Operations Research

Final Presentation

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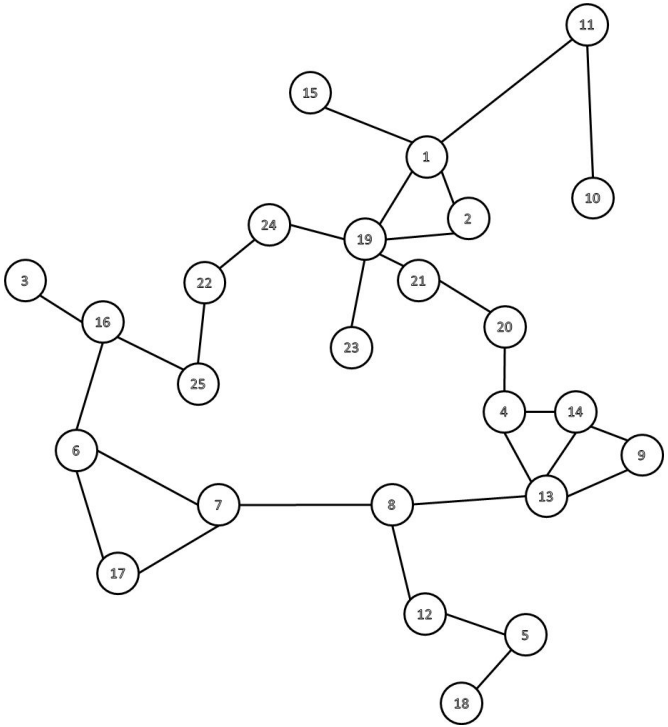
Agenda

- Research Topics
  - Traditional Allocation Problem
  - Allocation with connection : Articulation Point
- Operations Research (How we do it)
  - Data Input & Result Overview
  - Tarjan’s Algorithm and Dilemmas
  - Improving the Model : Optimization Limit
  - Steps
    - Find Articulation Points
    - Iteration & Solution
- Takeaway : Future Thoughts to Generalize Model

Research Topics

- In a city planning or a traffic problem, connection between areas or roads are sometimes more important than just benefits and costs.
- The operations research methods are based on improving current solution to approach to the optimization; besides traditional simplex method or gradient descent, we would like to introduce a **tree-based model** that takes **“Connection”** between cities into consideration.

Traditional Allocation Problem



Consider a City Network :

How to build a logistic center plan?

$$\max \sum_{j \in N} expected\ profit\ -\ costs\ under\ the\ plan$$

$$\text{s.t. Center Number} \leq \delta$$

$$\text{Longest Distance} \leq \beta\ ;\ \forall\ every\ center$$

$$\vdots$$

$$\vdots$$

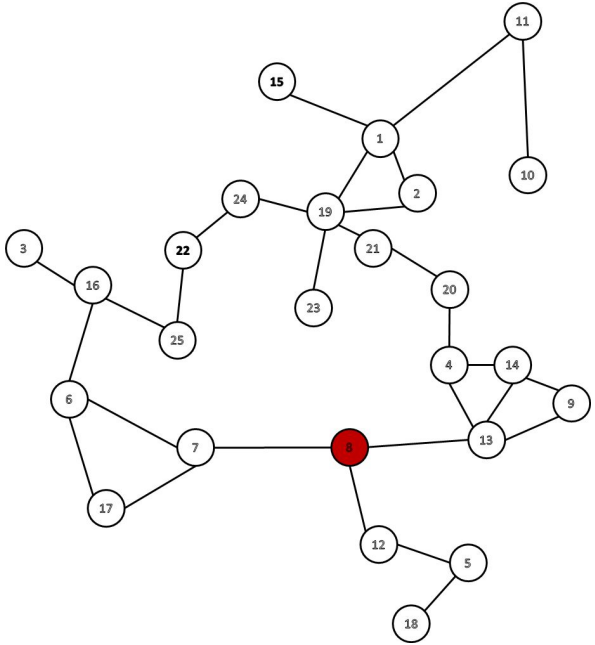
$$\text{Other Constraints ...}$$


Imagine a war is approaching....

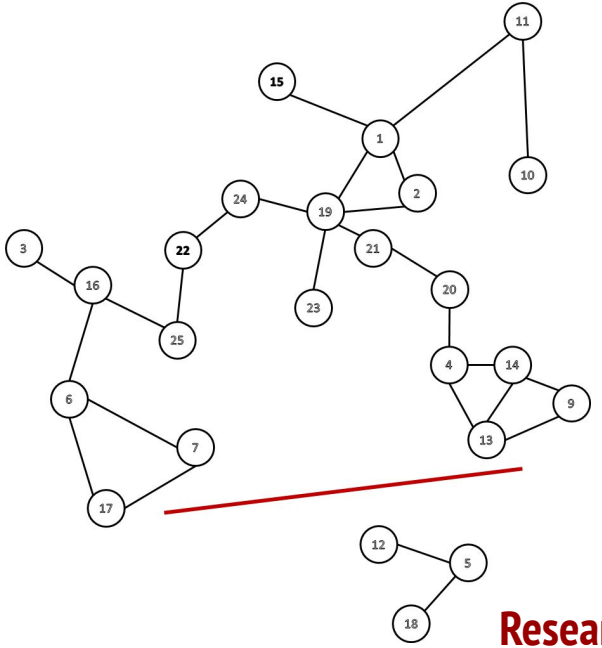
When emergency services in need...

Connections are first to consider

- Circles represent cities.
- Lines represent roads connecting cities.

