T,h(t) = f(t)/(1-F(t)) = 1/(2-t),0<t<2.

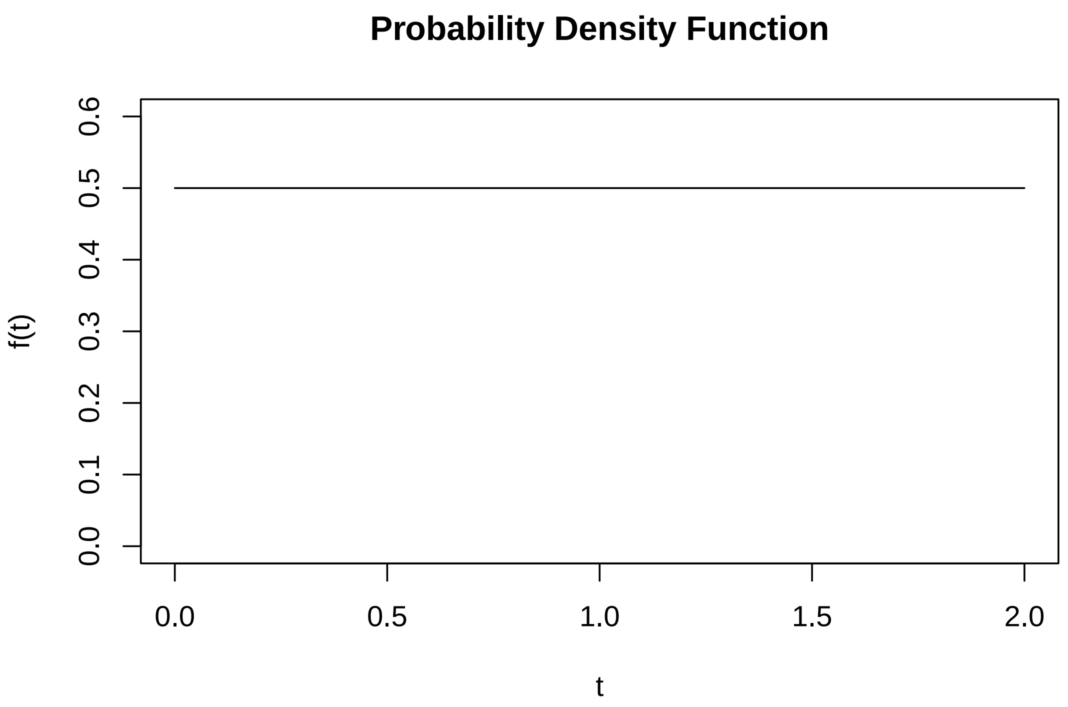
(b)

$$1 - \exp\left[-\int_0^t h(x)dx\right] = 1 - \exp\left(-\ln\frac{2}{2-t}\right) = 1 - (2-t)/2 = t/2 = F(t)$$

(c)

