In [3]:
```
downsampled_image = np.zeros((64, 64), dtype=np.uint8)


for i in range(0, 512, 8):
    for j in range(0, 512, 8):
        downsampled_image[i//8, j//8] = binarized_image[i,j]
```

將照片二值化後，將圖片從 512*512 改為 64*64，取左上角

In [5]:
```
top_bottom_zeros = np.zeros((downsampled_image.shape[0] + 2, downsampled_image.shape[1] + 2), dtype=downsampled_image.dtype)
top_bottom_zeros[1:-1, 1:-1] = downsampled_image
```

保留上下左右各一行避免超出邊界

In [6]:
```
def h(b, c, d, e):
    if(b == c and (d != b or e != b)):
        return 'q'
    elif(b == c and (d == b and e == b)):
        return 'r'
    elif(b!=c):
        return 's'
```

In [7]:
```
def f(a1,a2,a3,a4):
    if a1=='r' and a2=='r'and a3=='r'and a4=='r':
        return 5
    tmp=0
    if a1=='q':
        tmp+=1
    if a2=='q':
        tmp+=1
    if a3=='q':
        tmp+=1
    if a4=='q':
        tmp+=1
    return tmp
```

In [8]:
```
def yokoi(top_bottom_zeros):
    yokoi_image = np.zeros((66, 66), dtype=int)
    for i in range(1, 65):
        for j in range(1, 65):
            if top_bottom_zeros[i, j] == 1:
                a1 = h(top_bottom_zeros[i, j], top_bottom_zeros[i, j+1], top_bottom_zeros[i-1, j+1], top_bottom_zeros[i-1, j])
                a2 = h(top_bottom_zeros[i, j], top_bottom_zeros[i-1, j], top_bottom_zeros[i-1, j-1], top_bottom_zeros[i, j-1])
                a3 = h(top_bottom_zeros[i, j], top_bottom_zeros[i, j-1], top_bottom_zeros[i+1, j-1], top_bottom_zeros[i+1, j])
                a4 = h(top_bottom_zeros[i, j], top_bottom_zeros[i+1, j], top_bottom_zeros[i+1, j+1], top_bottom_zeros[i, j+1])

                yokoi_image[i, j] = f(a1, a2, a3, a4)
    return yokoi_image
```

使用上週實作的 yokoi function

In [9]:
```
def Pair_Relationship(yokoi_image):
    Relationship = [['q' for _ in range(66)] for _ in range(66)]
    res=[[0]*(66) for i in range(66)]
    for i in range(1, 65):
        for j in range(1, 65):
            if yokoi_image[i][j] == 1:
                if (yokoi_image[i+1][j] == 1) or (yokoi_image[i-1][j] == 1) or (yokoi_image[i][j+1] == 1) or (yokoi_image[i][j-1]
                    Relationship[i][j] = 'p'
#                 else:
#                     Relationship[i][j] = 'q'
#             else:
#                     Relationship[i][j] = 'q'
    return Relationship
```

實作 ppt 中的 pair_relation

Pair Relationship Operator

- H function: (m="1", means "edge" in Yokoi)
  - $h(a,m) = \begin{cases} 1, if\ a = m \\ 0, otherwise \end{cases}$
- Output:
  - $y = \begin{cases} q, if\ \sum_{n=1}^{4} h(x_n, m) < 1\ or\ x_0 \neq m \\ p, if\ \sum_{n=1}^{4} h(x_n, m) \geq 1\ and\ x_0 = m \end{cases}$

In [10]:
```
def h2(b,c,d,e):
    if b==c and (b!=d or b!=e):
        return 1
    else:
        return 0
def f2(a1,a2,a3,a4,x):
    if a1+a2+a3+a4==1:
        return 0
    else:
        return x
```

In [11]:
```
def Shrink(top_bottom_zeros,Relationship):
    new_image = top_bottom_zeros.copy()
    for i in range(1, 65):
        for j in range(1, 65):
            if  Relationship[i][j] == 'p':
                a1 = h2(top_bottom_zeros[i,j], top_bottom_zeros[i,j+1], top_bottom_zeros[i-1,j+1], top_bottom_zeros[i-1,j])
                a2 = h2(top_bottom_zeros[i,j], top_bottom_zeros[i-1,j], top_bottom_zeros[i-1,j-1], top_bottom_zeros[i,j-1])
                a3 = h2(top_bottom_zeros[i,j], top_bottom_zeros[i,j-1], top_bottom_zeros[i+1,j-1], top_bottom_zeros[i+1,j])
                a4 = h2(top_bottom_zeros[i,j], top_bottom_zeros[i+1,j], top_bottom_zeros[i+1,j+1], top_bottom_zeros[i,j+1])

                new_image[i,j] = f2(a1,a2,a3,a4,top_bottom_zeros[i,j])
    return new_image
```

實作 ppt 中 shrink 的部分

Connected Shrink Operator

- H function: (yokoi corner => “q”)
  - $h(b,c,d,e) = \begin{cases} 1, if\ b = c\ and\ (d \neq b\ or\ e \neq b) \\ 0, otherwise \end{cases}$
- Output:
  - $f(a_1,a_2,a_3,a_4,x) = \begin{cases} g, if\ exactly\ one\ of\ a_n = 1, n = 1\sim4 \\ x, otherwise \end{cases}$

In [12]:
```
yokoi_image = np.zeros((66, 66), dtype=int)
Relationship = [['' for _ in range(66)] for _ in range(66)]
new_image = np.zeros((66, 66), dtype=int)

yokoi_image = yokoi(top_bottom_zeros)
Relationship = Pair_Relationship(yokoi_image)
new_image = Shrink(top_bottom_zeros,Relationship)

save_image = (new_image * 255).astype(np.uint8)  # 將二值圖轉為 0 和 255
filename = f"iteration_1.bmp"  # 命名圖片文件
cv2.imwrite(filename, save_image)  # 保存圖片
print(f"Saved: {filename}")
```

執行確定結果

In [15]:
```
iteration = 2
while not np.array_equal(new_image, top_bottom_zeros):
    top_bottom_zeros = new_image.copy()
    yokoi_image = yokoi(top_bottom_zeros)
    Relationship = Pair_Relationship(yokoi_image)
    new_image = Shrink(top_bottom_zeros,Relationship)
    print('1')
    save_image = (new_image * 255).astype(np.uint8)  # 將二值圖轉為 0 和 255
    filename = f"iteration_{iteration}.bmp"  # 命名圖片文件
    cv2.imwrite(filename, save_image)  # 保存圖片
    print(f"Saved: {filename}")

    iteration += 1
```

反覆執行直到不再改變