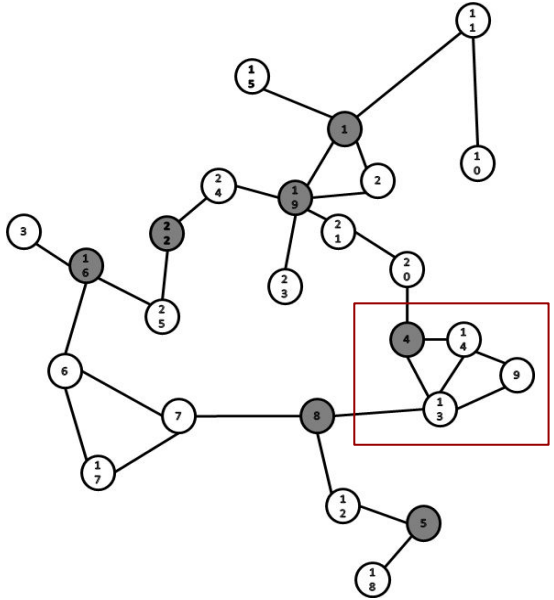


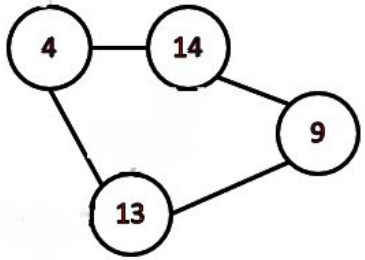
Articulation Point



However we can’t take all cities once because...

- **Capabilities** : We cannot build/explode in as many cities as we want



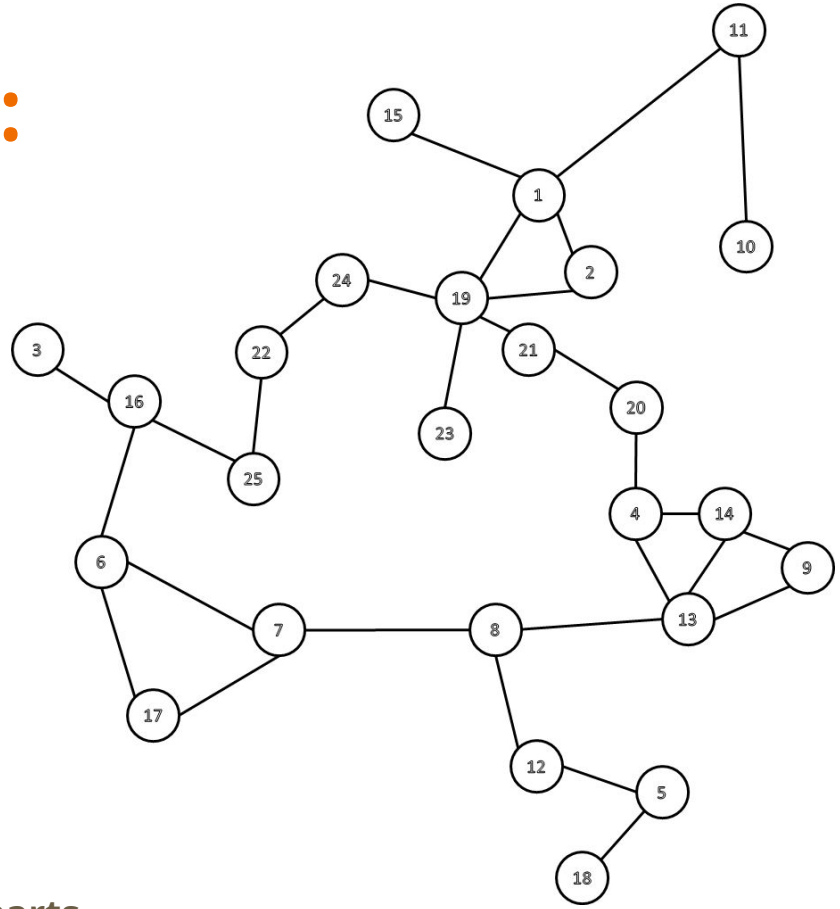


Stop when encoutering cycle

Allocation with Connection : Find Articulation Points

Let's take war for an example...

- Circles represent cities.
- Lines represent roads connecting cities.



1. For War, it blocks transportation between each parts
2. For Emergencies, it make sure the transportation flow unblocked

Research Goal

- Finding the articulation points, each with its own benefits defined by:
  - How many pieces it breaks the original graph?
  - How many total distance it can create?
- This is a **graph theory (a branch of operations research)** with **iterative searching solutions (DFS or BFS)** since whether the point is an articulation point will only be given at that state, we must iterate to find the optimization solution.

Related fields [edit]

Some of the fields that have considerable overlap with Operations Research and Management Science include[32]:

- Business analytics
- Data mining/Data science/Big data
- Decision analysis
- Decision intelligence
- Engineering
- Financial engineering
- Forecasting
- Game theory
- Geography/Geographic information science
- Graph theory
- Industrial engineering
- Logistics
- Mathematical modeling
- Mathematical optimization
- Probability and statistics
- Project management
- Policy analysis
- Simulation

Source: wikipedia

Data Overview

- Total number of cities : **<u>25</u>**
- Adjacency Matrix : **<u>25*25</u>**
- Distance Matrix : **<u>25*25</u>**
- Cost for each explode : λ
- Maximum Explosion : 10

<u>Objective Function :</u>

Pieces Number * Total Distance

- Connections

0	1	0	0	0	0	0	0	0	0	1	0	0	0	1	0
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0
0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1
0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	1	0	0	0	0	1	1	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0
0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
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0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
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0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

Data Overview

- Distance between cities

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
1	0	103	0	0	0	0	0	0	0	0	857	0	0	0	565	0	0	0	266	0	0	0	0	0	0
2	103	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	289	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	195	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0	237	189	0	0	0	0	0	468	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	104	0	0	0	0	0	673	0	0	0	0	0	0	0
6	0	0	0	0	0	0	298	0	0	0	0	0	0	0	0	611	645	0	0	0	0	0	0	0	0
7	0	0	0	0	0	298	0	724	0	0	0	0	0	0	0	0	648	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	724	0	0	0	0	839	567	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	162	85	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	631	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	857	0	0	0	0	0	0	0	0	631	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	104	0	0	839	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	237	0	0	0	567	162	0	0	0	0	121	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	189	0	0	0	0	85	0	0	0	121	0	0	0	0	0	0	0	0	0	0	0	0
15	565	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	0	0	195	0	0	611	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	444
17	0	0	0	0	0	645	648	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18	0	0	0	0	673	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19	266	289	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	277	0	377	175	0
20	0	0	0	468	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	300	0	0	0	0
21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	277	300	0	0	0	0	0
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	305	454
23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	377	0	0	0	0	0	0
24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	175	0	0	305	0	0	0
25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	444	0	0	0	0	0	454	0	0	0