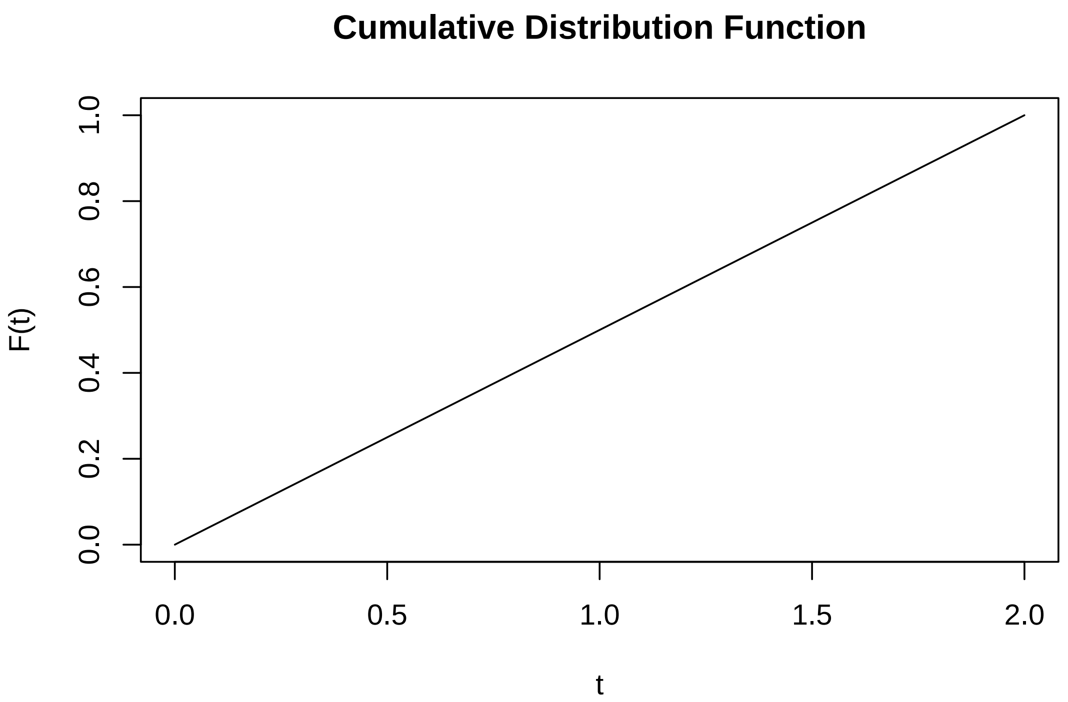
- pdf plot:

```r
pdf <- function(x){return(rep(1/2,length(x)))}
curve(pdf,from = 0,to = 2,xlab = "t",
      ylab="f(t)",ylim=c(0,0.6),
      main="Probability Density Function")
```



- cdf plot:

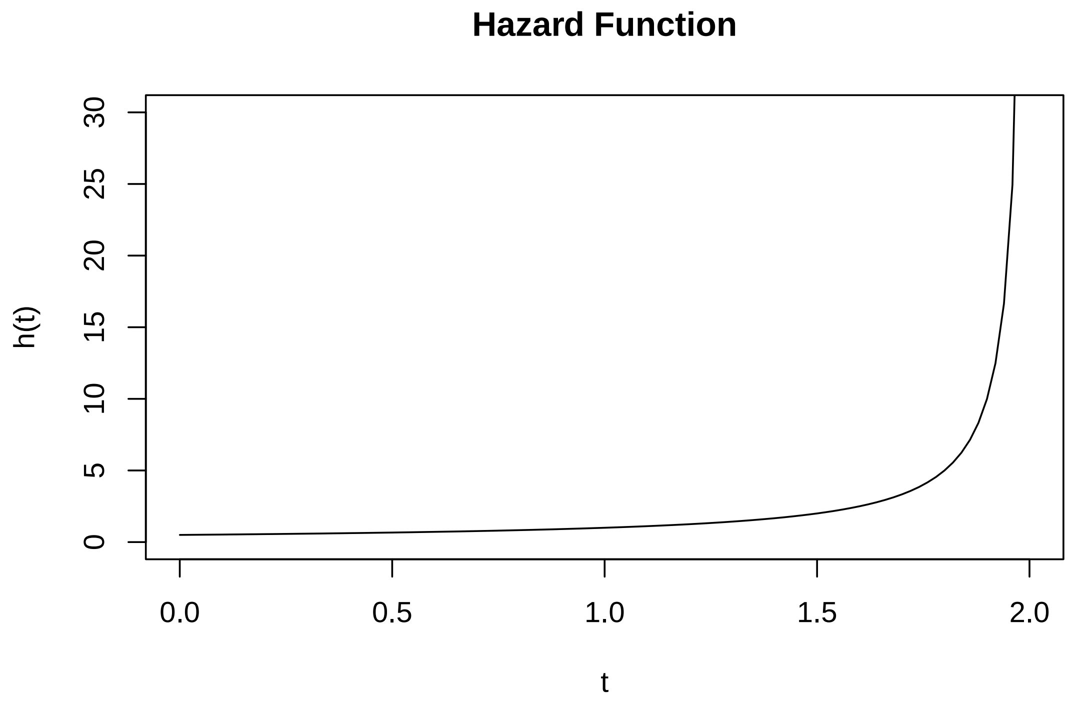
```
cdf = function(x){x/2}
curve(cdf,from = 0,to = 2,xlab = "t",
      ylab="F(t)",ylim=c(0,1),
      main="Cumulative Distribution Function")
```



