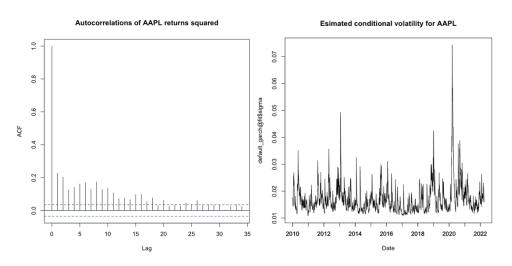
GARCH model estimation and simulation for AAPL

This report is to apply GARCH model for Apple Inc. (AAPL) from Jan 1, 2010 to Mar 31, 2022. Apple is a major internet technology company which develops widely used smartphones. All data in this report is from CRSP. The prices in this report were adjusted. The sample size T is 3083 days and the number of simulations S is 1000. The chosen GARCH model is GARCH(1,1) with the assumption that the mean of the returns is 0. Since the autocorrelation of the returns squared are significant for most lags as shown below, GARCH can be applied on this asset. The estimated conditional volatility is shown below. Through the graph it can be seen that the peak of volatility is at the beginning of 2020, which is around the start of Covid-19 pandemic.



The parameters of the GARCH model is: α =0.102, β =0.839, ω =1.83×10⁻⁵. Also, a tGARCH model is applied to measure the difference and the value of the parameters (especially ω) are holding different values. According to the result of likelihood ratio test, two models perform differently.

By using Monte-Carlo method, after 1000 times of simulation, the predicted returns and volatilities in 1000 days are illustrated. Using the portfolio value 1000 and the probability 5%, the VaR^{5%} and 5% ES of the simulation are shown below and the red line denotes the value of VaR^{5%} and corresponding ES in the past T days. After feeding the result of simulation back to the model, the parameters (ω , α and β) converge to the original parameters. It is specific that the simulated VaR and ES do not converge to the real values in the past, which is 26.32 and 40.57. The simulated VaR(28.78) is higher and ES(37.27) is lower. The 95% empirical confidence bounds of VaR and ES are [28.65, 28.91] and [37.08, 37.46] respectively. Through the GARCH(1,1) model, it can be predicted that in the following approximately 4 years, AAPL will have a higher VaR^{5%} and lower 5% ES, which means in the worst 5% situations, the lost may be higher than before.