Mathematics Formulation

- Objective Function and Constraints

$$\max \text{sum}_t(Ptj * Dtj) \quad t \in T,\ j \in \text{length}(At)$$

$$\text{s.t.} \quad At = B(S(At-1)) \ ; \ \forall t = 2,\ldots,T$$

$$A1 = B(G)$$

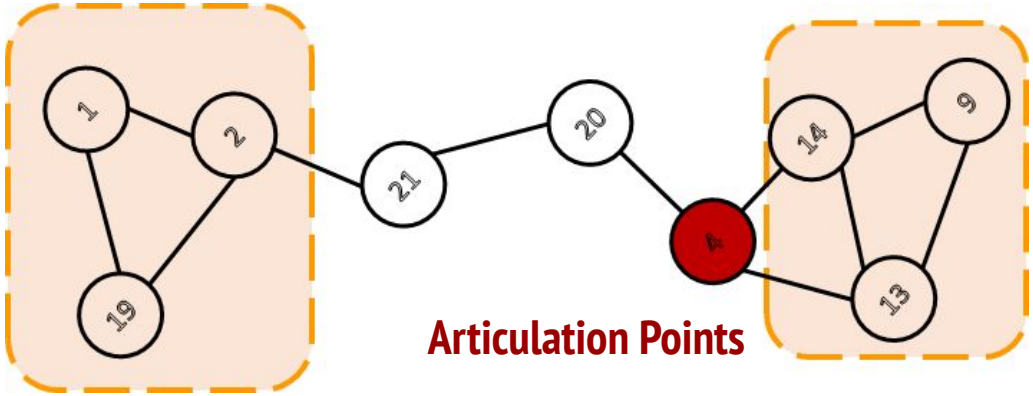
$$0 < t \leq T$$

$$j \in At$$

- t = current level
- At = articulation set at t
- $j \in \{1,\ldots,25\}$, index
- S(A) = a function that generates subplots from the given articulation points
- B(Fragment) = a function that finds all articulation points
- Ptj = number of pieces of articulation j at level t
- Dtj = total distance an articulation j at level t
- G = original graph
- T = maximum iterations

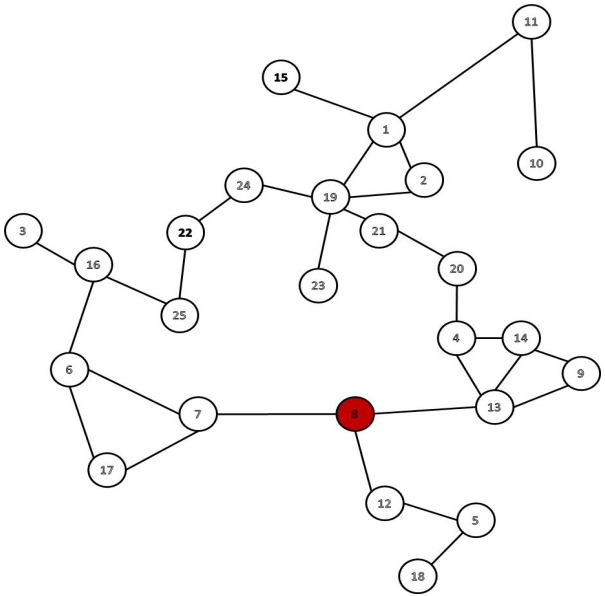
Articulation Points & Biconnected Components

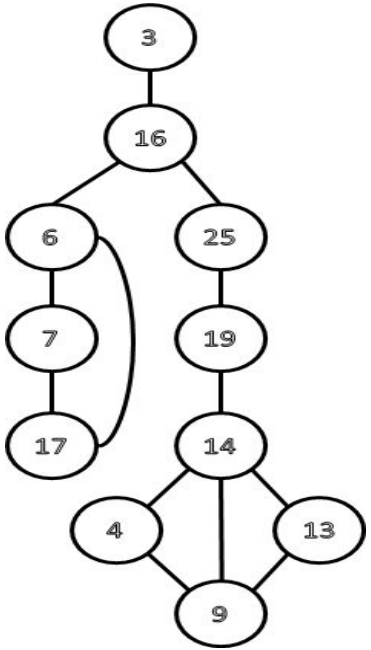
- Articulation Point disconnects an undirected graph
- Biconnected Component does not generate articulation points