(d)

- Hazard function plot:

```r
# correct
hf = function(x){1/(2-x)}
curve(hf,from = 0.0001,to = 1.9999,xlab = "t",
      ylab="h(t)",
      main="Hazard Function",
      ylim=c(0,30))
```

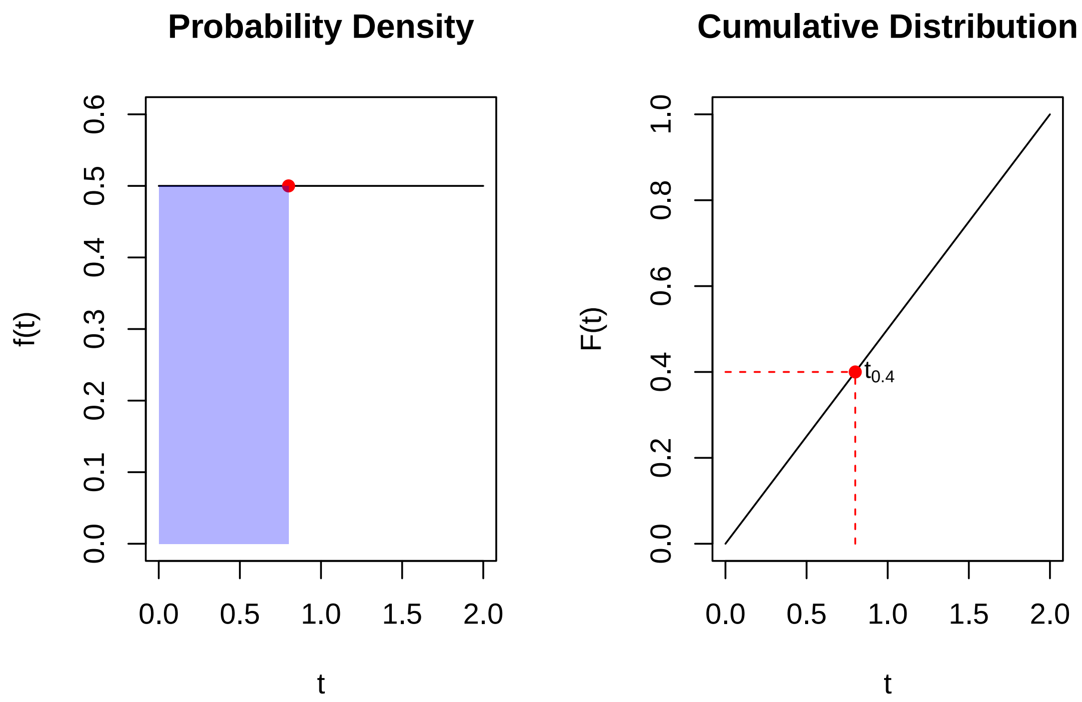


由於 $h(t) = 1/(2-t)$ 於 $t = 2$ 是 undefined，當 $t \to 2^-$，會使得 h(t) 急遽增加，由上面圖型可看出。

(e)

```r
# correct
par(mfrow=c(1,2))
curve(pdf,from = 0,to = 2,xlab = "t",
      ylab="f(t)",ylim=c(0,0.6),
      main="Probability Density")
points(x = 0.8,y=0.5,pch=16,col="red")
rect(xleft = 0, xright = 0.8,
     ybottom = 0, ytop =0.5,
     border = NA, col = adjustcolor("blue", alpha = 0.3))
```

```
curve(cdf,from = 0,to = 2,xlab = "t",