How to find articulation points : Tarjan's Algorithm

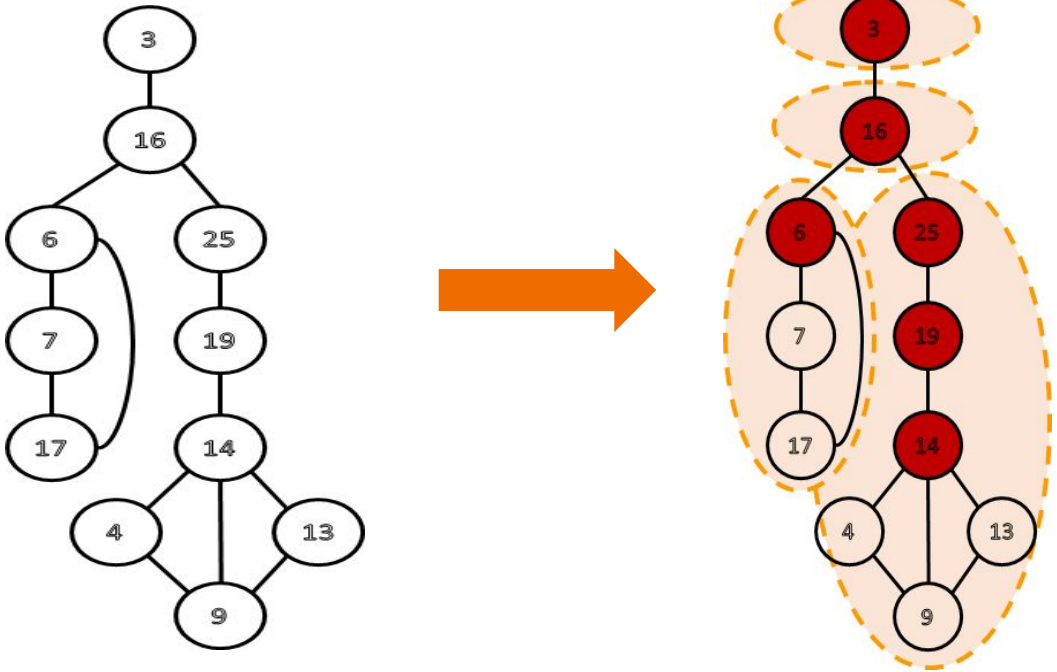
- The algorithm is to find all **Biconnected Components** and thus we have all the **Articulation Points**





How to find articulation points : Tarjan's Algorithm

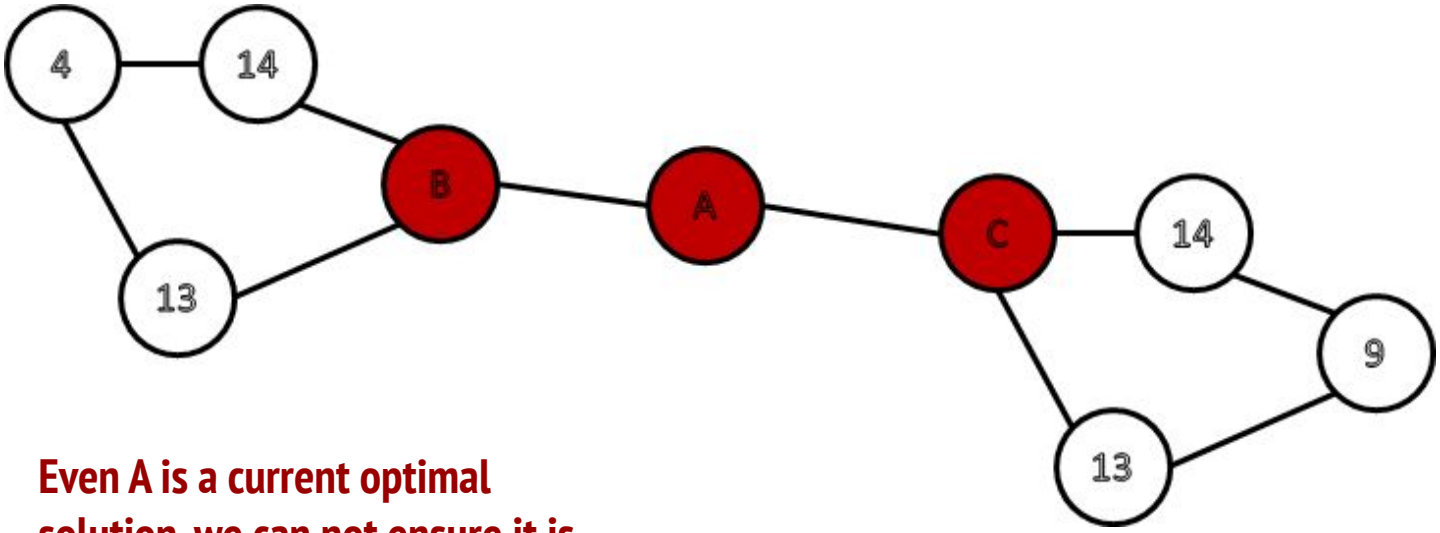
- Step 2 : DFS each point and find highest ancestor for each point



If the ancestor of a point is itself, it's a biconnected component

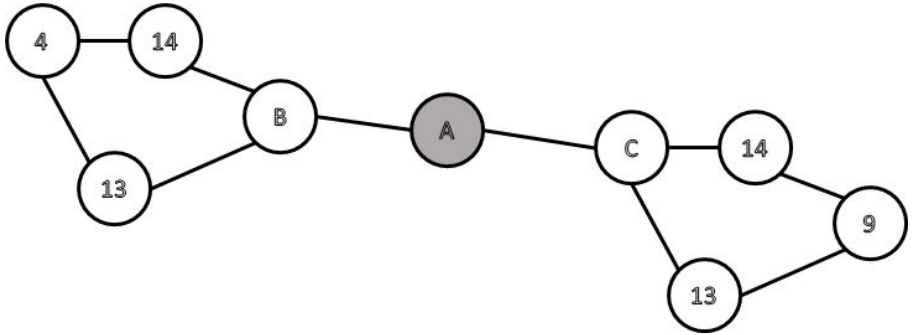
The overlapping part of each BCC is an articulation point

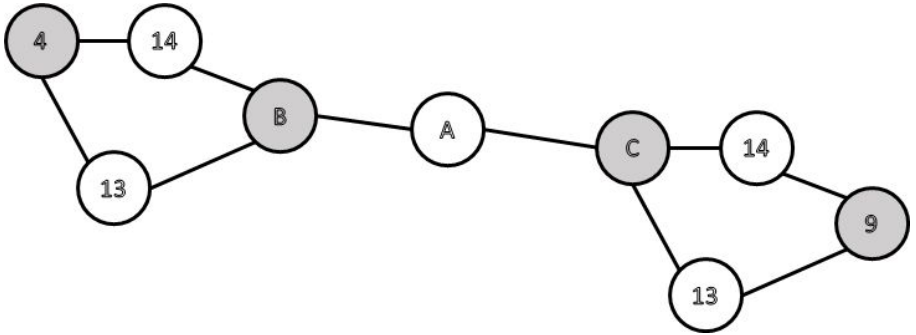
Algorithm Dilemma : Not optimization solution



➡ Even A is a current optimal solution, we can not ensure it is also a global solution.