      ylab="F(t)",ylim=c(0,1),
      main="Cumulative Distribution")
points(x = 0.8,y=0.4,pch=16,col="red")
text(0.95,0.4,expression(t[0.4]),cex=0.8)
segments(x0=0.8,y0=0,x1=0.8,y1=0.4,lty=2,col = "red")
segments(x0=0,y0=0.4,x1=0.8,y1=0.4,lty=2,col = "red")
```



By Probability integral transformation,$F(t) \sim U(0,1)$.

$$\int_0^{t_p} dx = p \Rightarrow t_p = p.\text{So,}t_{0.4} = 0.4.$$

在前面的小題中，得知 $T \sim U(0,1/2)$，以及觀察其 cdf plot ，可看出 T 的 p 百分位數對應到 t=2p。F(t) 一樣是 uniform distribution，如果 T 乘 2 倍就和 F(t) 的分佈一樣，那對應的 p 百分位數就會除以 2，得到 p。

(f)

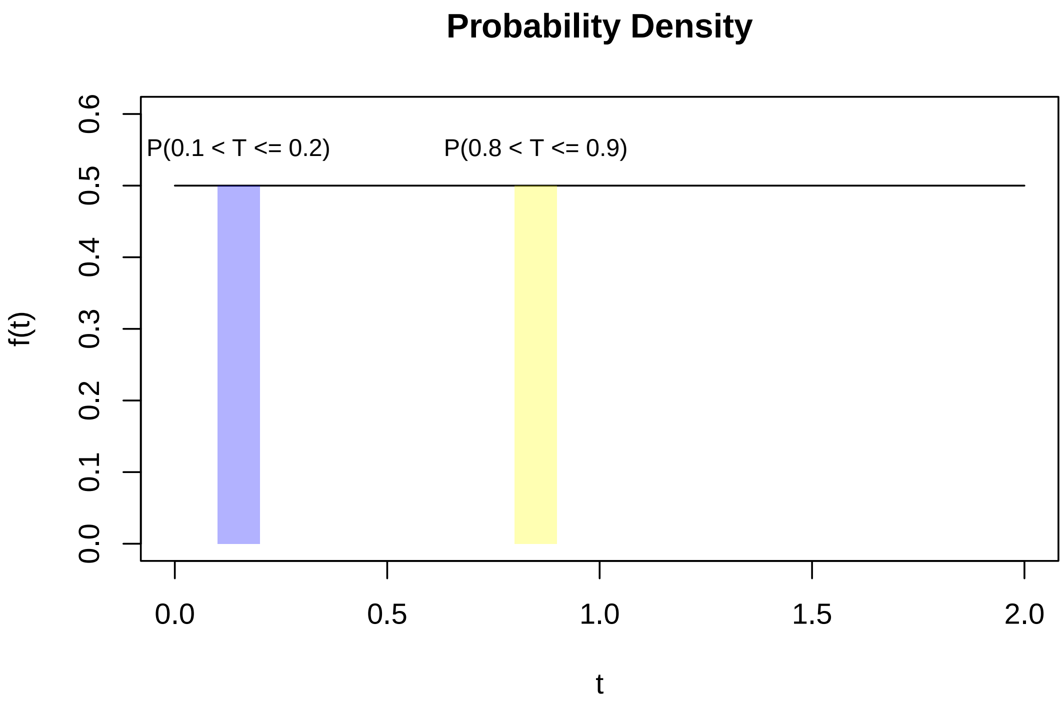
$$Pr(0.1 < T \leq 0.2) = F(0.2) - F(0.1) = 0.05$$