Algorithm Dilemma : Not optimization solution

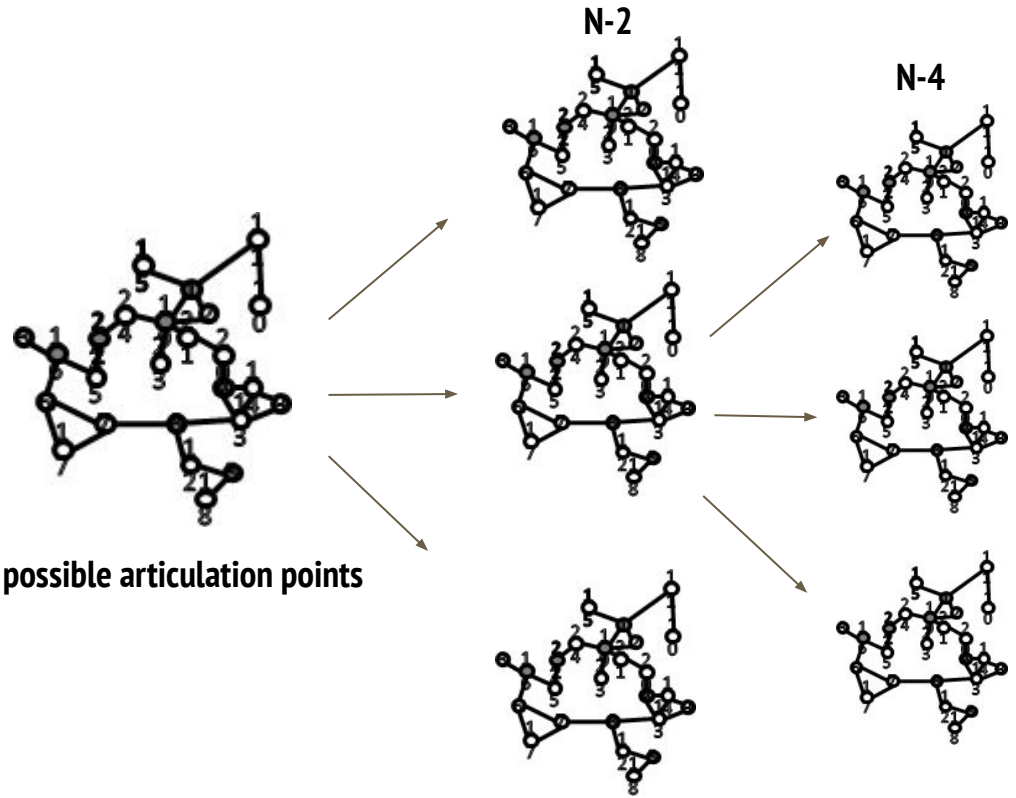




No one knows which division is better considered pieces and distance

➡ Iteration Solutions

Another Dilemma : Is it solvable?

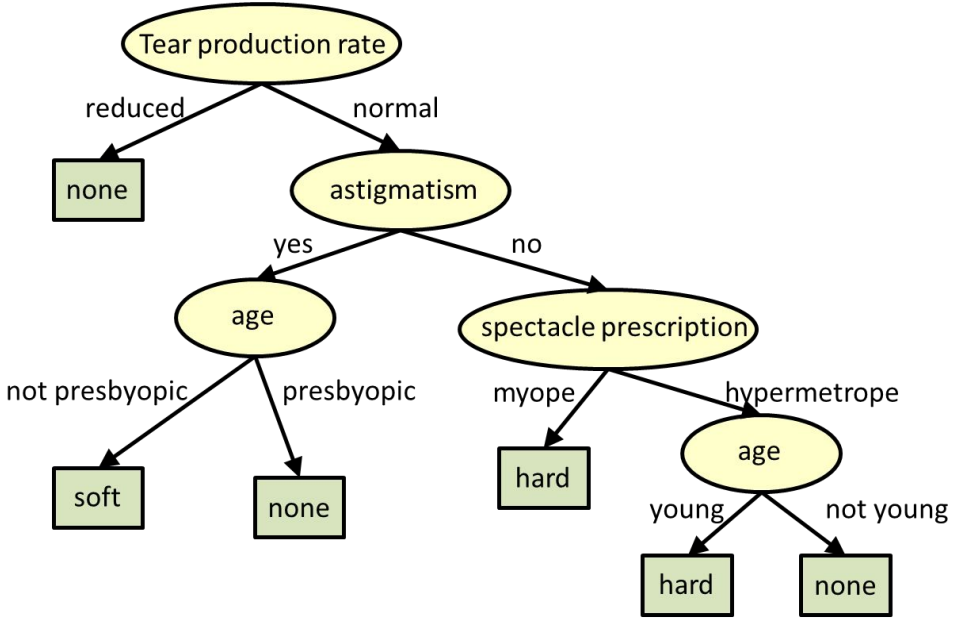


Time Complexity?

$$2N * (2n - 2) * \cdots 2 = O(2^n n!)$$

... TSP only O(n!)