$$Pr(0.8 < T \leq 0.9) = F(0.9) - F(0.8) = 0.05$$

由 pdf plot，因為圖形是一個矩形，可看出這兩個機率值等於 area $= 0.1(\text{difference}) \times 1/2(\text{density})$。

```
# correct
curve(pdf,from = 0,to = 2,xlab = "t",
```

```r
      ylab="f(t)",ylim=c(0,0.6),
      main="Probability Density")
rect(xleft = 0.1, xright = 0.2,
     ybottom = 0, ytop =0.5,
     border = NA, col = adjustcolor("blue", alpha = 0.3))
text(0.15,0.55,"P(0.1 < T <= 0.2)",cex=0.8)
rect(xleft = 0.8, xright = 0.9,
     ybottom = 0, ytop =0.5,
     border = NA, col = adjustcolor("yellow", alpha = 0.3))
text(0.85,0.55,"P(0.8 < T <= 0.9)",cex=0.8)
```



(g)

$$Pr(0.1 < T \leq 0.2 | T > 0.1) = \frac{0.05}{1 - F(0.1)} = 0.05/0.95 = 1/19,$$

comparing to $h(0.1) \times \Delta t = \dfrac{f(0.1)}{1 - F(0.1)} \times 0.1 = \dfrac{0.1}{1.9} = 1/19$,equality exactly.

$$Pr(0.8 < T \leq 0.9 | T > 0.8) = \frac{0.05}{1 - F(0.8)} = 0.05/0.60 = 1/12,$$

comparing to $h(0.8) \times \Delta t = \dfrac{f(0.8)}{1 - F(0.8)} \times 0.1 = \dfrac{0.1}{1.2} = 1/12$,equality exactly.

(h)

From the definition of h(t),

$$h(t) = \lim_{\Delta t \to 0} \frac{P(t < T < t + \Delta t | T > t)}{\Delta t},$$

the h(t) gives the "propensity" that a unit will fail in the next small interval of time,conditional on that is has survived to time t.So,$h(t) \times \Delta t \approx P(t < T < t + \Delta t | T > t)$. The result of part (g) is obtained accroding to this.

Problem 4

$$L(t_0) = E(U) = \frac{1}{1 - F(t_0)} \int_{t_0}^{\infty} [1 - F(z)] dz.$$

(a)

$$L(t)(1 - F(t)) = \int_t^{\infty} [1 - F(z)] dz$$

Differential the above with respect to t,

$$L^{'}(t)(1 - F(t)) - f(t)L(t) = -(1 - F(t)) \Rightarrow \frac{1 + L'(t)}{L(t)} = \frac{f(t)}{1 - F(t)}.$$

$\because H(t) = \int_0^t \frac{f(z)}{1 - F(z)} dz = -\ln [1 - F(t)]$

$\therefore F(t) = 1 - \exp [-\int_0^t \frac{f(z)}{1 - F(z)} dz] = 1 - \exp [-\int_0^t \frac{1 + L'(z)}{L(z)} dz].$

(b)

$$S(t) = 1 - F(t) = \exp [-\int_0^t \frac{1 + L'(t)}{L(t)} dz]$$

$$h(t) = \frac{f(t)}{1 - F(t)} = \frac{1 + L'(t)}{L(t)}$$

$$f(t) = S(t) \times h(t) = \frac{1 + L'(t)}{L(t)} \times \exp [-\int_0^t \frac{1 + L'(t)}{L(t)} dz]$$

Problem 5

```
#install.packages("binom")
library("binom")
r=1000
p=seq(0,1,length=r)
n=50
cp=0
for(i in 1:length(p)){
```

```
  cp[i]=binom.coverage(p[i],n,0.95,method = "lrt")$coverage
}
plot(p,cp,type="l",ylim=c(0.85,1),main="LRT interval (n=50)")
abline(h=0.95,col=2)
```