Improving Model : Tree-model Adjustment



Solving This is Like a Decision Tree

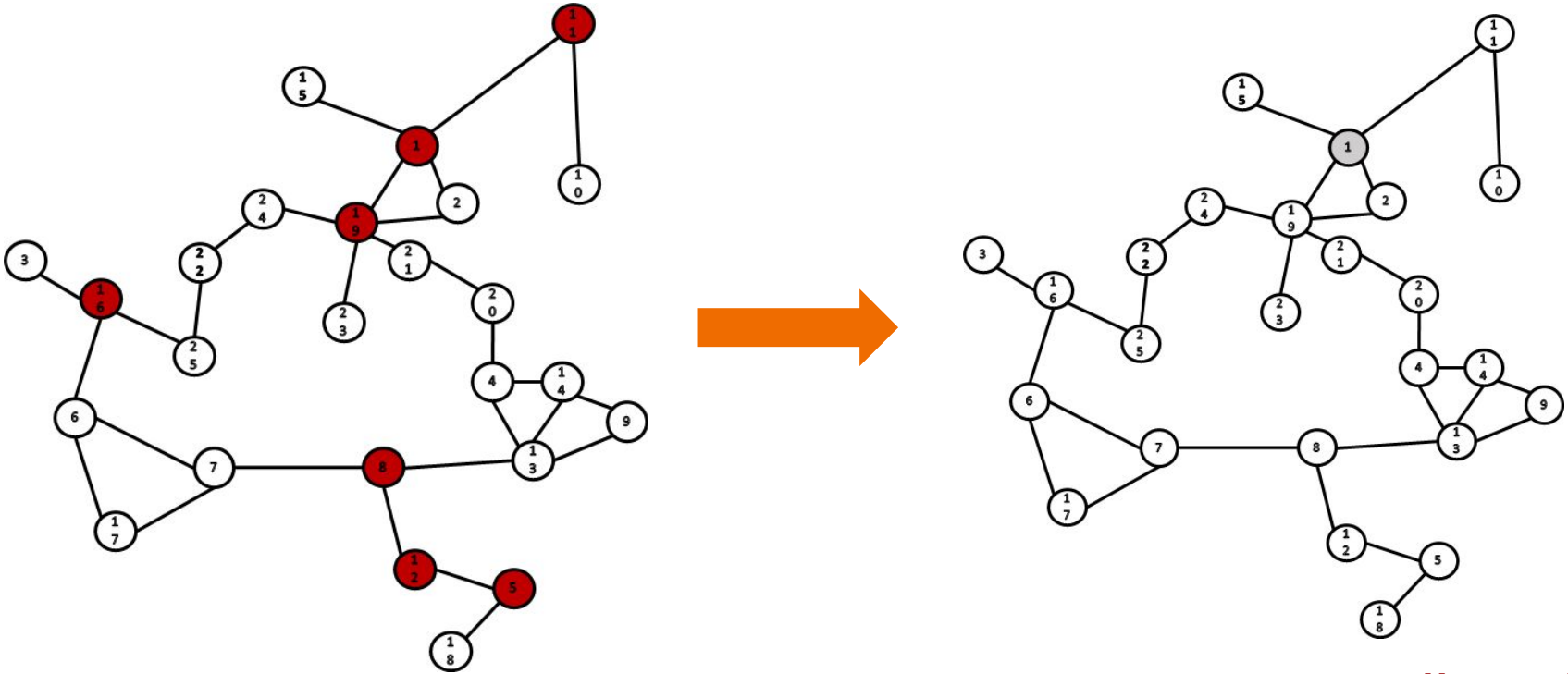
- **Max_Depth** : Stop if tree depth larger than this number (we can launch infinite missiles)
- **Min_Leaf_Split** : Do not accept the solution if profit less than this number (cost of each launch)
- **Tolerance** : If profit increase less than this number, stop generating new solution (cost of each launch)

Solvable when data is small but not satisfying

Steps : Virtual Code

```
void CityPlanning (adj matrix : graph, distance) {
    /* Articulation Set = [Articulation Point, Sub_graph, Layer] */
    Articulation_Set = Get_articulation_points(graph);
    while Articulation_Set not empty {  /* BFS Find Solution */
        sub_graph = Articulation Set.top()[1];
        solution_set = Get_articulation_points(sub_graph);
        Articulation_Set.pop() ; Articulation Set.push(solution set)
    }
}
```

Steps : Visualization



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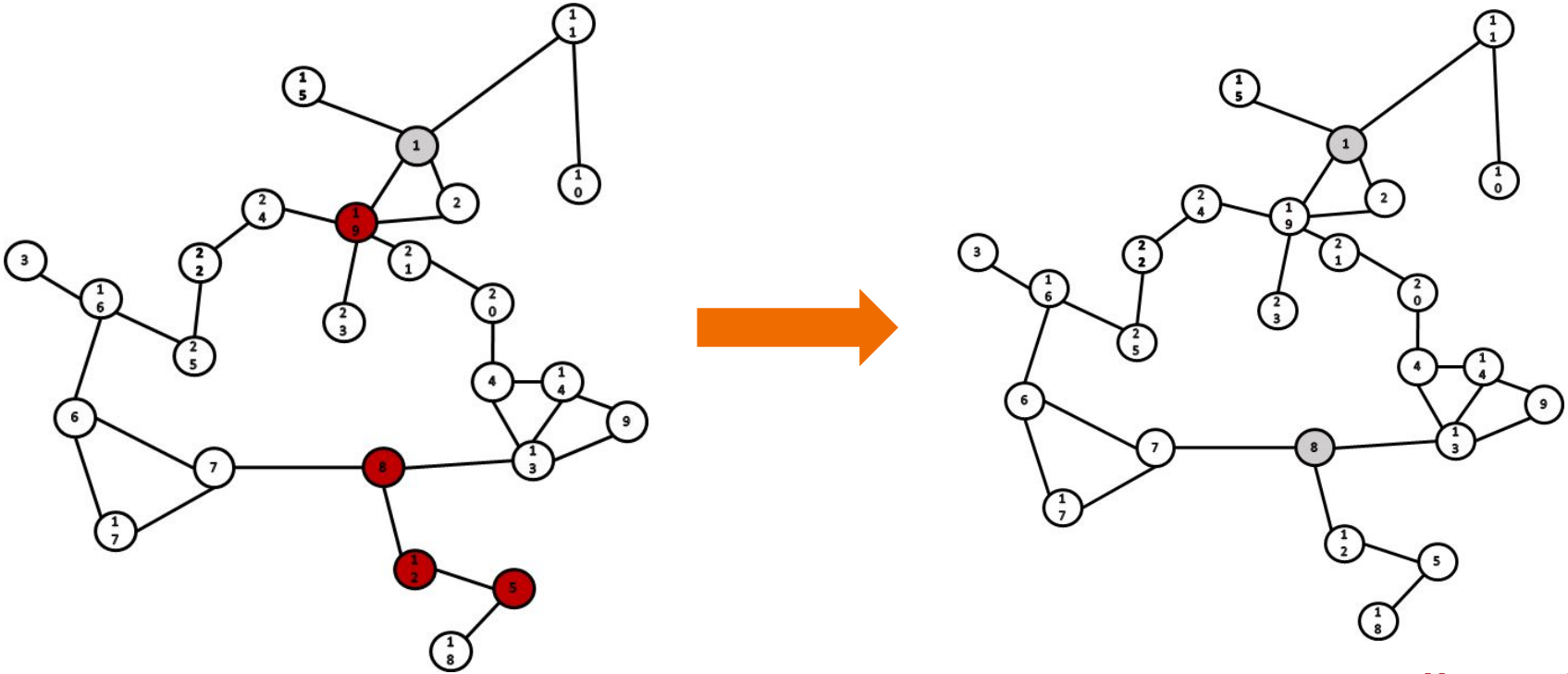
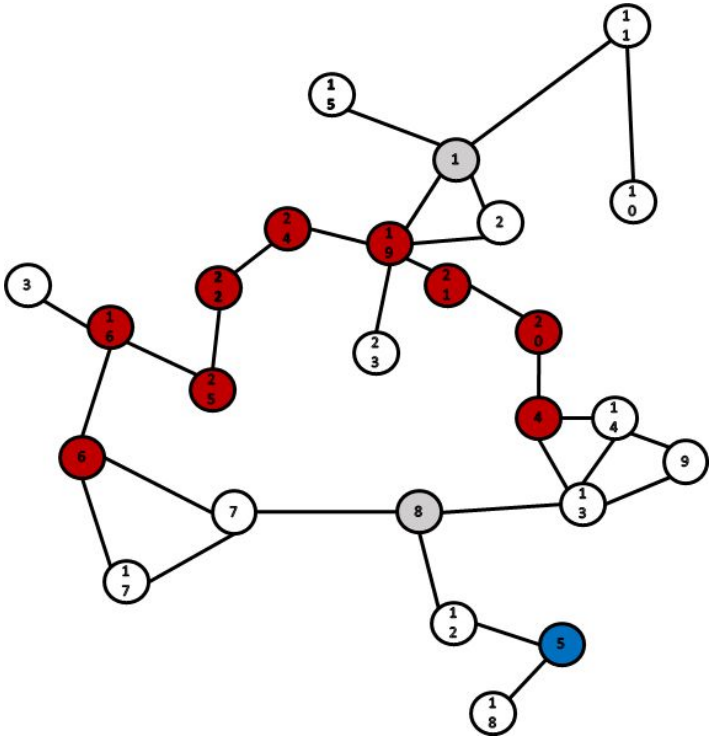
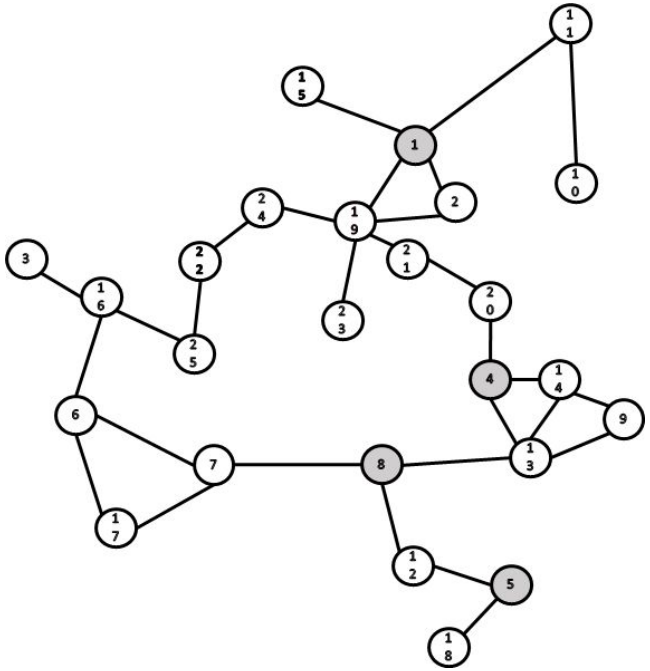


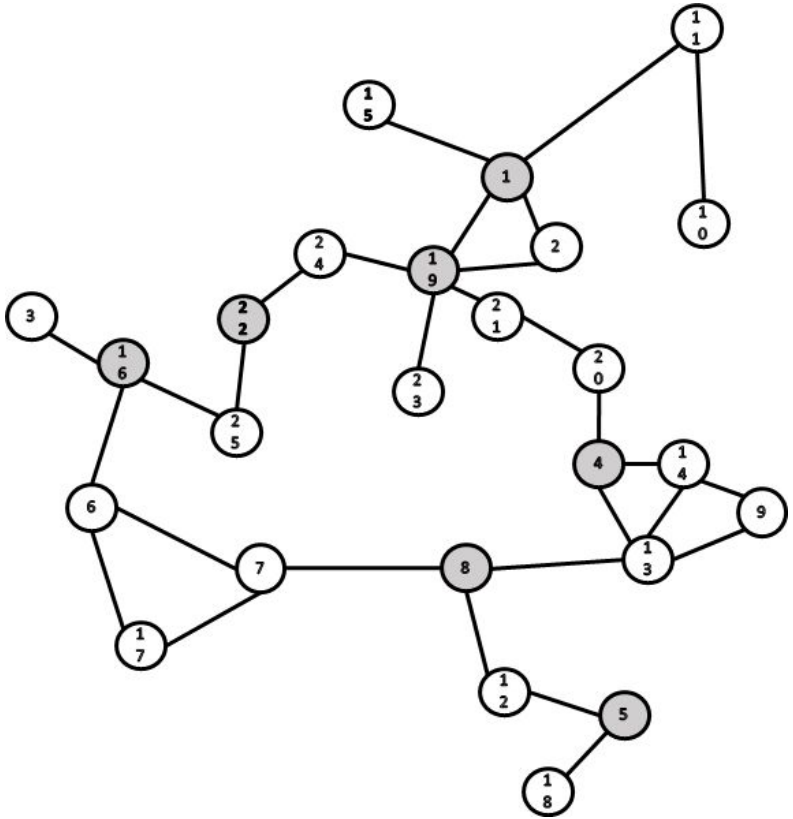
Steps : Visualization





Solution Interpretation

- Order : [1, 8, 19, 5, 4, 14, 20]
- Total Profit : 32714
- Interpretation : We follow the order to explode each city, and we do not run out all of our missiles because launching eighth missiles do not benefit in our planning.



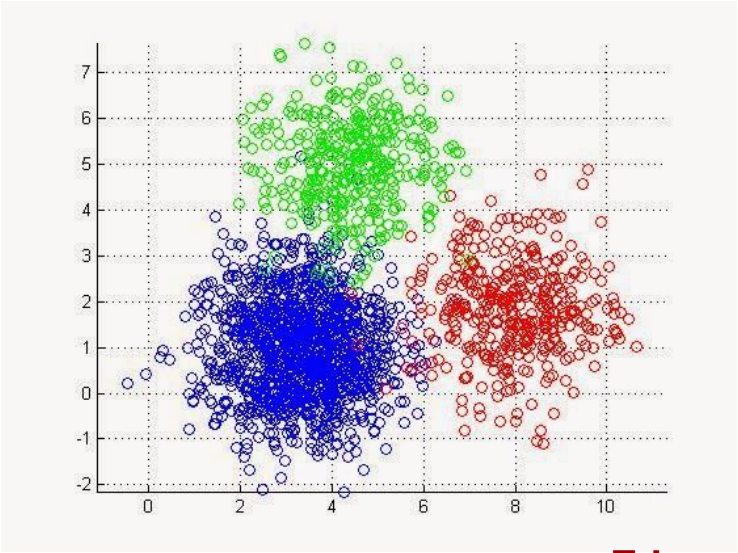
Takeaway : Future Improvement

- The model is not generalized enough, especially for big data
- Idea of the planning is like K-Means, which have :

$$l(G_1, \dots, G_n) = \sum_{j=1}^{k} \sum_{i \in G_j} \|x_i - \mu_j\|_2^2$$

$$\bigcup_{j=1}^{k} G_j = \{1, 2, \dots, n\}; G_j \cap G_j' = \phi$$

$$\mu_j = \frac{1}{|G_j|} \sum_{i \in G_j} x_i$$



Takeaway : Apply Community Detection

- A Community is a subgraph that contains nodes more **densely linked**

$$Q = \sum_{c \in \mathcal{C}} \left(\frac{I_c}{m} - \left(\frac{D_c}{2m} \right)^2 \right)$$

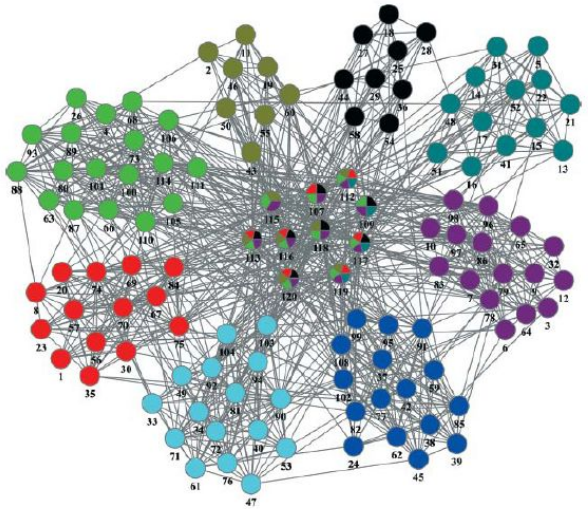
m = number of edges

I_c = number of edges in community c

D_c = sum of degree of all vertex in community

By BMLPA Algorithm, it could be done in O(nlogn)

overlapping communities

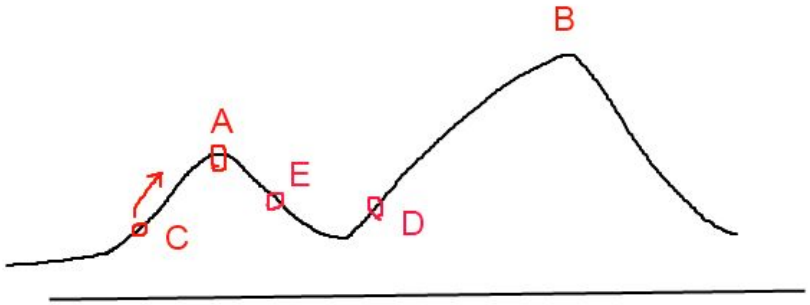


Takeaway : Simulated Annealing (Other Searching)

- SA algorithm can somewhat improve the TSP dilemma.

Accept new state by a specific probability (not definitely optimized solution)

$$
\begin{aligned}
&if\ \Delta E > 0: \\
&\quad Accept\ new\ state \\
&else: \\
&\quad Accept\ by\ probability\ e^{\frac{\Delta E}{T}}
\end{aligned}
$$



More possible searching methods

Thank You!

Final Presentation

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